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CHARACTERISTICS OF THE POLISH STOCK MARKET CORRELATIONS

Summary: In this paper a network structure of the Polish Stock Market (PSM), one of the emerging markets, is studied. The conceptions: Minimum Spanning Tree (MST) and Weighted Random Graph (WRG), constructed among companies listed on this stock exchange, are compared. In these models every vertex is a stock and the weight assigned to each edge in WRG is the cross-correlation coefficient. The Influence-Strength (IS) is defined at every vertex in both models: in WRG as the sum of the weights on the edges upon that vertex, in MST as the vertex degree. The IS distribution follows a power law with exponent $r = 1.6$ in WRG and $\delta = 2.2$ in MST. The both results show that there must be a few stocks whose price fluctuations can powerfully influence the price dynamics of other stocks in the same market. In both cases these are the same companies.

Key words: emerging markets, scale-free network, complex system, random graph, financial correlations.

1. Introduction

Financial markets are well-defined complex systems. They are studied by mathematicians, economists and recently also by physicists. Stock price changes of individual companies are influenced by others. Thus, the cross-correlation coefficients between different companies are one of the most important quantities in understanding the cooperative behavior in stock market. The various economic factors affect fluctuations of the stock prices. It is the main problem to construct a dynamic equation, and predict the evolution of the stock price change in the future. The cross-correlation coefficient is computed between all the possible pairs of stocks present in the portfolio in a given time period. K. Hyun-Joo, L. Youngki, K. In--mook et al. [2002] have researched the stock price behaviors between the Standard and Poor's 500 (S&P 500) companies based on WRG, where all vertices (companies) are fully connected. R.N. Mantegna [1999] investigating the portfolio of the stocks used to compute the Dow Jones Industrial Average (DJIA) index, the portfolio of stocks and the S&P 500 which introduced MST connecting the n stocks of the portfolio. This paper investigates previously unstudied Warsaw Stock Exchange in the time period from 1 January 2007 to 31 December 2007. It analyses only the companies which were present on the Polish Stock Market for the entire investigated period. With this constraint the portfolio is composed of 252 stocks.

2. The stock network models

2.1. The WRG model

In this model, every company listed on the market corresponds to one vertex, and every edge is used to describe the influence relationship among stock price fluctuations. The weight assigned to every edge is the cross-correlation coefficient between two companies.

2.2. The MST model

The MST was introduced in graph theory quite long time ago [Boruvka 1926; Kruskal 1956]. It was rediscovered several times [Papadimitrou, Steigliz 1982; West 1996] later. To analyze the stock market correlations it was applied by Mantegna [1999] and later on by several authors [Bonanno et al. 2003; Bonanno et al. 2004; Bonanno et al. 2001; Onella et al. 2003; Onella et al. 2003a]. To construct the MST graph the following measure is chosen

$$d(i, j) = \sqrt{2\left(1 - w_{i,j}\right)}$$

that satisfies the standard axioms of metric. The distance d between two stocks is smaller if their correlation coefficient ($w_{i,j}$) is closer to unity. One node (vertex) in the graph corresponds to every stock. The two nodes, A and B, are connected with a line (edge, "leg"), if $d(A,B)$ is the smallest. In the next step we are looking for another two closest nodes and again they are connected with a line. This procedure is repeated until a connected graph, which is a tree, is obtained.

3. Definition and distribution of the cross-correlation coefficients

Let $X_i\left(t\right)$ be the stock-price of a company i (i = 1, …, N) at time t. Then the return of the stock-price after a time interval Δt is defined as

$$Y_i(t) = \ln X_i(t) - \ln X_i(t - \Delta t),$$

where Δt = 1 day. The time-dependence of the mean value might be caused by external economic environments such as bank interest, inflation index, exchange rate, etc, which fluctuates from time to time. In order to avoid it, the following quantity is introduced

$$C_i(t) = Y_i(t) - \frac{1}{N}\sum_t Y_i(t) \,,$$

where $C_i(t)$ means the relative return of a company *i* to its mean value over the all 252 companies at time *t*. The cross-correlation coefficients are defined as

$$w_{i,j} = \frac{\langle C_i C_j \rangle - \langle C_i \rangle \langle C_j \rangle}{\sqrt{\left(\langle C_i^2 \rangle - \langle C_i \rangle^2\right)\left(\langle C_j^2 \rangle - \langle C_j \rangle^2\right)}},$$

where the brackets mean a temporal average over the studied period. The cross-correlation coefficient $w_{i,j}$ is the weight assigned to the edge connected between vertices *i* and *j*, and varies from –1 to 1. The distribution of the cross-correlation coefficient is shown in Figure 1.

