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EFFECT OF HALL CURRENTS ON THERMAL INSTABILITY OF COMPRESSIBLE DUSTY VISCOELASTIC FLUID IN POROUS MEDIUM

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Abstract: The thermal instability of compressible Walters B′ viscoelastic fluid in porous medium is considered to include the effects of Hall currents and suspended particles. Following the linearized stability theory and normal mode analysis, the dispersion relation is obtained. For the case of stationary convection, Hall currents and suspended particles are found to have destabilizing effects whereas compressibility and magnetic field have stabilizing effects on the system. The medium permeability, however, has stabilizing and destabilizing effects on thermal instability in contrast to its destabilizing effect in the absence of magnetic field. The magnetic field, Hall currents and viscoelasticity parameter are found to introduce oscillatory modes in the system.

1. INTRODUCTION

The problem of thermal convection in a horizontal layer of electrically conducting fluid in the presence of magnetic field has been discussed in detail by CHANDRASEKHAR [1] in his celebrated monograph. It is shown that a uniform magnetic field inhibits the onset of thermal convection. If an electric field is applied at right angles to the magnetic field, the whole current will not flow along the electric field. This tendency of the electric current of flow across an electric field in the presence of a magnetic field is called Hall effect. The Hall effect is likely to be important in many geophysical and astrophysical situations as well as in flows of laboratory plasma. SHERMAN and SUTTON [2] have considered the effect of Hall current on the efficiency of a magneto-fluid-dynamic generator. GUPTA [3] studied the problem of thermal instability in the presence of Hall currents and found that Hall currents have a destabilizing effect on the thermal instability of a horizontal layer of a conducting fluid in the presence of a uniform vertical magnetic field. The use of the Boussinesq approximation has been made throughout, which states that the variations of density in the equations of motion can safely be ignored everywhere except in its association with the external force. The approximation is well justified in the case of incompressible fluids.

When the fluids are compressible, the equations governing the system become quite complicated. To simplify them, Boussinesq tried to justify the approximation for compressible fluids when the density variations arise principally from thermal effects. SPIEGEL and VERONIS [4] have simplified the set of equations governing the flow of compressible fluids under the following assumptions:

(a) the depth of the fluid layer is much less than the scale height, as defined by them; and

(b) the fluctuations in temperature, density and pressure, introduced due to motion, do not exceed their total static variations.

Under the above approximations, the flow equations are the same as those for incompressible fluids, except that the static temperature gradient is replaced by its excess over the adiabatic one and C_v is replaced by C_p.

In geophysical situations, the fluid is often not pure but contains suspended particles. SCANLON and SEGEL [5] have considered the effects of suspended particles on the onset of Bénard convection and found that the critical Rayleigh number is reduced because of the heat capacity of the particles. The suspended particles were thus found to destabilize the layer. PALANISWAMY and PURUSHOTHAM [6] have studied the stability of shear flow of stratified fluids with fine dust and found the effects of fine dust to increase the region of instability. The fluids have been considered to be Newtonian and the medium has been considered to be non-porous in all the above studies.

There is growing importance of non-Newtonian fluids in geophysical fluid dynamics, chemical technology and petroleum industry. BHATIA and STEINER [7] have studied the problem of thermal instability of Maxwellian viscoelastic fluid in the presence of rotation and have found that the rotation has a destabilizing influence in contrast to the stabilizing effect on an ordinary viscous (Newtonian) fluid. The thermal instability of an Oldroydian viscoelastic fluid acted on by a uniform rotation has been studied by SHARMA [8]. There are many elastico-viscous fluids that cannot be characterized by Maxwell's or Oldroyd's constitutive relations. One such class of viscoelastic fluids is Walters B′ fluid [9] having relevance and importance in geophysical fluid dynamics, chemical technology, and petroleum industry. WALTERS [10] reported that the mixture of polymethyl methacrylate and pyridine at 25 °C containing 30.5 g of polymer per litre with density of 0.98 g per litre behaves much the same as the Walters B′ viscoelastic fluid. Polymers are used in the manufacture of spacecrafts, aeroplanes, tyres, belt conveyers, ropes, cushions, seats, foams, plastics engineering equipment, contact lens, etc. Walters B′ viscoelastic fluid forms the basis for the manufacture of many such important and useful products. CHAKRABORTY and SENGUPTA [11] have studied the flow of unsteady viscoelastic (Walters B′ liquid) conducting fluid through two porous concentric non-conducting infinite circular cylinders rotating with different angular velocities in the presence of uniform axial magnetic field. SHARMA and KUMAR [12] studied the stability of the plane interface separating two viscoelastic (Walters B′) superposed fluids of uniform densities. In another study, SHARMA and KUMAR [13] studied Rayleigh–Taylor instability of superposed conducting Walters B′ viscoelastic fluids in hydromagnetics. KUMAR [14] has considered the thermal instability of a layer of Walters B′ viscoelastic fluid acted on by a uniform rotation and found that for stationary convection, rotation has a stabilizing effect. KUMAR et al. [15] have considered the stability of plane interface separating the Walters B′ viscoelastic superposed fluids of uniform densities in the presence of suspended particles.

