# Ring core fibre as an optical amplifier 

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#### Abstract

The ring shape of doped rare earth core fibre (M-profile fibre) offers the possibility of stimulating luminescence from inside of the fibre with higher efficiency of coupling. In this type of fibre, the inner and outer claddings have lower than the ring core refraction indexes. The pump light is guided by the multimode inner cladding and the laser light is guided by the doped rare earth ions ring -core. The pump absorption is almost complete because of the geometry of that kind of fibre construction. In this paper, a ring core doped $\mathrm{Dy}^{3+}$ ions fibre is presented. In the first part of this report, an analysis of light transfer between inner cladding and ring-core in ring-core optical fibre is presented. In the second part, the parameters of $\mathrm{Dy}^{3+}$ ions doped ring core optical fibre manufactured by author are described. It seems that this kind of fibre may be used for optical amplifier construction.


Keywords: ring-core optical fibre, rare earth ions, amplifier.

## 1. Analysis of the ring core fibre material and geometrical parameters on radiation distribution inside the fibre

In the ring-core optical fibre the pump light is guided by the multimode inner cladding [1], [2]. For this reason an analysis of energy distribution inside the fibre is necessary. The laser light is guided by the doped rare earth ions ring-core.

The objective of the present study is a ring shaped core optical fibre (Fig. 1). The following assumptions have been made:

1. The geometry of the elements of a ring core light fibre is known: $r_{1}$ - inner cladding radius, $r_{2}$ - ring-core outer radius, $r_{3}$ - outer radius of the fibre, $l$ - length of the fibre.
2. The particular layers of the fibre are made of materials of constant and known refractive index: $n_{1}$ - refractive index of the inner cladding material, $n_{2}$ - refractive index of the ring-core, $n_{3}$ - refractive index of the outer cladding. The dependence $n_{1}<n_{2}>n_{3}$ is accepted.
3. The front surface of the inner cladding is illuminated from the direction inclined at the angle $\alpha$ with respect to the optical fibre axis. The intensity of lighting $E_{0}$ of the light source is known.


Fig. 1. Cross-section of a ring core optical fibre.
4. The elementary luminous flux is carried by a vector. Elements of all surfaces cooperating with this vector are flat.
5. The symmetry axis of the light fibre coincides with the axis of rectangular coordinate system. The light source is displaced in the plane $x, z$ from the adopted coordinate system.

The angle $\alpha$ is a parameter constant for the entire front surface of the fibre. It can be changed arbitrarily from $0^{\circ}$ to $90^{\circ}$ (Fig. 2). On the front surface of the fibre the lighting intensity amounts to $E_{p}=E_{o} \cos \alpha$ (where: $E_{o}$ - normal intensity of lighting, $\alpha$ - angle of incidence of the light beam).

An elementary light ray incident on an element of the light fibre front and represented by the vector $\mathbf{P}$ is equal to:

$$
\begin{equation*}
|\mathbf{P}|=E_{o} \Delta x \Delta y \cos \alpha \tag{1}
\end{equation*}
$$

where $\Delta x, \Delta y$-dimensions of the sides of a rectangular elementary surface.
If the diameter of inner cladding $\left(2 r_{1}\right)$ is divided into $n_{R}$ elements, the dimensions $\Delta x$ and $\Delta y$ can be assumed as equal to:

$$
\begin{equation*}
\Delta x=\Delta y=\frac{2 r_{1}}{n_{R}} \tag{2}
\end{equation*}
$$

The coordinates of the point $P$ can be determined from the dependence:

$$
\left.\begin{array}{l}
x_{P}=i \Delta x+\frac{\Delta x}{2}-r_{1} \\
y_{P}=j \Delta y+\frac{\Delta y}{2}-r_{1} \tag{3}
\end{array}\right\}
$$

where: $i=0 \ldots n_{R}$ - successive calculation points in the direction of the $x$ axis, $j=0 \ldots n_{R}-$ successive calculation points in the direction of the $y$ axis.


Fig. 2. Course of an elementary luminous flux in a ring-core optical fibre.
If $x_{P}^{2}+y_{P}^{2}>r_{1}^{2}$ then the point $P$ under consideration lies outside the front of the inner cladding surface and in the calculations this elementary luminous flux is not analysed any longer.

The knowledge of the angle of incidence $\alpha$ allows us to determine the angle of refraction $\beta$, lying also in the plane parallel to $x, y$ from the equation

$$
\begin{equation*}
\beta=\operatorname{atan} \frac{\sin \alpha}{\sqrt{n_{1}^{2}-\sin ^{2} \alpha}} \tag{4}
\end{equation*}
$$

Knowing the angles $\alpha$ and $\beta$ we can determine Fresnel's coefficient of reflection $\rho_{P}$ at the point $P$ from the expression

$$
\begin{equation*}
\rho_{P}=\frac{1}{2}\left[\frac{\sin ^{2}(\alpha-\beta)}{\sin ^{2}(\alpha+\beta)}+\frac{\tan ^{2}(\alpha-\beta)}{\tan ^{2}(\alpha+\beta)}\right] \tag{5}
\end{equation*}
$$

and in the case where $\alpha=0$ (the flux runs parallel to the symmetry axis of the fibre), from:

$$
\begin{equation*}
\rho_{P}=\left(\frac{n_{1}-1}{n_{1}+1}\right)^{2} \tag{6}
\end{equation*}
$$