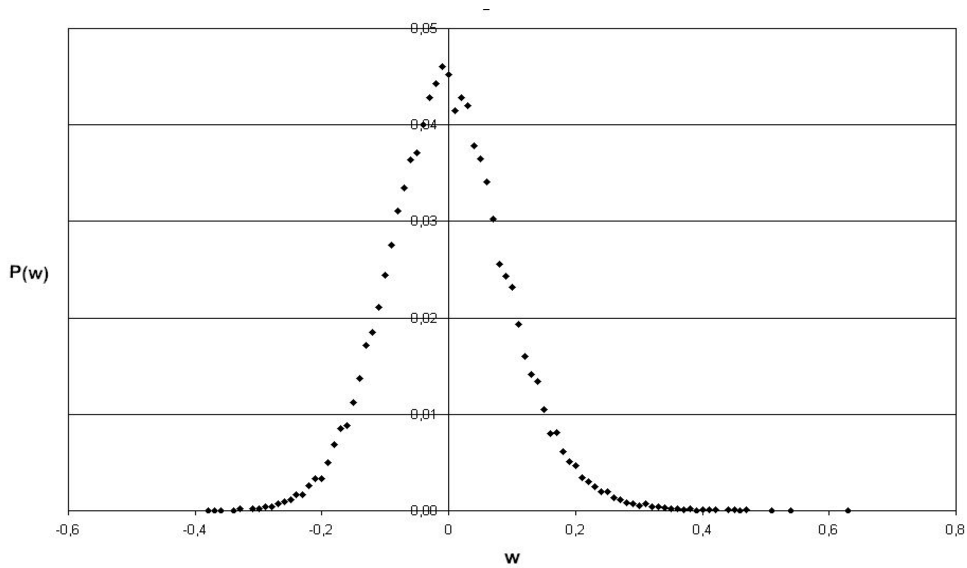

Figure. 1. Distribution of the cross-correlation coefficient

Source: own work.

4. Scale-free behavior of the Influence-Strength (IS)

4.1. WRG

Shares on the market are connected one with another. The price fluctuations of a stock always influence the price behaviors of other stocks in the same market. In order to know how powerfully a given stock influences others, based on the research of K. Hyun-Joo, L. Youngki, K. In-mook et al. [2001], there is defined the IS of a given stock i as

$$q_i = \sum_{j=i} w_{i,j}$$

where $w_{i,j}$ is the cross-correlation coefficient between stocks i and j, q_i is the IS of the stock i to other stocks in the same market. Since the weight $w_{i,j}$ is distributed between [–1, 1], the IS of a certain vertex could be negative. In order to avoid such a situation, the absolute value of IS for every vertex is taken. The probability distribution of IS is drawn in the logarithmic scale (log-log) as in Figure 2.

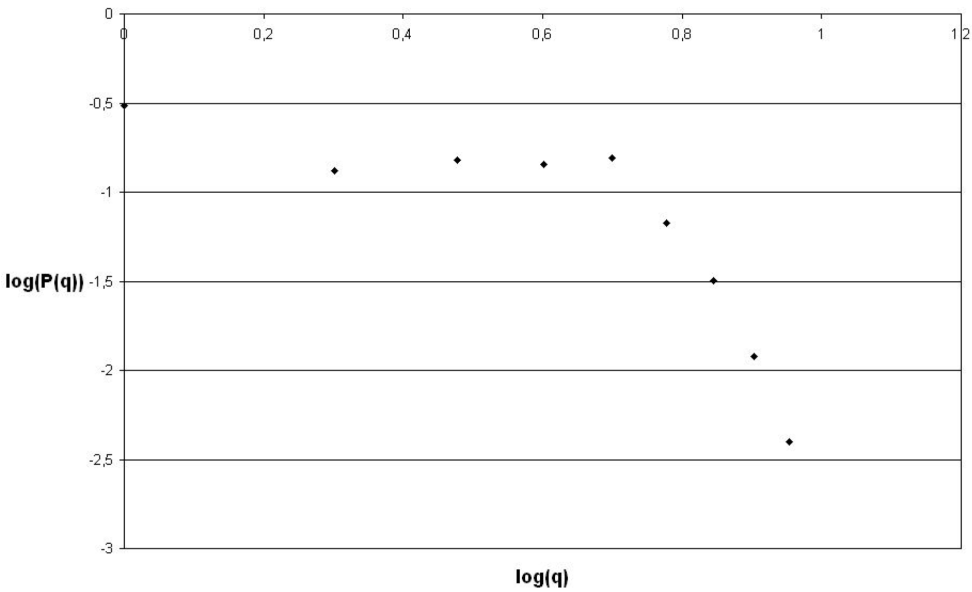

Figure 2. Probability distribution of the IS for WRG in log-log scale

Source: own work.