The flow through porous media is of considerable interest for petroleum engineers and geophysical fluid dynamicists. A great number of applications in geophysics may be found in the books by PHILLIPS [16], INGHAM and POP [17], and NIELD and BEJAN [18]. When the fluid slowly percolates through the pores of a macroscopically homogeneous and isotropic porous medium, the gross effect is represented by Darcy's law. As a result of this macroscopic law, the usual viscous term in the equations of fluid motion is replaced by the resistance term $-\frac{1}{k_1}\left(\mu - \mu' \frac{\partial}{\partial t}\right)\vec{q}$, where μ and μ' are associated with the viscosity and viscoelasticity, k_1 is the medium permeability and $\vec{q}$ is the Darcian (filter) velocity of the fluid. LAPWOOD [19] has studied the stability of convective flow in hydromagnetics in a porous medium using Rayleigh's procedure. The Rayleigh instability of a thermal boundary layer in flow through porous medium has been considered by WOODING [20]. KUMAR [21] has studied the problem of thermal convection in Walters B′ viscoelastic fluid in porous medium in the presence of suspended particles. KUMAR et al. [22] have considered the effect of suspended particles on the thermal instability of Walters B′ viscoelastic fluid in hydromagnetics in porous medium. In another study, KUMAR and SINGH [23] studied the instability of the plane interface between two Walters B′ viscoelastic superposed fluids permeated with suspended particles and uniform rotation in porous medium.

Keeping in mind the importance of various parameters like medium permeability, compressibility, Hall currents, suspended particles; in geophysics (e.g., Earth's molten core), soil sciences, groundwater hydrology, atmospheric physics, astrophysics, and various applications mentioned above, our interest, in the present paper, is to bring out the effect of suspended particles on thermal instability of compressible Walters B′ viscoelastic fluid in porous medium to include the effect of Hall currents. In this complex model, our main purpose is to examine the behaviour of various important parameters on instability problem. The present study can serve as a theoretical support for experimental investigations, e.g., in evaluating the influence of impurifications in Walters B′ viscoelastic fluid on thermal convection phenomena in porous medium.

2. FORMULATION OF THE PROBLEM AND PERTURBATION EQUATIONS

Consider an infinite horizontal, compressible Walters B′ viscoelastic fluid-particle layer of thickness d confined between the planes $z = 0$ and $z = d$ in the presence of uniform vertical magnetic field $\vec{H}(0, 0, H)$ in porous medium. This layer is heated from below such that a steady adverse temperature gradient $\beta(=|dT/dz|)$ is maintained. The equations of motion and continuity for the fluid are

$$\frac{\rho}{\varepsilon}\left[\frac{\partial \vec{v}}{\partial t}+\frac{1}{\varepsilon}(\vec{v}.\nabla)\,\vec{v}\right]=-\nabla p-\rho g\vec{\lambda}-\frac{1}{k_1}\left(\mu-\mu'\frac{\partial}{\partial t}\right)\vec{v}+\frac{KN}{\varepsilon}(\vec{u}-\vec{v})+\frac{\mu_e}{4\pi}(\nabla\times\vec{H})\times\vec{H}, \quad (1)$$

$$\varepsilon\frac{\partial \rho}{\partial t}+\nabla.(\rho\vec{v})=0, \quad (2)$$

where ρ, μ, μ', p and $\vec{v}(u, v, w)$ denote the density, viscosity, viscoelasticity, pressure and the velocity of the pure fluid, respectively. Here, $\vec{u}(l, r, s), N(\bar{x}, t)$ denote the velocity and number density of the suspended particles, ε is the medium porosity, k_1 is the medium permeability, μ_e is the magnetic permeability, g is the acceleration due to gravity, $\bar{x}=(x, y, z), \vec{\lambda}=(0, 0, 1)$ and $K=6\pi\mu\eta', \eta'$ being the particle radius, is the Stokes' drag coefficient. Assuming uniform particle size, spherical shape and small relative velocities between the fluid and the particles, the presence of particles adds an extra force term, in the equations of motion (1), proportional to the velocity difference between the particles and the fluid.