The luminous flux $\mathbf{R}$ reflected from the front surface is equal to:

$$
\begin{equation*}
|\mathbf{R}|=\rho_{P}|\mathbf{P}| . \tag{7}
\end{equation*}
$$

The remaining part of the elementary luminous flux $\left|\mathbf{S}_{\mathbf{1}}\right|$ enters the inner cladding of the ring-core light fibre

$$
\begin{equation*}
\left|\mathbf{S}_{\mathbf{1}}\right|=\left(1-\rho_{P}\right)|\mathbf{P}| \tag{8}
\end{equation*}
$$

where the vector $\mathbf{S}_{\mathbf{1}}$ has the components: $S_{1 x}=\left|\mathbf{S}_{\mathbf{1}}\right| \sin \beta, S_{1 y}=0, S_{1 z}=\left|\mathbf{S}_{\mathbf{1}}\right| \cos \beta$.

In order to determine the point $S_{1}$ at which the vector $\mathbf{S}_{\mathbf{1}}$ hits the side surface of the inner cladding, we must solve the system of equations when $y_{S 1}=y_{P}$ :
i) equation for a straight line passing through the point $P$ and parallel to the vector $\mathbf{S}_{\mathbf{1}}$

$$
\begin{equation*}
\frac{x_{S 1}-x_{P}}{S_{1 x}}=\frac{z_{S 1}}{S_{1 z}} \tag{9}
\end{equation*}
$$

ii) equation of the cylindrical core surface

$$
\begin{equation*}
x_{S 1}^{2}+y_{S 1}^{2}=r_{1}^{2} \tag{10}
\end{equation*}
$$

Hence:

$$
\begin{align*}
& x_{S 1}=\sqrt{r_{1}^{2}-y_{P}^{2}}  \tag{11}\\
& z_{S 1}=\frac{S_{1 z}}{S_{1 x}}\left(\sqrt{r_{1}^{2}-y_{P}^{2}}-x_{P}\right) .
\end{align*}
$$

If $z_{S 1}>l(l-$ length of the light pipe $)$, the light ray meets the exit surface of the inner cladding at the point $Q_{1}$ and the coordinates of this point are equal to:

$$
\left.\begin{array}{l}
x_{Q 1}=\frac{S_{1 x}}{S_{1 z}} l+x_{P} \\
y_{Q 1}=\frac{S_{1 y}}{S_{1 z}} l+y_{P}  \tag{12}\\
z_{Q 1}=l
\end{array}\right\}
$$

If $z_{S 1}<l$, the other coordinates of the point $S_{1}$ can be determined from the dependence:

$$
\left.\begin{array}{l}
x_{S 1}=\frac{S_{1 x}}{S_{1 z}} z_{S 1}+x_{P} \\
y_{S 1}=\frac{S_{1 y}}{S_{1 z}} z_{S 1}+y_{P} \tag{13}
\end{array}\right\}
$$

At the point $S_{1}$ the unit vector $\mathbf{N}_{\mathbf{S 1}}$, normal to the side surface of the inner cladding is equal to:

$$
\mathbf{N}_{\mathbf{S} 1}\left(N_{S 1 x}, N_{S 1 y}, N_{S 1 z}\right)=\mathbf{N}_{\mathbf{S} 1}\left(-\frac{x_{S 1}}{r_{1}},-\frac{y_{S 1}}{r_{1}}, 0\right)
$$

The angle of incidence $\delta_{1}$ of the vector $\mathbf{S}_{\mathbf{1}}$ on the side surface of the inner cladding is determined from:

$$
\begin{equation*}
\cos \delta_{1}=\cos \left(-\mathbf{S}_{\mathbf{1}} \cdot \mathbf{N}_{\mathbf{S} \mathbf{1}}\right)=\frac{-\mathbf{S}_{\mathbf{1}} \cdot \mathbf{N}_{\mathbf{S} \mathbf{1}}}{\left|\mathbf{S}_{\mathbf{1}}\right|}=\frac{-S_{1 x} N_{S 1 x}-S_{1 y} N_{S 1 y}}{\left|\mathbf{S}_{\mathbf{1}}\right|} \tag{14}
\end{equation*}
$$

Now, we can calculate the angle of refraction $\varepsilon_{1}$ of the luminous flux $\mathbf{T}_{1}$ running in the ring-core of the light fibre being analysed

$$
n_{2} \sin \varepsilon_{1}=n_{1} \sin \delta_{1}
$$

Hence:

$$
\begin{equation*}
\varepsilon_{1}=\operatorname{acos} \sqrt{1-\frac{n_{1}^{2}}{n_{2}^{2}}\left(1-\cos ^{2} \delta_{1}\right)} . \tag{15}
\end{equation*}
$$

