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DISTRIBUTIVE JUSTICE AND SCHUMPETERIAN INNOVATIVE EVOLUTION – AN AXIOMATIC APPROACH IN THE CONTEXT OF SOCIAL COHESION

Abstract

The aim of the paper is to examine axiomatically the rules of distributive justice as a component of the broad idea of social cohesion in the context of the Schumpeterian evolution.

The methodology of studying Schumpeterian evolution in the dynamic axiomatic Arrow–Debreu set-up is now extended in the two new directions, neglected before. Firstly, the relationship between the circular flow and economic development, which did not find any satisfactory solution in Schumpeter's work, is studied. Secondly, the rules of distributive justice can be viewed as the components of a broad idea of social cohesion as defined by the Council of Europe, so that some aspects of it can be analyzed more rigorously. In particular, it turns out that the innovative Schumpeterian evolution based on the principle of creative destruction does not ruin the social structure built on the Rawlsian rules of distributive justice. Consequently, an evolutionary process of sustaining and enhancing l-justice relation as well as of improving the social cohesion in this aspect is constructed.

1. Introduction

The aim of the paper is to examine axiomatically the rules of distributive justice as a component of the broad idea of social cohesion in the context of the Schumpeterian evolution.

The methodology of studying Schumpeterian evolution in the dynamic axiomatic Arrow–Debreu set-up suggested in [8] and developed later in [6; 7; 9], is based on modelling the two fundamental forms of economic life distinguished by Schumpeter [13], i.e. the circular flow and economic development as specific extensions of the production system being a part of the Debreu [4] economy with a private ownership, so that the analysis takes static as well as dynamic forms. In this framework some problems of poverty, freedom of choice and distributive justice have also been elaborated [10]. In particular, it has been shown that the rules of circular flow preserve and/or improve the positions of all individuals in society structured by the Rawlsian principles of justice. This research program will be now extended in the two new directions, neglected before.

Firstly, the relationship between the circular flow and economic development, which did not find any satisfactory solution in Schumpeter's work, is studied. In particular, the sufficient conditions to guarantee the preservation of the rules of circular flow of the consumption sphere within the innovative development of the whole economy are specified. It enables us to reconsider the Rawlsian difference principle and its generalization to the leximin scheme as a relation of *l*-justice [5; 11] in this framework.

Secondly, the rules of distributive justice can be viewed as the components of a broad idea of social cohesion as defined by the Council of Europe [3], so that some aspects of it can be analysed more rigorously. In particular, it turns out that the innovative Schumpeterian evolution based on the principle of creative destruction does not ruin the social structure built on the Rawlsian rules of distributive justice. Consequently, an evolutionary process of sustaining and enhancing *l*-justice relation as well as of improving the social cohesion in this aspect is constructed.

The paper is organized as follows. The section below is focused on the key categories of Schumpeterian innovative evolution, distributive justice and social cohesion, which are briefly outlined. The third section presents the formal model of a Debreu economy as a state of the Schumpeterian innovative evolution. The last section deals with the analysis of the rules of distributive justice in the context mentioned above, which leads to the conclusion that they are not violated there, as the components of social cohesion.

2. Innovative evolution – distributive justice – social cohesion

2.1. Schumpeterian innovative evolution

Let us recall now that at the core of the Schumpeterian economic thinking there lie distinction and analysis of two forms of economic activity [13]: circular flow and economic development. The former, referring to the periodicity of economic life is based on the Walrasian general equilibrium model and interpreted as comparative statics within which an economic system tends to an equilibrium defining commodities prices and quantities. In this approach, economic life goes smoothly along the beaten tracks keeping the governing rules unchanged year after year. Indeed, Schumpeter wrote [13, p. 108]: "In the circular flow, from which we always start, the same products are produced every year in the same way. For every supply there waits somewhere in the economic system a corresponding demand, for every demand the corresponding supply. All goods are dealt in at determined prices with only insignificant oscillations, so that every unit of money may be considered as going the same way in every period. ... There is no market for the bearers of the original productive services themselves, ..., and there is also no price for them within the normal circular flow". Consequently, the process reminds us (cf. [13, p. 61]) of the blood circulation in a living organism and can be interpreted as a stagnation of the economic life.