Using a linear regression equation is easy to calculate that the IS follows a power-law distribution as follows

$$P(q) \sim q^{-r},$$

where the coefficient r = 1.6, that means the exponent of the power-law distribution is 1.6.

4.2. MST

In earlier researches [Bonanno et al. 2003; Bonanno et al. 2004; Bonanno et al. 2001; Mantegna 1999; Onella et al. 2003; Onella et al. 2003a] the MST was used for identifying many groups of companies which were homogeneous from an economic point of view, i.e. belonged to the same economic sector, e.g. energy industry, metal industry, food and drink industry. MST which was obtained from the portfolio of 252 stocks of the Warsaw Stock Exchange is shown in Figure 3.

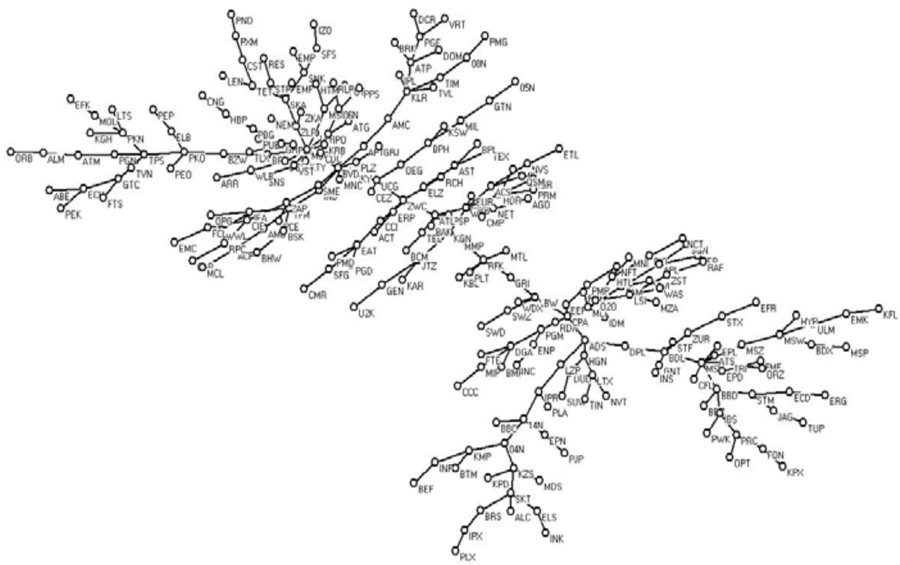

Figure 3. Minimum spanning tree connecting the 252 stocks from the Warsaw Stock Exchange
The 252 stocks are labeled by their tick symbols [Warsaw Stock Exchange…].

Source: own work.

In this paper the IS of a given stock i is introduced as

$$s_i = \deg(v_i),$$

where v_i denotes vertex (stock) degree i in MST. The obtained probability distribution of IS is drawn in the logarithmic scale (log-log) as in Figure 4.

It is easy to notice that the IS follows a power-law distribution as follows

$$P(s) \sim s^{-\delta},$$

where the coefficient δ = 2.2, that means the exponent of the power-law distribution is 2.2.