Knowing the angles $\delta_{1}$ and $\varepsilon_{1}$ we can determine the coefficient of Fresnel's reflections $\rho_{S 1}$ at the point $S_{1}$ :

$$
\begin{equation*}
\rho_{S 1}=\frac{1}{2}\left[\frac{\sin ^{2}\left(\delta_{1}-\varepsilon_{1}\right)}{\sin ^{2}\left(\delta_{1}+\varepsilon_{1}\right)}+\frac{\tan ^{2}\left(\delta_{1}-\varepsilon_{1}\right)}{\tan ^{2}\left(\delta_{1}+\varepsilon_{1}\right)}\right] \tag{16}
\end{equation*}
$$

At the point $S_{1}$ some part of the luminous flux undergoes refraction $\mathbf{T}_{\mathbf{1}}$

$$
\begin{equation*}
\left|\mathbf{T}_{\mathbf{1}}\right|=\left(1-\rho_{S 1}\right)\left|\mathbf{S}_{\mathbf{1}}\right|=\left(1-\rho_{S 1}\right)\left(1-\rho_{P}\right)|\mathbf{P}| \tag{17}
\end{equation*}
$$

and the remaining part $\mathbf{V}_{\mathbf{1}}$ is equal to:

$$
\begin{equation*}
\left|\mathbf{V}_{\mathbf{1}}\right|=\rho_{S 1}\left|\mathbf{S}_{\mathbf{1}}\right|=\left(1-\rho_{P}\right) \rho_{S 1}|\mathbf{P}| \tag{18}
\end{equation*}
$$

The vector $\mathbf{V}_{\mathbf{1}}$ reflected at the point $S_{1}$ is determined from the relation

$$
\begin{equation*}
\mathbf{V}_{\mathbf{1}}=\left[\mathbf{S}_{\mathbf{1}}+2\left|\mathbf{S}_{\mathbf{1}}\right| N_{S 1} \cos \delta_{1}\right] \rho_{S 1} \tag{19}
\end{equation*}
$$

and its components are equal to:

$$
\left.\begin{array}{l}
V_{1 x}=S_{1 x}-2\left|\mathbf{S}_{\mathbf{1}}\right| \cos \delta_{1} \frac{x_{S 1}}{r_{1}} \\
V_{1 y}=S_{1 y}-2\left|\mathbf{S}_{\mathbf{1}}\right| \cos \delta_{1} \frac{x_{S 1}}{r_{1}}  \tag{20}\\
V_{1 z}=S_{1 z}
\end{array}\right\}
$$

Equation for the vector $\mathbf{T}_{\mathbf{1}}$ :

$$
\mathbf{T}_{\mathbf{1}}=\left(\frac{n_{1}}{n_{2}} \mathbf{S}_{\mathbf{1}}+\frac{n_{1}}{n_{2}}\left|\mathbf{S}_{\mathbf{1}}\right| \mathbf{N}_{\mathbf{S} 1} \cos \delta_{1}-\left|\mathbf{S}_{\mathbf{1}}\right| \mathbf{N}_{\mathbf{S} 1} \cos \varepsilon_{1}\right)\left(1-\rho_{S 1}\right)
$$

enables to determine its components:

$$
\begin{align*}
& T_{1 x}=\left(\frac{n_{1}}{n_{2}} S_{1 x}-\frac{n_{1}}{n_{2}}\left|\mathbf{S}_{\mathbf{1}}\right| \frac{x_{S 1}}{r_{1}} \cos \delta_{1}+\left|\mathbf{S}_{\mathbf{1}}\right| \frac{x_{S 1}}{r_{1}} \cos \varepsilon_{1}\right)\left(1-\rho_{S 1}\right) \\
& T_{1 y}=\left(\frac{n_{1}}{n_{2}} S_{1 y}-\frac{n_{1}}{n_{2}}\left|\mathbf{S}_{\mathbf{1}}\right| \frac{y_{S 1}}{r_{1}} \cos \delta_{1}+\left|\mathbf{S}_{\mathbf{1}}\right| \frac{y_{S 1}}{r_{1}} \cos \varepsilon_{1}\right)\left(1-\rho_{S 1}\right)  \tag{21}\\
& T_{1 z}=\frac{n_{1}}{n_{2}} S_{1 y}\left(1-\rho_{S 1}\right)
\end{align*}
$$

The vector $\mathbf{T}_{\mathbf{1}}$ hits the side surface at the point $T_{1}$ being the boundary between the ring-core with the refractive index $n_{2}$ and outer cladding with the index $n_{3}$. The coordinates of the point $T_{1}$ can be determined from the system of equations:

$$
\left.\begin{array}{l}
x_{T 1}^{2}+y_{T 1}^{2}=r_{2}^{2}  \tag{22}\\
x_{T 1}=\frac{T_{1 x}}{T_{1 z}}\left(z_{T 1}-z_{S 1}\right)+x_{S 1} \\
y_{T 1}=\frac{T_{1 y}}{T_{1 z}}\left(z_{T 1}-z_{S 1}\right)+y_{S 1}
\end{array}\right\}
$$