The force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid. The buoyancy force on the particles is negligibly small. Interparticle reactions are ignored for we assume that the distances between particles are quite large compared with their diameters. If mN is the mass of particles per unit volume, then the equations of motion and continuity for the particles are

$$mN\left[\frac{\partial \vec{u}}{\partial t}+\frac{1}{\varepsilon}(\vec{u}\cdot\nabla)\vec{u}\right]=KN\,(\vec{v}-\vec{u}), \quad (3)$$

$$\varepsilon\frac{\partial N}{\partial t}+\nabla.(N\vec{u})=0. \quad (4)$$

Let C_v, C_p, C_{pt}, T and q denote the heat capacity of fluid at constant volume, the heat capacity of fluid at constant pressure, the heat capacity of particles, the temperature and the "effective thermal conductivity" of the pure fluid, respectively. Assuming that the particles and the fluid are in thermal equilibrium, the equation of heat conduction gives

$$[\rho C_v\varepsilon+\rho_s C_s(1-\varepsilon)]\frac{\partial T}{\partial t}+\rho C_v(\vec{v}.\nabla)T+mNC_{pt}\left(\varepsilon\frac{\partial}{\partial t}+\vec{u}.\nabla\right)T=q\nabla^2 T\,, \quad (5)$$

where ρ_s, C_s are the density and the heat capacity of the solid matrix, respectively.

Maxwell's equations in the presence of Hall currents give

$$\nabla.\vec{H} = 0, \tag{6}$$

$$\varepsilon \frac{\partial \vec{H}}{\partial t} = \nabla \times (\vec{v} \times \vec{H}) + \varepsilon\eta\nabla^2 \vec{H} - \frac{c\varepsilon}{4\pi N' e} \nabla \times [(\nabla \times \vec{H}) \times \vec{H}], \tag{7}$$

where η, c, N' and e denote the resistivity, the speed of light, the electron number density and the charge of an electron, respectively.

The initial state of the system is taken to be a quiescent layer (no settling) with a uniform particle distribution N_0 and is given by

$$\vec{v} = (0,0,0), \quad \vec{H} = (0,0,H), \quad \vec{u} = (0,0,0), \tag{8}$$

$$T = T(z), \quad p = p(z), \quad \rho = \rho(z) \quad \text{and } N = N_0, \text{ a constant}, \tag{9}$$

where, following SPIEGEL and Veronis [4], we have

$$T(z) = -\beta z + T_0,$$

$$p(z) = p_m - g \int_0^z (\rho_m + \rho_0)\, dz,$$

$$\rho(z) = \rho_m [1 - \alpha_m (T - T_m) + K_m (p - p_m)],$$

$$\alpha_m = -\left(\frac{1}{\rho} \frac{\partial \rho}{\partial T} \right)_m,$$

$$K_m = \left(\frac{1}{\rho} \frac{\partial \rho}{\partial p} \right)_m.$$

SPIEGEL and VERONIS [4] expressed any state variable, say X, in the form

$$X = X_m + X_0(z) + X'(x, y, z, t), \tag{10}$$

where X_m stands for the constant space distribution of X, X_0 is the variation in X in the absence of motion and $X'(x, y, z, t)$ stands for the fluctuations in X due to the motion of the fluid. Also, ρ_m and p_m stand for the constant space distribution of ρ and p and ρ_0 and T_0 stand for the density and temperature of the fluid at the lower boundary. Following SPIEGEL and VERONIS [4] assumptions and results for compressible fluids, the flow equations are found to be the same as those of incompressible fluids except that the static temperature gradient β is replaced by its excess over the adiabatic $(\beta - g/Cp)$.

Let $\delta p, \delta\rho, \theta, \vec{v}(u, v, w), \vec{u}(l, r, s), \vec{h}(h_x, h_y, h_z)$ and N denote the perturbations in fluid pressure, density, temperature, velocity, particle velocity, magnetic field $\vec{H}$, and particle number density N_0, respectively. Then the linearized hydromagnetic perturbation equations of the viscoelastic fluid-particle layer, under SPIEGEL and VERONIS [4] assumptions, are

$$\frac{1}{\varepsilon}\frac{\partial \vec{v}}{\partial t} = -\frac{1}{\rho_m}\nabla\delta p - g\left(\frac{\delta\rho}{\rho_m}\right)\vec{\lambda} - \frac{1}{k_1}\left(\nu - \nu'\frac{\partial}{\partial t}\right)\vec{v} + \frac{KN_0}{\varepsilon\rho_m}(\vec{u}-\vec{v}) + \frac{\mu_e}{4\pi\rho_m}(\nabla\times\vec{h})\times\vec{H}, \quad (11)$$

$$\nabla.\vec{v} = 0\,, \quad (12)$$

$$mN_0\frac{\partial \vec{u}}{\partial t} = KN_0(\vec{v}-\vec{u})\,, \quad (13)$$