On the other hand, economic life goes beyond these narrow patterns of continuous evolution, taking shape of a development described by Schumpeter in the following way [13, p. 64]: "Development in our sense is a distinct phenomenon, entirely foreign to what may be observed in the circular flow or in the tendency towards equilibrium. It is spontaneous and discontinuous change in the channels of the flow, disturbance of equilibrium, which forever alters and displaces the equilibrium state previously existing".

The driving force and the key to the understanding of the phenomenon of economic development is the creative destruction rule. The rule is the synthesis of two opposite tendencies present in the economic life development: creative innovations and elimination of existing, old products and organizational structures by new ones. Indeed, Schumpeter [13, pp. 82-83] wrote: "The fundamental impulse that acts and keeps the capitalistic engine in motion comes from the new consumers' goods, the new methods of production, the new forms of industrial organization that capitalist enterprise creates. ... The opening up of new markets, foreign or domestic, and the organizational development ... illustrate the same process of industrial mutation – if I may use that biological term – that revolutionizes the economic structure from within, incessantly destroying the old one, incessantly creating a new one".

Such an understanding of economic development is full of dynamics and discontinuity, it continuously destructs the existing old economic structures, substituting them with new ones all the time. It has to be remembered, though, that while the fundamental force explaining the rules of circular flow are consumers' needs and preferences, economic development is initiated by producers who become innovators and entrepreneurs (cf. [13, p. 65]).

2.2. Distributive justice

The Rawlsian conception of distributive justice is based on the idea that its rules can be established by the representative members of society acting in the *original position* behind a *veil of ignorance*. To use Rawls's words, it means that [11, p. 7] "The guiding idea is that the principles of justice for the basic structure of society are the object of the original agreement. They are the principles that free and rational persons concerned to further their own interests would accept in an initial position of equality as defining the fundamental terms of their association. These principles are to regulate all further agreements; they specify the kinds of social cooperation that can be entered and the forms of government that can be established. This way of regarding the principles of justice I shall call justice as fairness".

In formulating such a contract as the result of a thought experiment, the representative members of society abstract from its social structure, institutions, positions they hold, generation they belong to, being not even born. What is more, the principles of justice are constructed within the space of primary goods. These are defined to be [11, p. 62] "things that every rational man is presumed to want. These goods normally have a use whatever a person's rational plan of life. For simplicity, assume that the chief primary goods at the disposition of society are rights and liberties, powers and opportunities, income and wealth. (Later on … the primary good of self-respect has a central place). These are the primary social goods. Other primary goods such as health and vigor, intelligence and imagination, are natural goods …".

Those rules in question are set-up in a lexicographical ordering which starts with the first principle defined as follows [11, p. 302]: "Each person is to have an equal right to the most extensive total system of equal basic liberties compatible with a similar system of liberty for all".

However, from the perspective of economic science the second rule referred to as the difference principle seems to be of more importance. It says that [11, p. 302]: "Social and economic inequalities are to be arranged so that they are both:

(a) to the greatest benefit of the least advantaged, consistent with the just savings principle, and

(b) attached to offices and positions open to all under conditions of fair equality of opportunity".