If $z_{T 1}>l$, the vector $\mathbf{T}_{\mathbf{1}}$ hits the exit surface of the light fibre and the coordinates of this point $Q_{2}$ are equal to:

$$
\left.\begin{array}{l}
x_{Q 2}=\frac{T_{1 x}}{T_{1 z}}\left(l-z_{S 1}\right)+x_{S 1} \\
y_{Q 2}=\frac{T_{1 y}}{T_{1 z}}\left(l-z_{S 1}\right)+y_{S 1}  \tag{23}\\
z_{Q 2}=l
\end{array}\right\}
$$

If $z_{T 1}<l$, we can determine the other coordinates of the point $T_{1}$ from the Eq. (22).

At the point $T_{1}$, at the boundary of two media with the coefficients $n_{2}$ and $n_{3}$, the unit vector $\mathbf{N}_{\mathbf{T} 1}$ normal to the boundary surface is equal to:

$$
\mathbf{N}_{\mathbf{T} 1}\left(N_{T 1 x}, N_{T 1 y}, N_{T 1 z}\right)=\mathbf{N}_{\mathbf{T} 1}\left(-\frac{x_{T 1}}{r_{2}},-\frac{y_{T 1}}{r_{2}}, 0\right)
$$

The angle of incidence $\varphi_{1}$ of the luminous flux (vector $\mathbf{T}_{\mathbf{1}}$ ) on the boundary surface is described by the relation:

$$
\begin{equation*}
\cos \varphi_{1}=\cos \left(-\mathbf{T}_{\mathbf{1}} \cdot \mathbf{N}_{\mathbf{T} \mathbf{1}}\right)=\frac{-\mathbf{T}_{\mathbf{1}} \mathbf{N}_{\mathbf{T} \mathbf{1}}}{\left|\mathbf{T}_{\mathbf{1}}\right|}=\frac{-T_{1 x} N_{T 1 x}-T_{1 y} N_{T 1 y}}{\left|\mathbf{T}_{\mathbf{1}}\right|} \tag{24}
\end{equation*}
$$

At the point $T_{1}$ we must check if there occurs the phenomenon of the complete inner reflection. This phenomenon takes place if the following condition is satisfied:

$$
\begin{equation*}
\frac{n_{3}}{n_{2}}<\sqrt{1-\cos ^{2} \varphi_{1}} \tag{25}
\end{equation*}
$$

The vector $\mathbf{F}_{\mathbf{1}}$ of the inside reflection is described by the relation:

$$
\mathbf{F}_{\mathbf{1}}=\mathbf{T}_{\mathbf{1}}+2\left|\mathbf{T}_{\mathbf{1}}\right| \mathbf{N}_{\mathbf{T} \mathbf{1}} \cos \varphi_{1}
$$

Hence, the components of this vector are equal to:

$$
\left.\begin{array}{l}
F_{1 x}=T_{1 x}+2\left|\mathbf{T}_{\mathbf{1}}\right| N_{T 1 x} \cos \varphi_{1} \\
F_{1 y}=T_{1 y}+2\left|\mathbf{T}_{\mathbf{1}}\right| N_{T 1 y} \cos \varphi_{1}  \tag{26}\\
F_{1 z}=T_{1 z}
\end{array}\right\}
$$

When determining the point $F_{1}$ the following system of equations is applicable:

$$
\left.\begin{array}{l}
x_{F 1}^{2}+y_{F 1}^{2}=r_{1}^{2}  \tag{27}\\
x_{F 1}=\frac{F_{1 x}}{F_{1 z}}\left(z_{F 1}-z_{T 1}\right)+x_{T 1} \\
y_{F 1}=\frac{F_{1 y}}{F_{1 z}}\left(z_{F 1}-z_{T 1}\right)+y_{T 1}
\end{array}\right\}
$$

If the coordinate of point $F_{1}: z_{F 1}>l$, the luminous flux meets the entrance surface of the fibre at the point $Q_{3}$ with the coordinates:

$$
\left.\begin{array}{l}
x_{Q 3}=\frac{F_{1 x}}{F_{1 z}}\left(l-z_{T 1}\right)+x_{T 1} \\
y_{Q 3}=\frac{F_{1 y}}{F_{1 z}}\left(l-z_{T 1}\right)+y_{T 1}  \tag{28}\\
z_{Q 3}=l
\end{array}\right\}
$$

If $z_{F 1}<l$, the other coordinates can be determined from relation (27). However, it happens occasionally that the luminous flux $\mathbf{F}_{\mathbf{1}}$ by-passes the inner cladding of the fibre and again meets the outer side surface of the ring. In this case, in order to determine the point of intersection of the vector $\mathbf{F}_{\mathbf{1}}$ with the side surface of the ring, the radius $r_{2}$ should be substituted into the equation of the ring in relation (27) and then the calculations should be carried out as above.