$$\varepsilon\frac{\partial N}{\partial t} + \nabla.(N_0\vec{u}) = 0\,, \quad (14)$$

$$(E+h\varepsilon)\frac{\partial\theta}{\partial t} = \left(\beta - \frac{g}{C_p}\right)(w+hs) + \kappa\,\nabla^2\theta\,, \quad (15)$$

$$\nabla\,.\,\vec{h} = 0\,, \quad (16)$$

$$\varepsilon\frac{\partial\vec{h}}{\partial t} = \nabla\times(\vec{v}\times\vec{H}) + \varepsilon\eta\nabla^2\vec{h} - \frac{c\varepsilon}{4\pi N'e}\nabla\times[(\nabla\times\vec{h})\times\vec{H}]\,, \quad (17)$$

where $\alpha_m = \frac{1}{T_m} = \alpha$ (say), $\nu = \frac{\mu}{\rho_m}$, $\kappa = \frac{q}{\rho_m C_v}$, and $\frac{g}{C_p}$, ν and κ stand for the adiabatic gradient, kinematic viscosity and thermal diffusivity, respectively.

Also, $h = \frac{fC_{pt}}{C_v}$, $f = \frac{mN_0}{\rho_m}$ and $E = \varepsilon + \frac{(1-\varepsilon)\rho_s C_s}{\rho_m C_v}$.

The linearized dimensionless perturbation equations relevant to the problem are

$$N_{p_1}^{-1}\frac{\partial u}{\partial t} = -\frac{\partial}{\partial x}\delta p - \frac{1}{P}\left(1 - A\frac{\partial}{\partial t}\right)u + \omega(l-u) + N_Q\left(\frac{\partial h_x}{\partial z} - \frac{\partial h_z}{\partial x}\right), \quad (18)$$

$$N_{p_1}^{-1}\frac{\partial v}{\partial t} = -\frac{\partial}{\partial y}\delta p - \frac{1}{P}\left(1 - A\frac{\partial}{\partial t}\right)v + \omega(r-v) + N_Q\left(\frac{\partial h_y}{\partial z} - \frac{\partial h_z}{\partial y}\right), \quad (19)$$

$$N_{p_1}^{-1}\frac{\partial w}{\partial t} = -\frac{\partial}{\partial z}\delta p - \frac{1}{P}\left(1 - A\frac{\partial}{\partial t}\right)w + N_R\theta + \omega\,(s-w)\,, \quad (20)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0\,, \quad (21)$$

$$\left(\tau\frac{\partial}{\partial t}+1\right)l=u\,,\ \left(\tau\frac{\partial}{\partial t}+1\right)r=v\,,\ \left(\tau\frac{\partial}{\partial t}+1\right)s=w\ \ , \tag{22}$$

$$\frac{\partial M}{\partial t}+\frac{\partial l}{\partial x}+\frac{\partial r}{\partial y}+\frac{\partial s}{\partial z}=0\,, \tag{23}$$

$$\frac{\partial h_x}{\partial x}+\frac{\partial h_y}{\partial y}+\frac{\partial h_z}{\partial z}=0\,, \tag{24}$$

$$N_{p_2}N_{p_1}^{-1}\frac{\partial h_x}{\partial t}=\varepsilon^{-1}\frac{\partial u}{\partial z}+\nabla^2 h_x-M_1\frac{\partial}{\partial z}\left(\frac{\partial h_z}{\partial y}-\frac{\partial h_y}{\partial z}\right), \tag{25}$$

$$N_{p_2}N_{p_1}^{-1}\frac{\partial h_y}{\partial t}=\varepsilon^{-1}\frac{\partial v}{\partial z}+\nabla^2 h_y-M_1\frac{\partial}{\partial z}\left(\frac{\partial h_x}{\partial z}-\frac{\partial h_z}{\partial x}\right), \tag{26}$$

$$N_{p_2}N_{p_1}^{-1}\frac{\partial h_z}{\partial t}=\varepsilon^{-1}\frac{\partial w}{\partial z}+\nabla^2 h_z-M_1\frac{\partial}{\partial z}\left(\frac{\partial h_y}{\partial x}-\frac{\partial h_x}{\partial y}\right), \tag{27}$$

$$(E+h\varepsilon)\frac{\partial \theta}{\partial t}=\left(\frac{G-1}{G}\right)(w+hs)+\nabla^2\theta\,, \tag{28}$$