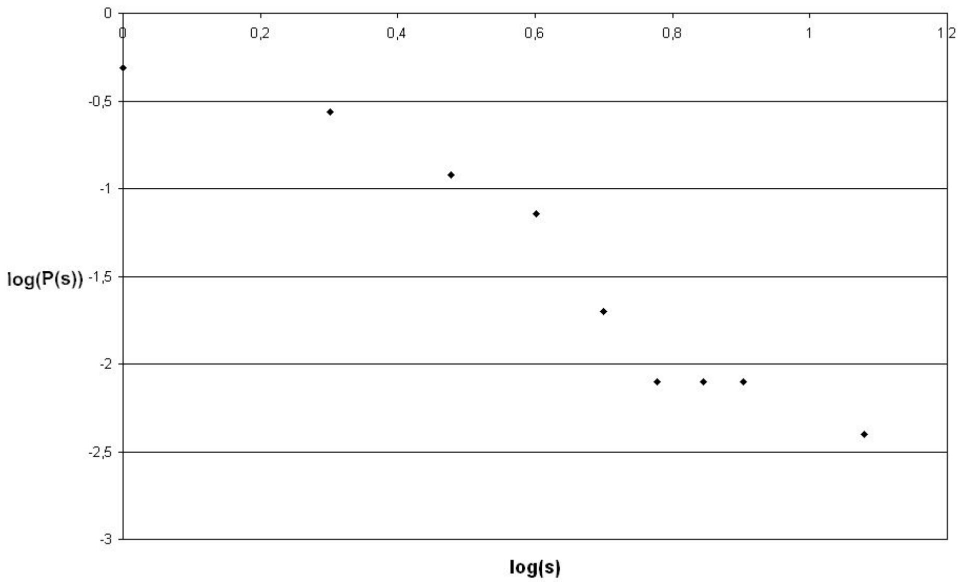

Figure. 4. Probability distribution of the IS for MST in log-log scale

Source: own work.

5. WRG and MST – comparing models

IS, both in the MST model as well as in the WRG model, follows power-law distribution. This property reveals that most of the vertices of the WRG and MST have a small IS index, and at the same time there are also a few vertices which IS indices are very large. This implies that in the PSM there are a few companies whose stocks' price fluctuations can powerfully influence the price dynamics of other stocks in the market. It is obvious that if a powerful vertex which IS index is very large has strong vibrations in its price changes, there will be a series of great fluctuations in other stocks' price dynamics in the PSM. The rich-get-richer phenomenon appears in both models very strongly.

It is observed that companies having the highest IS in the MST model also in the WRG model have the greatest IS value among all companies. In other words, the strength of every company is comparable in both models. This observation can be very important from the point of view of stock exchange investors worldwide, because with a very high probability it is possible to assess the stock market behavior tracing only a few strongest results according to the MST and WRG models of companies. On the other hand, many companies which have low IS can be omitted from a point of view of a stock market behavior. They have no influence on its fluctuations.

In this paper the spectral property of matrix $W = \left(w_{i,j}\right)$, the matrix of the cross-correlation coefficients, is also studied, based on Random Matrix Theory (RMT) [Laloux et al. 1999; Plerou et al. 1999]. Figure 5 shows the probability distribution of the eigenvalues of the matrix *W*.

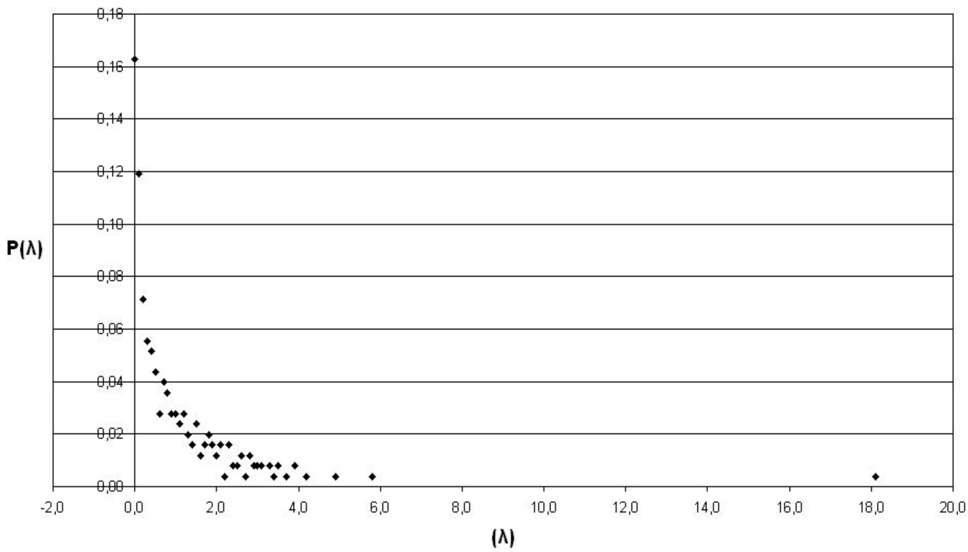

Figure 5. Distribution of the eigenvalues of the cross-correlation coefficient matrix

Source: own work.

It is easy to notice that the most of the eigenvalues of the matrix are small, though some eigenvalues are absolutely large. This indicates again that in a stock market there exist a few companies having strong influence on the price dynamics of other stocks in the same market.