If at the point $T_{1}$ the condition (25) is not satisfied, the inside reflection does not occur, then the luminous flux will penetrate into the area of the cladding $\left(n_{3}\right)$, but we must take into consideration Fresnel's reflection. The angle of refraction $\psi_{1}$ is determined from the relation:

$$
\begin{equation*}
\cos \psi_{1}=\sqrt{1-\frac{n_{2}^{2}}{n_{3}^{2}}\left(1-\cos ^{2} \varphi_{1}\right)} \tag{29}
\end{equation*}
$$

and the reflection coefficient $\rho_{T 1}$ - in a similar way as from relation (16):

$$
\begin{equation*}
\rho_{T 1}=\frac{1}{2}\left[\frac{\sin ^{2}\left(\varphi_{1}-\psi_{1}\right)}{\sin ^{2}\left(\varphi_{1}+\psi_{1}\right)}+\frac{\tan ^{2}\left(\varphi_{1}-\psi_{1}\right)}{\tan ^{2}\left(\varphi_{1}+\psi_{1}\right)}\right] . \tag{30}
\end{equation*}
$$

Knowing the reflection coefficient $\rho_{T 1}$ it is possible to determine the values of the elementary luminous fluxes $\mathbf{F}_{\mathbf{1}}$ (Fresnel's reflection) and $\mathbf{U}_{\mathbf{1}}$ (the main luminous flux):

$$
\left.\begin{array}{l}
\mathbf{F}_{\mathbf{1}}=\rho_{T 1} \mathbf{T}_{\mathbf{1}}  \tag{31}\\
\mathbf{U}_{\mathbf{1}}=\left(1-\rho_{T 1}\right) \mathbf{T}_{\mathbf{1}}
\end{array}\right\}
$$

The luminous flux $\mathbf{U}_{\mathbf{1}}$, running in the jacked, escapes from it through the side surface and is not analysed any longer.

In the case where at the point $T_{1}$ a complete inner reflection takes place (condition (25) is satisfied) and $z_{F 1}<l$, the main luminous flux $\mathbf{F}_{1}$ will enter the inner cladding. In this case, the angle of incidence $\mu_{1}$ should be determined from the relation:

$$
\begin{equation*}
\cos \mu_{1}=\cos \left(\mathbf{F}_{\mathbf{1}} \cdot \mathbf{N}_{\mathbf{F} \mathbf{1}}\right)=\frac{F_{1 x} N_{F 1 x}+F_{1 y} N_{F 1 y}}{\left|\mathbf{F}_{\mathbf{1}}\right|} \tag{32}
\end{equation*}
$$

and the vector $\mathbf{N}_{\mathbf{F} 1}$ is determined as follows:

$$
\mathbf{N}_{\mathbf{F} 1}\left(N_{F 1 x}, N_{F 1 y}, N_{F 1 z}\right)=\mathbf{N}_{\mathbf{F} 1}\left(-\frac{x_{F 1}}{r_{1}},-\frac{y_{F 1}}{r_{1}}, 0\right) .
$$

Next, the angle of refraction $\eta_{1}$, occurring when the main ray $\mathbf{S}_{\mathbf{2}}$ penetrates into the inner cladding:

$$
\begin{equation*}
\cos \eta_{1}=\sqrt{1-\frac{n_{2}^{2}}{n_{1}^{2}}\left(1-\cos ^{2} \mu_{1}\right)} \tag{33}
\end{equation*}
$$

and the coefficient of Fresnel's reflection is determined

$$
\begin{equation*}
\rho_{F 1}=\frac{1}{2}\left[\frac{\sin ^{2}\left(\mu_{1}-\eta_{1}\right)}{\sin ^{2}\left(\mu_{1}+\eta_{1}\right)}+\frac{\tan ^{2}\left(\mu_{1}-\eta_{1}\right)}{\tan ^{2}\left(\mu_{1}+\eta_{1}\right)}\right] . \tag{34}
\end{equation*}
$$

The luminous fluxes $\mathbf{S}_{\mathbf{2}}$ (main) and $\mathbf{G}_{\mathbf{1}}$ (Fresnel's reflection) are determined from the relations:

$$
\left.\begin{array}{l}
\left|\mathbf{S}_{\mathbf{2}}\right|=\left(1-\rho_{F 1}\right)\left|\mathbf{F}_{\mathbf{1}}\right|  \tag{35}\\
\left|\mathbf{G}_{\mathbf{1}}\right|=\rho_{F 1}\left|\mathbf{F}_{\mathbf{1}}\right|
\end{array}\right\}
$$

The luminous flux $\mathbf{G}_{\mathbf{1}}$ behaves in a similar way as $\mathbf{T}_{\mathbf{1}}$ and its further course can be analysed in accordance with the relation (17) and the next ones.