where $N_{p_1}=\dfrac{\varepsilon\nu}{\kappa}$ is the modified Prandtl number, $N_{p_2}=\dfrac{\varepsilon\nu}{\eta}$ is the modified magnetic Prandtl number, $N_R=\dfrac{g\alpha\beta d^4}{\nu\kappa}$ is the Rayleigh number, $N_Q=\dfrac{\mu_e H^2 d^2}{4\pi\rho_m\nu\eta}$ is the Chandrasekhar number, $M=\dfrac{\varepsilon N}{N_0}$, $M_1=\dfrac{cH}{4\pi N'e\eta}$ is the Hall parameter, $\omega=\dfrac{KN_0d^2}{\rho_m\nu\varepsilon}$, $\tau=\dfrac{m\kappa}{Kd^2}$, $A=\left(\dfrac{\nu}{\nu'}\right)\dfrac{\kappa}{d^2}$, $f=\dfrac{mN_0}{\rho_m}=\tau\omega N_{p_1}$ is the mass fraction, $G=\dfrac{C_p\beta}{g}$ and $P=\dfrac{k_1}{d^2}$. Here, physical variables have been scaled using d, $\dfrac{d^2}{\kappa}$, $\dfrac{\kappa}{d}$, $\dfrac{\rho\nu\kappa}{d^2}$, βd and $\dfrac{H\kappa}{\eta}$ as the length, time, velocity, pressure, temperature and magnetic field scale factors, respectively.

Now, we consider the case where both the boundaries are free and the medium adjoining the fluid as non-conducting. The case of two free boundaries is slightly artificial, except in stellar atmospheres [24] and in certain geophysical situations where it is most appropriate. However, the case of two free boundaries allows us to obtain analytical solution without affecting the essential features of the problem. The boundary conditions appropriate for the problem are

$$w = \frac{\partial^2 w}{\partial z^2} = \theta = 0, \quad \xi = \frac{\partial \zeta}{\partial z} = 0, \qquad \text{at } z = 0 \text{ and } z = 1, \tag{29}$$

and h_x, h_y, h_z are continuous with an external vacuum field. (30)

Here, $\zeta = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$ and $\xi = \left(\frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}\right)$ are the z-components of vorticity and current density, respectively.

Equations (18)–(28), after eliminating u, v and δp, can be expressed as

$$\left[L_1 + \frac{L_2}{P}\left(1 - A\frac{\partial}{\partial t}\right)\right]\nabla^2 w = L_2 N_Q \nabla^2 \frac{\partial h_z}{\partial z} + L_2 N_R \nabla_1^2 \theta, \tag{31}$$

$$\left[L_1 + \frac{L_2}{P}\left(1 - A\frac{\partial}{\partial t}\right)\right]\zeta = L_2 N_Q \frac{\partial \xi}{\partial z}, \tag{32}$$

$$\left[N_{p_2} N_{p_1}{}^{-1} \frac{\partial}{\partial t} - \nabla^2\right]\xi = \varepsilon^{-1}\frac{\partial \zeta}{\partial z} + M_1 \frac{\partial}{\partial z}(\nabla^2 h_z), \tag{33}$$

$$\left[N_{p_2} N_{p_1}^{-1} \frac{\partial}{\partial t} - \nabla^2\right]h_z = \varepsilon^{-1}\frac{\partial w}{\partial z} - M_1 \frac{\partial \xi}{\partial z}, \tag{34}$$

$$L_2\left[(E + h\varepsilon)\frac{\partial}{\partial t} - \nabla^2\right]\theta = \left(\frac{G-1}{G}\right)\left[\tau\frac{\partial}{\partial t} + \overline{H}\right]w, \tag{35}$$

where $L_1 = N_{p_1}^{-1}\left(\tau\frac{\partial^2}{\partial t^2} + F\frac{\partial}{\partial t}\right)$, $F = f + 1$, $L_2 = \tau\frac{\partial}{\partial t} + 1$, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, $\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, $\overline{H} = h + 1$.

3. THE DISPERSION RELATION

Analysing the disturbances in terms of normal modes by seeking solutions whose dependence on x, y and t is given by

$$[w, \theta, h_z, \zeta, \xi] = [W(z), \Theta(z), K(z), Z(z), X(z)]\exp(ik_x x + ik_y y + nt), \tag{36}$$

where n is the growth rate and $\alpha = (\alpha_x^2 + \alpha_y^2)^{\frac{1}{2}}$ is the resultant wave number of the disturbance.