6. Conclusion

This paper deals with the cross correlations in stock price changes among 252 companies in PSM by presenting the WRG model and the MST model. The cross correlation coefficients $w_{i,j}$ describe generic correlations among different companies. Next the IS is defined in both models, which probability distribution follows a power-law in both cases: in WRG with exponent $r = 1.6$ and in MST with exponent $\delta = 2.2$. This implies that in a stock market there must be a few companies which stocks' price fluctuations can powerfully influence the price dynamics of other stocks in the same market. This property is also confirmed by the distribution of the eigenvalues of the cross-correlation coefficient matrix. Next, these two models are compared. IS in both models is analysed. The result of this analysis is the statement that IS of every company is in both models comparable. This interesting property can be very useful not only for professionals, but also for an amateur player on the stock market. However, a question concerning the assumptions of applying these models still remains. The credibility of results depends on the data set size (in the studied case on the quantity of listed companies on the market). It can be exactly said that the credibility of results grows with the data set size. A possibility of predicting the behavior of the stock exchange on the basis of historical data is a major advantage of models presented by the author of the article. Since the mentioned models are the forecasting models some deviations from expected results can happen what can be perceived by some investors as a defect in such models.

Literature

Bonanno G., Caldarelli G., Lillo F., Mantegna R.N., *Topology of correlation-based minimal spanning trees in real and model markets*, ”Physical Review E” 2003, no. 68, p. 130-133.

Bonanno G., Caldarelli G., Lillo F., Micciche S., Vandewalle N., Mantegna R.N., *Networks of equities in financial markets*, “European Physical Journal B” 2004, no. 38, p. 363-371.

Bonanno G., Lillo F., Mantegna R.N., *High-frequency cross- correlation in a set of stocks*, “Quantitive Finance” 2001, p. 96-104.

Boruvka O., *O jistém problému minimálním (About a certain minimal problem),* Práce mor. přírodověd. spol. v Brně, 1926, no. 3, p. 37-58.

Hyun-Joo K., Youngki L., In-mook K. et al., *Weighted scale-free network in financial correlations*, “Journal of the Physical Society of Japan” 2002, no. 72, p. 2133-2136.

Kruskal J., *On the shortest spanning subtree of a graph and the traveling salesman problem,* “Proceedings Of the American Mathematical Society” 1956, no. 7, p. 48-50.

Laloux L., Cizeau P., Bouchaud J.P., Potters M., *Noise dressing of financial correlation matrices,* “Physical Review Letters” 1999, no. 83, p. 1467.

Mantegna R.N., *Hierarchical structure in financial markets*, “European Physical Journal B” 1999, no. 11, p. 193-197.

Onella J.P., Chakraborti A., Kaski K., Kertesz J., *Dynamic asset trees and Black Monday*, “Physica A” 2003, no. 324, p. 247-252.

Onella J.P., Chakraborti A., Kaski K., Kertesz J., Kanto A., *Dynamics of market correlations: Taxonomy and portofolio analysis*, “Physical Review E” 2003a, no. 68, 056110-1.

Papadimitrou C.H., Steigliz K., *Combinatorial Optimization*, Prentice-Hall, Englewood Cliffs 1982.

Plerou V., Gopikrishnan P., Rosenow B., Amaral L.A.N., Stanleyl H.E., *Scaling of the distributions of fluctuations of financial market indices*, “Physical Review Letters” 1999, no. 83, p. 1471-1475.

Warsaw Stock Exchange Website, http://www.gpw.pl.

West D.B., *Introduction to Graph Theory*, Prentice-Hall, Englewood Cliffs 1996.

CHARAKTERYSTYKA KORELACJI NA POLSKIM RYNKU AKCJI

Streszczenie: W niniejszej pracy analizowana jest struktura Giełdy Papierów Wartościowych w Warszawie. Porównywane są koncepcje Minimalnego Drzewa Rozpinającego (MST) i Wagowego Grafu Losowego (WRG), które zostały skonstruowane ze spółek notowanych na giełdzie. W tych modelach każdy wierzchołek oznacza akcję i waga przypisana do każdej krawędzi w WRG jest współczynnikiem korelacji. Siła wpływu (IS) jest w każdym wierzchołku w obu modelach zdefiniowana: w WRG jako suma wag krawędzi incydentnych do wierzchołka, w MST jako stopień wierzchołka. Rozkład IS jest potęgowy z wykładnikiem $r = 1{,}6$ w WRG i $\delta = 2{,}2$ w MST. Oba rezultaty pokazują, że na GPW musi być parę spółek, których wahania cenowe mogą silnie wpłynąć na dynamikę cen innych akcji. W obu przypadkach są to te same spółki.