The main luminous flux $\left|\mathbf{S}_{\mathbf{2}}\right|$ meets the side surface of the inner cladding at the point $S_{2}$, thus it is necessary to determine the coordinates of this point, using the following system of equations:

$$
\left.\begin{array}{l}
x_{S 2}^{2}+y_{S 2}^{2}=r_{1}^{2} \\
x_{S 2}=\frac{H_{1 x}}{H_{1 z}}\left(z_{S 2}-z_{F 1}\right)+x_{F 1}  \tag{36}\\
y_{S 2}=\frac{H_{1 y}}{H_{1 z}}\left(z_{S 2}-z_{F 1}\right)+y_{F 1}
\end{array}\right\}
$$

Further course of the rays refracted and reflected at the point $S_{2}$ is analysed in the same way as for the point $S_{1}$.

The vector $\mathbf{V}_{\mathbf{1}}$, determined by Eqs. (18)-(20), meets the surface of the ring-core at the point $V_{1}$. The coordinates of this point can be determined by solving the system of equations:

$$
\left.\begin{array}{l}
x_{V 1}^{2}+y_{V 1}^{2}=r_{1}^{2} \\
x_{V 1}=\frac{V_{1 x}}{V_{1 z}}\left(z_{V 1}-z_{S 1}\right)+x_{S 1}  \tag{37}\\
y_{V 1}=\frac{V_{1 y}}{V_{1 z}}\left(z_{V 1}-z_{S 1}\right)+y_{S 1}
\end{array}\right\}
$$

If the coordinate $z_{V 1}>l$, the ray $\mathbf{V}_{1}$ hits the exit surface of the inner cladding at the point $Q$. The coordinates of the point $Q$ are determined in a similar way as in the case of the ray $\mathbf{S}_{\mathbf{1}}$, Eq. (12). In the opposite case the other coordinates of this point must be found. In order to calculate the reflection coefficient $\rho_{V 1}$ we must know the angle of incidence $\kappa_{1}$ and the angle of $\lambda_{1}$. Knowing that

$$
\cos \kappa_{1}=\frac{V_{1 x}}{\left|\mathbf{V}_{\mathbf{1}}\right|} \frac{x_{V 1}}{r_{1}}+\frac{V_{1 y}}{\left|\mathbf{V}_{\mathbf{1}}\right|} \frac{y_{V 1}}{r_{1}}
$$

these angles are determined from the relations:

$$
\begin{align*}
& \kappa_{1}=\operatorname{atan} \sqrt{\frac{1-\cos ^{2} \kappa_{1}}{\cos \kappa_{1}}},  \tag{38}\\
& \lambda_{1}=\operatorname{atan} \sqrt{\frac{1-\cos ^{2} \kappa_{1}}{\frac{n_{2}^{2}}{n_{1}^{2}}-1+\cos ^{2} \kappa_{1}}} . \tag{39}
\end{align*}
$$

Hence,

$$
\begin{equation*}
\rho_{V 1}=\frac{1}{2}\left[\frac{\sin ^{2}\left(\kappa_{1}-\lambda_{1}\right)}{\sin ^{2}\left(\kappa_{1}+\lambda_{1}\right)}+\frac{\tan ^{2}\left(\kappa_{1}-\lambda_{1}\right)}{\tan ^{2}\left(\kappa_{1}+\lambda_{1}\right)}\right] \tag{40}
\end{equation*}
$$

Knowing the value of the reflection coefficient $\rho_{V 1}$ makes it possible to determine the division of the elementary luminous flux at the point $V_{1}$ into the reflected part $\mathbf{H}_{\mathbf{1}}$ and the refracted part $\mathbf{T}_{2}$ :

$$
\left.\begin{array}{rl}
\left|\mathbf{H}_{\mathbf{1}}\right| & =\rho_{V 1}\left|\mathbf{V}_{\mathbf{1}}\right|  \tag{41}\\
\left|\mathbf{T}_{\mathbf{2}}\right| & =\left(1-\rho_{V 1}\right)\left|\mathbf{V}_{\mathbf{1}}\right|
\end{array}\right\}
$$

A similar situation is found at the points $V_{2}, V_{3}$, etc. The refracted beam $\mathbf{T}_{2}$ at the point $V_{1}$ becomes also completely reflected at $T_{2}$ and will hit the point $F_{2}$. Thus, the phenomena occurring at the point $V_{1}$ will be analogous to phenomena occurring at the other points $V$. Calculations of the points $F$ and $V$ are repeated many times.

The refracted beam $\mathbf{H}_{1}$ behaves similarly as $\mathbf{V}_{\mathbf{1}}$, thus its further analysis is known.
The final result of the calculations is the fact of hitting the exit surface of the optical fibre by the elementary luminous flux. Part of this flux escapes through the inner cladding, and part through the ring core. In order to determine the distribution of the luminous flux the exit face of the fibre was divided into elements having the shape of squares each with $m$ elements along the diameter of the ring (Fig. 3).