Equations (31)–(35), with the help of expression (36) become

$$\left[L_1 + \frac{L_2}{P}\{1 - An\}\right](D^2 - \alpha^2)W = L_2 N_Q (D^2 - \alpha^2)DK - L_2 N_R \alpha^2 \Theta, \tag{37}$$

$$\left[L_1 + \frac{L_2}{P}\{1 - An\}\right]Z = L_2 N_Q DX, \tag{38}$$

$$\lfloor N_{p_2} N_{p_1}^{-1} n - (D^2 - \alpha^2)\rfloor X = \varepsilon^{-1} DZ + M_1 (D^2 - \alpha^2 DK, \tag{39}$$

$$[N_{p_2} N_{p_1}{}^{-1} n - (D^2 - \alpha^2]\, K = \varepsilon^{-1} DW - M_1 DX, \tag{40}$$

$$L_2[(E + h\varepsilon)n - (D^2 - \alpha^2)]\,\Theta = \left(\frac{G-1}{G}\right)[\tau n + \overline{H}]\, W, \tag{41}$$

where $D = \dfrac{d}{dz}$, $L_1 = N_{p_1}^{-1}(\tau n^2 + Fn)$ and $L_2 = \tau n + 1$.

Eliminating Z, X, K and Θ between (37)–(41), we obtain

$$\left[L_1 + \frac{L_2}{P}(1 - An)\right][(D^2 - \alpha^2) - (E + h\varepsilon)](D^2 - \alpha^2)W + L_2 N_Q[(D^2 - \alpha^2) - (E + h\varepsilon)n]$$

$$\left[\{(D^2 - \alpha^2) - N_{p_2} N_{p_1}^{-1} n\} M_1^{-1} \varepsilon^{-1} + L_2 N_Q \left[L_1 + \frac{L_2}{P}(1 - An)\right]^{-1} M_1^{-1} \varepsilon^{-2} D^2\right] D^2$$

$$\left[M_1 (D^2 - \alpha^2) D^2 + M_1^{-1}\{(D^2 - \alpha^2) - N_{p_2} N_{p_1}^{-1} n\}^2 + L_2 N_Q \left[L_1 + \frac{L_2}{P}(1 - An)\right]^{-1}\right.$$

$$\left.\varepsilon^{-1} M_1^{-1}\{(D^2 - \alpha^2) - N_{p_2} N_1^{-1} n\} D^2\right]^{-1} (D^2 - \alpha^2)W = \left(\frac{G-1}{G}\right) N_R \alpha^2 (\tau n + \overline{H}). \tag{42}$$

Using the boundary conditions (29) and (30), it can be shown that all the even order derivatives of W vanish at the boundaries. Hence, the proper solution of (42) characterizing the lowest mode is

$$W = W_0 \sin \pi z, \tag{43}$$

where W_0 is constant.

On substituting the solution (43) in (42), we get the dispersion relation as

$$N_R=\left(\frac{G}{G-1}\right)\frac{(\pi^2+\alpha^2)[(\pi^2+\alpha^2)+(E+h\varepsilon)n]}{\alpha^2(\tau\, n+\overline{H})}\left[\left(L_1+\frac{L_2}{P}\{1-An\}\right)\right]$$

$$+L_2N_Q\pi^2\left[\left[\{(\pi^2+\alpha^2)+N_{p_2}\,N_{p_1}^{-1}n\}M_1^{-1}\varepsilon^{-1}+L_2N_Q\left(L_1+\frac{L_2}{P}\{1-An\}\right)^{-1}M_1^{-1}\varepsilon^{-2}\pi^2\right]\right.$$

$$\left[M_1\pi^2(\pi^2+\alpha^2)+M_1^{-1}\{(\pi^2+\alpha^2)+N_{p_2}N_{p_1}^{-1}n\}^{-2}+L_2N_Q\left(L_1+\frac{L_2}{P}(1-An)\right)^{-1}\right.$$

$$\left.\left.\varepsilon^{-1}M_1^{-1}\{(\pi^2+\alpha^2)+N_{p_2}N_{p_1}^{-1}n\}\,n\right]^{-1}\right]. \tag{44}$$

4. STATIONARY CONVECTION

When the instability sets in as stationary convection, the marginal state will be characterized by $n = 0$ and the dispersion relation (44) reduces to

$$N_R=\left(\frac{G}{G-1}\right)\frac{(\pi^2+\alpha^2)^2}{\alpha^2\overline{H}}\left[\frac{1}{P}+\frac{N_Q\pi^2\varepsilon^{-1}\{(\pi^2+\alpha^2)+N_QP\varepsilon^{-1}\pi^2\}}{(\pi^2+\alpha^2)\{M_1^2\pi^2+(\pi^2+\alpha^2)+N_QP\varepsilon^{-1}\pi^2\}}\right]. \tag{45}$$

Thus for stationary convection, the viscoelastic parameter A vanishes with n, and Walters B′ viscoelastic fluid behaves like a Newtonian fluid. Also, for fixed values of P, N_Q, M_1 and $\overline{H}$, let the non-dimensional number G accounting for the compressibility effects be also kept as fixed, then we have

$$\overline{N_R^C}=\left(\frac{G}{G-1}\right)N_R^C, \tag{46}$$

where N_R^C and $\overline{N_R^C}$ denote the critical Rayleigh numbers in the absence and presence of compressibility, respectively. Since critical Rayleigh number is positive and finite, so $G > 1$ and we obtain stabilizing effect of compressibility, i.e., its effect is to postpone the onset of thermal convection in fluid-particle layer.