Fig. 3. Escape of luminous fluxes from the ring-core optical fibre.
Based on relations (12), (23) and (28) the coordinates of the point where the elementary luminous flux hits the exit surface of the fibre are determined. On this surface there also appear Fresnel's reflections, hence it is necessary to know the value of the incidence angle $\beta_{Q}$ and that of the refraction angle $\gamma_{Q}$. These angles are determined from the dependence

$$
\begin{equation*}
\beta_{Q}=\operatorname{atan} \frac{\sqrt{S_{x}^{2}+S_{y}^{2}}}{S_{z}} \tag{42}
\end{equation*}
$$

and for the luminous fluxes leaving the inner cladding:

$$
\begin{equation*}
\gamma_{Q}=\operatorname{atan} \frac{n_{1} \sin \beta_{Q}}{\sqrt{1-n_{1}^{2} \sin ^{2} \beta_{Q}}} \tag{43}
\end{equation*}
$$

and for the fluxes leaving the ring core

$$
\begin{equation*}
\gamma_{Q}=\operatorname{atan} \frac{n_{2} \sin \beta_{Q}}{\sqrt{1-n_{2}^{2} \sin ^{2} \beta_{Q}}} \tag{44}
\end{equation*}
$$

Next, the reflection index $\rho_{Q}$ is calculated from the relation

$$
\begin{equation*}
\rho_{Q}=\frac{1}{2}\left[\frac{\sin ^{2}\left(\beta_{Q}-\gamma_{Q}\right)}{\sin ^{2}\left(\beta_{Q}+\gamma_{Q}\right)}+\frac{\tan ^{2}\left(\beta_{Q}-\gamma_{Q}\right)}{\tan ^{2}\left(\beta_{Q}+\gamma_{Q}\right)}\right] . \tag{45}
\end{equation*}
$$

Now, we can determine the value of the elementary luminous flux $|\mathbf{Q}|$ leaving the fibre

$$
\begin{equation*}
|\mathbf{Q}|=\left(1-\rho_{Q}\right)|\mathbf{S}| . \tag{46}
\end{equation*}
$$

The above calculations are carried out in the same way for all partial elementary luminous fluxes $\mathbf{S}, \mathbf{V}, \mathbf{T}$ and $\mathbf{F}$.

It is necessary to sum up all the elementary luminous fluxes leaving the inner cladding $\Phi_{R}$ and the ring-core $\boldsymbol{\Phi}_{P}$ :


Fig. 4. Relative values of luminous fluxes leaving inner cladding (1) and ring-core (2) as a function of outer ring-core radius $r_{2}\left(n_{1}=2.17 ; n_{2}=2.20 ; n_{3}=1.52 ; r_{1}=0.5 \mathrm{~mm} ; r_{3}=r_{2}+0.2 \mathrm{~mm}\right)$.

$$
\begin{align*}
& \Phi_{R}=\sum\left|\mathbf{Q}_{\mathbf{R}}\right|  \tag{47}\\
& \Phi_{P}=\sum\left|\mathbf{Q}_{\mathbf{P}}\right| \tag{48}
\end{align*}
$$

In this way, the operation of a ring-core optical fibre is illustrated.
For numerical calculations the following data have been accepted: $r_{1}=0.5 \mathrm{~mm}$; $r_{2}$ was changed from 0.7 do $1.5 \mathrm{~mm} ; r_{3}=r_{2}+0.2 \mathrm{~mm}$; length $l$ was changed from 30 to $150 \mathrm{~mm} ; n_{1}=2.17 ; n_{2}=2.20 ; n_{3}=1.52$. The result is presented in Fig. 4.

## 2. Parameters of the fibre

Heavy metal oxide (HMO) glasses are very attractive hosts for rare earth ions. Among the HMO glasses one of the most interesting are $\mathrm{PbO}-\mathrm{Bi}_{2} \mathrm{O}_{3}-\mathrm{Ga}_{2} \mathrm{O}_{3}-\mathrm{BaO}$ glasses because of their high transparency (up to $70 \%$ ) within the $0.5-7.6 \mu \mathrm{~m}$ IR spectral range [3]. Mainly PbO and $\mathrm{Bi}_{2} \mathrm{O}_{3}$ oxides determine optical properties of this family of glasses [4]. Moreover, the HMO glass structure is flexible enough to accept rare earth elements.

The glass composition: $\mathrm{PbO}(0.50)-\mathrm{Bi}_{2} \mathrm{O}_{3}(0.25)-\mathrm{Ga}_{2} \mathrm{O}_{3}(0.20)-\mathrm{BaO}(0.05)(\mathrm{mol} \%)$ doped with $0.5 \%$ of $\mathrm{Dy}_{2} \mathrm{O}_{3}$ for the ring-core fibre construction was used [4]. The molten mass of glass was poured into a vertically whirling brass cylinder and after forming was distressed at a temperature of $350^{\circ} \mathrm{C}$. In this way, the glass attained the ring shape. For the inner cladding glass, the composition $\mathrm{PbO}(0.40)-\mathrm{Bi}_{2} \mathrm{O}_{3}(0.28)-$ $\mathrm{Ga}_{2} \mathrm{O}_{3}(0.25)-\mathrm{CdO}(0.07)(\mathrm{mol} \%)$ was chosen. The glass rod with refractive index of $n_{1}=2.17(\lambda=630 \mathrm{~nm})$, thermal expansion coefficient $\alpha_{1}=115 \times 10^{-7} \mathrm{~K}^{-1}$ was obtained. The outer radius of the rod fitted the inner radius of the ring-core glass.

Because of a very low viscosity of prepared glasses it was impossible to use "classic" drawing methods (double crucible, rod in tube, etc.) As a first operation, cylindrical rod for inner cladding was insert into the ring core shaped glass. Both


Fig. 5. Step index ring-core optical fibre doped by $\mathrm{Dy}^{3+}$ ions. Outer diameter of the fibre $150 \mu \mathrm{~m}$.


Fig. 6. IR absorption spectra of the fibre.
glasses were heated in a fine fit crucible. The nozzle of the crucible was earlier connected with a capillary made from $\mathrm{SiO}_{2}-\mathrm{Na}_{2} \mathrm{O}-\mathrm{CaO}$ glass, used for outer cladding. At a temperature of about $625^{\circ} \mathrm{C}$ the core and inner cladding glasses flew into the capillary and then a two-step drawing process was possible.

The wide IR transparency (up to $7.6 \mu \mathrm{~m}$ ) is a result of a relatively low phonon energy (about $700 \mathrm{~cm}^{-1}$ ) defined by the large atomic mass of the cations, such as Pb , $\mathrm{Ga}, \mathrm{Bi}, \mathrm{Ba}$. The core glass has refractive index $n_{2}=2.20(\lambda=630 \mathrm{~nm})$, thermal expansion coefficient $\alpha_{2}=120 \times 10^{-7} \mathrm{~K}^{-1}$ and measured non-linear coefficient $n_{2}=8.5 \times 10^{-19} \mathrm{~m}^{2} / \mathrm{W}$. Figure 6 presents IR absorption spectra of the ring core fibre doped with $0.5 \%$ of $\mathrm{Dy}^{3+}$ (in weight units). We can see five absorption bands: $910 \mathrm{~nm}(1), 1160 \mathrm{~nm}(2), 1320 \mathrm{~nm}(3), 1700 \mathrm{~nm}$ (4) and 2780 nm (5). The observed bands correspond to the optical excitation from the ground ${ }^{6} \mathrm{H}_{15 / 2}$ state to the excited ${ }^{6} \mathrm{~F}_{7 / 2},{ }^{6} \mathrm{H}_{7 / 2},{ }^{6} \mathrm{H}_{9 / 2},{ }^{6} \mathrm{H}_{11 / 2},{ }^{6} \mathrm{H}_{13 / 2}$ states.


Fig. 7. Core luminescence spectra $\mathrm{PbO}-\mathrm{Bi}_{2} \mathrm{O}_{3}-\mathrm{Ga}_{2} \mathrm{O}_{3}-\mathrm{BaO}-\mathrm{Dy}^{3+}$.


Fig. 8. Ring-core optical fibre with special channel for inner stimulation.

Luminescence spectra of $\mathrm{PbO}-\mathrm{Bi}_{2} \mathrm{O}_{3}-\mathrm{Ga}_{2} \mathrm{O}_{3}-\mathrm{BaO}-\mathrm{Dy}^{3+}$ fibre pumped at $1.06 \mu \mathrm{~m}$ YAG-Nd laser at room temperature are presented in Fig. 7. Luminescence bands at 1890 nm (1) and 3020 nm (2) are due to ${ }^{6} \mathrm{H}_{11 / 2} \rightarrow{ }^{6} \mathrm{H}_{15 / 2}$ and ${ }^{6} \mathrm{H}_{11 / 2} \rightarrow{ }^{6} \mathrm{H}_{13 / 2}$ transitions, respectively. Measured lifetimes of $\mathrm{Dy}^{3+}$ levels and their quantum efficiency are as follows: $\tau=200 \mathrm{~ms}, \eta=41 \%$ (1) and $\tau=3200 \mathrm{~ms}, \eta=13.8 \%$ (2).

The first experiments showed that too many energy propagates through the inner cladding. To reduce it a ring-core optical fibre with special inner stimulation channel was constructed [5] (see Fig. 8).

## 3. Conclusions

The following conclusions can be drawn:

1. The ring core optical fibre can be used to divide the luminous flux between the inner cladding and cylindrical core proportionally. The range of those changes is relatively wide and depends on inner cladding and core ratio (Fig. 4).
2. The measured lifetime of $\mathrm{Dy}^{3+}$ level was about $6211 \mu \mathrm{~s}$ and is not far from that of the crystalline materials. The relatively high quantum efficiency of the ${ }^{6} \mathrm{H}_{13 / 2} \rightarrow$ ${ }^{6} \mathrm{H}_{15 / 2}$ transition is probably caused by an increased multiphonon relaxation due to the low energy differences between the ground and first excited states.
3. The ring core $\mathrm{Dy}^{3+}$ doped optical fibre was made (Fig. 5). In spite of 25 years of experience in glass technology and optical fibre drawing the author had to solve some problems connected with low viscosity of the glasses used. Particularly, the method of forming whirling glass elements seems to be suitable for construction of optical fibres from the glasses characterised by very low viscosity. This method will be developed by the author.

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