To study the effects of suspended particles, medium permeability, magnetic field and Hall currents, we examine the natures of $\frac{dN_R}{d\overline{H}}, \frac{dN_R}{dP}, \frac{dN_R}{dN_Q}$ and $\frac{dN_R}{dM_1}$, respectively.

From (45), we have

$$\frac{dN_R}{d\overline{H}} = -\left(\frac{G}{G-1}\right)\frac{(\pi^2+\alpha^2)^2}{\alpha^2\overline{H}^2}\left[\frac{1}{P} + \frac{N_Q\pi^2\varepsilon^{-1}\{(\pi^2+\alpha^2)+N_QP\varepsilon^{-1}\pi^2\}}{(\pi^2+\alpha^2)\{M_1^2\pi^2+(\pi^2+\alpha^2)+N_QP\varepsilon^{-1}\pi^2\}}\right], \quad (47)$$

which is negative. The effect of suspended particles is, thus, destabilizing on thermal instability of compressible fluid-particle layer in the presence of Hall currents through porous medium.

It is evident from (45) that

$$\frac{dN_R}{dP} = \left(\frac{G}{G-1}\right)\frac{(\pi^2+\alpha^2)^2}{\alpha^2\overline{H}}\left[-\frac{1}{P^2} + \frac{(N_Q\pi^2\varepsilon^{-1})M_1^2\pi^2}{(\pi^2+\alpha^2)\{M_1{}^2\pi^2+(\pi^2+\alpha^2)+N_QP\varepsilon^{-1}\pi^2\}^2}\right], \quad (48)$$

which is positive if

$$P\left[M_1\pi-(\pi^2+\alpha^2)^{\frac{1}{2}}\right] > \frac{(\pi^2+\alpha^2)^{\frac{1}{2}}[M_1^2\pi^2+(\pi^2+\alpha^2)]}{N_Q\varepsilon^{-1}\pi^2},$$

and is negative if

$$P\left[M_1\pi-(\pi^2+\alpha^2)^{\frac{1}{2}}\right] < \frac{(\pi^2+\alpha^2)^{\frac{1}{2}}[M_1^2\pi^2+(\pi^2+\alpha^2)]}{N_Q\varepsilon^{-1}\pi^2}.$$

Thus, the medium permeability has both stabilizing and destabilizing effects depending upon the values of the various parameters. In the absence of magnetic field, the medium permeability has a destabilizing effect, since for this case

$$\frac{dN_R}{dP} = -\left(\frac{G}{G-1}\right)\frac{(\pi^2+\alpha^2)^2}{\alpha^2\overline{H}P^2}, \quad (49)$$

which is always negative. The medium permeability, thus, succeeds in stabilizing the thermal instability of compressible fluid-particle layer for certain wave numbers in the presence of magnetic field, which was unstable in the absence of magnetic field.

Now from (45), we get

$$\frac{dN_R}{dN_Q} = \left(\frac{G}{G-1}\right)\frac{(\pi^2+\alpha^2)\pi^2\varepsilon^{-1}}{\alpha^2\overline{H}}[M_1^2\pi^2+(\pi^2+\alpha^2)+N_QP\varepsilon^{-1}\pi^2]^{-1}$$

$$\left\lfloor\{(\pi^2+\alpha^2)+N_QP\varepsilon^{-1}\pi^2\}+M_1^2\pi^4N_QP\varepsilon^{-1}\{M_1^2\pi^2+(\pi^2+\alpha^2)+N_QP\varepsilon^{-1}\pi^2\}^{-1}\right\rfloor, \quad (50)$$

which is always positive. This shows that magnetic field has a stabilizing effect on the system.

To find the effect of Hall currents, from (45), we have

$$\frac{dN_R}{dM_1} = -2\left(\frac{G}{G-1}\right)\frac{(\pi^2+\alpha^2)N_Q\varepsilon^{-1}M_1\pi^4\{(\pi^2+\alpha^2)+N_QP\varepsilon^{-1}\pi^2\}}{\alpha^2\overline{H}\{M_1^2\pi^2+(\pi^2+\alpha^2)+N_QP\varepsilon^{-1}\pi^2\}^2}, \tag{51}$$

which is always negative. Therefore, the effect of Hall currents is destabilizing on the thermal convection in compressible fluid-particle layer in porous medium.

5. EXISTENCE OF OSCILLATORY MODES

Multiplying (37) by W^*, the complex conjugate of W, integrating over the range of z and using (38)–(47) together with the boundary conditions (29) and (30), we obtain

$$\left[L_1+\frac{L_2}{P}(1-An)\right]I_1 + L_2N_Q\varepsilon N_{p_2}N_{p_1}^{-1}(nI_2+n^*I_5)+L_2N_Q\varepsilon\,(I_3+I_6)$$

$$+\frac{L_2}{L_2^*}\left[L_1^*+\frac{L_2^*}{P}(1-An^*)\right]I_4 = L_2L_2^*N_R\alpha^2\left(\frac{G}{G-1}\right)\left(\frac{1}{\tau\, n^*+\overline{H}}\right)\{I_7+(E+h\varepsilon)n^*I_8\}, \tag{52}$$

where

$$\left.\begin{aligned}
I_1 &= \int_0^1 (|DW|^2+\alpha^2|W|^2)\,dz, & I_2 &= \int_0^1 |X|^2\,dz,\\
I_3 &= \int_0^1 (|DX|^2+\alpha^2|X|^2)\,dz, & I_4 &= \int_0^1 |Z|^2\,dz,\\
I_5 &= \int_0^1 (|DK|^2+\alpha^2|K|^2)\,dz, & I_6 &= \int_0^1 (|D^2K|^2+2\alpha^2|DK|^2+\alpha^4|K|^2)dz,\\
I_7 &= \int_0^1 (|D\Theta|^2+\alpha^2|\Theta|^2)dz, & I_8 &= \int_0^1 |\Theta|^2\,dz,
\end{aligned}\right\}, \tag{53}$$

which are all positive definite. Putting $n = in_0$, where n_0 is real, in (52) and equating imaginary parts on both sides, we obtain either

$$n_0 = 0, \tag{54}$$

or

$$n_0^2 = -\tau^{-2}\Bigg[\left(N_{p_1}^{-1}\overline{H}F - \frac{\tau}{P} + \frac{\tau}{P}A\right)I_1 + N_Q\varepsilon N_{p_2}N_{p_1}^{-1}\overline{H}(I_2 - I_5)$$

$$-N_Q\varepsilon\tau(I_3 + I_6) - \left(N_{p_1}^{-1}\overline{H}F + \frac{\tau}{P} - \frac{\tau}{P}A\right)I_4 + N_R\alpha^2\left(\frac{G}{G-1}\right)\{\tau I_7 + (E + h\varepsilon)I_8\}\Bigg]$$

$$\Bigg[\left\{N_{p_1}^{-1}(\overline{H} + 1 - F) - \frac{\tau}{P} + \frac{\tau}{P}A\right\}I_1 + N_Q\varepsilon N_{p_2}N_{p_1}^{-1}\overline{H}(I_2 - I_5) - N_Q\varepsilon\tau(I_3 + I_6)$$

$$+\left\{N_{p_1}^{-1}(1 - \overline{H} - F) - \frac{\tau}{P} + \frac{\tau}{P}A\right\}I_4 + N_R\alpha^2\left(\frac{G}{G-1}\right)\{\tau I_7 + (E + h\varepsilon)I_8\}\Bigg]^{-1}. \qquad (55)$$

In the absence of magnetic field

$$n_0{}^2 = \frac{\left[\left(N_{p_1}^{-1}\overline{H}F - \frac{\tau}{P} + \frac{\tau}{P}A\right)I_1 + N_R\alpha^2\left(\frac{G}{G-1}\right)\{\tau I_7 + (E + h\varepsilon\}I_8\right]}{\tau^2\left[\left\{N_{p_1}^{-1}(\overline{H} + 1 - F) - \frac{\tau}{P} + \frac{\tau}{P}A\right\}I_1 + N_R\alpha^2\left(\frac{G}{G-1}\right)\{\tau I_7 + (E + h\varepsilon)I_8\}\right]}. \qquad (56)$$

From (54)–(56) we can draw the following conclusions:

(I) The presence of magnetic field (and therefore Hall currents) and viscoelasticity effects introduce oscillatory modes in the system.

(II) In the absence of magnetic field, the expression for n_0^2 given by (56), is negative if

$$C_{pt} > C_v\left[1 + \frac{\varepsilon m}{f k_1 K d^2}\{\nu d^2 - \nu' k\}\right], \qquad (57)$$

for all positive N_R. Now, since n_0 is real and n_0^2 is negative, therefore, we must have $n_0 = 0$. It is thus established that n is real for positive N_R in the absence of magnetic field if the inequality (57) holds true and that the principle of exchange of stabilities is valid for this case. However, if the inequality (57) is violated, then the oscillatory modes may come into play even in the absence of magnetic field.

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