In this context three comments seem to be necessary. Firstly, the *just savings principle* refers to the *original position*, where no one representative member of society knows to which generation will be included. Thus all persons will want to contract to prevent one generation from squandering resources to the detriment of future generations. Secondly, it is widely discussed (cf. [21]) how to select the least advantaged members of society. On the one hand, as Rawls argues, in the original position members of society have limited knowledge of future events. Thus they have to make decisions under uncertainty, and being rational agents, they would adopt what in the decision theory is called a *least worst* or *maximin* rule. On the other hand, in our axiomatic Arrow–Debreu framework, we will restrict employing the maximum rule, applied by Rawls to all primary goods, to these of economic significance, i.e. ones referring to wealth and income. Specifically, *via* the Sen's capability approach to economic inequality, poverty and freedom of choice [16; 17] we will focus on comparing the budget sets of consumers, which are interpreted as opportunity sets (cf., e.g., [20]) being components of the Debreu economy with a private ownership. Thirdly, the Rawlsian rules of distributive justice can be viewed as a component of the comprehensive idea of social cohesion sketched below.

2.3. Social cohesion

Social cohesion is a meaningful concept which can be defined in many various ways (cf. [3, p. 23]). These approaches “vary according to period, culture and prevailing political ideas and differ from one another mainly in terms of the role of the players involved, the areas of life or groups concerned and, finally, the methods they employ to foster this cohesion”.

In this context we want to focus on a benchmark definition of social cohesion suggested by the Council of Europe. According to it, the social cohesion of a modern society is understood as [3, p. 23] “society’s ability to secure the long-term well-being of all its members, including equitable access to available resources, respect to human dignity with due regard for diversity, personal and collective autonomy and responsible participation”.

It is also worth emphasizing that social cohesion is not interpreted here as a “natural” phenomenon in modern societies but [3, p. 27] “results from ‘inter-relations’ between free individuals and private and public institutions within a framework of standards and laws recognized as legitimate by community”.

Moreover, it is easy to see that the components of the definition presented above lead us to the following research problems:

1) guaranteeing of long-term well-being requires a dynamic analysis,

2) equitable access to available resources can be studied within the Rawlsian idea of distributive justice,

3) autonomies in question and responsible participation require some specific conditions and postulates to be preserved.

What is more, we share the opinion that [3, p. 23] “the changes that continually occur in such an open and plural society must, in turn, be the subject of in-depth analysis to ensure they do not lead to ‘cohesion crises’ or, indeed, forms of cohesion that are weaker because they are based on exclusion rather than the consensual inclusion of individual and groups”. In our framework this danger will be referred to the principle of creative destruction to govern the Schumpeterian innovative evolution.

3. The model

3.1. The static analysis

A dynamical system approach to the Arrow–Debreu theory of general equilibrium [6; 8] enables us to study the model of a private ownership economy as described in [4] in the form of a multi-range relational system which includes, in combination, the production system and the consumption one.

The production system is represented by a two-range relational system $P = (B, \mathbf{R}^l, y, p, \eta, \pi)$, where: $B = \{b_1, \ldots, b_n\}$ is a finite set of the producers, $\mathbf{R}^l$ is a commodity/price space, $y \subset B \times P_0(\mathbf{R}^l)$ is a correspondence of production sets, $p \in \mathbf{R}^l$ is a price system, $\eta \subset B \times P_0(\mathbf{R}^l)$ is a correspondence of supply, $\pi: B \to \mathbf{R}$ is a maximal profit function. In short, the production system will be denoted: $P = (B, \mathbf{R}^l; Ch_p)$, where $Ch_p = (y, p, \eta, \pi)$ is the characteristic of the system P.

Similarly, the consumption system is represented by a three-range relational system $C = (A, \mathbf{R}^l, \mathrm{P}; x, e, \varepsilon, p, \beta, \varphi)$, where $A = \{a_1, \ldots, a_m\}$ is a finite set of the consumers, $\mathbf{R}^l$ is a commodity/price space, $\mathrm{P} \subset P(\mathbf{R}^{2l})$ is the class of all preference relations defined on the commodity space $\mathbf{R}^l$, $x \subset A \times P_0(\mathbf{R}^l)$ is a correspondence of consumption sets, $e \subset A \times \mathbf{R}^l$ is an initial endowment mapping, $\varepsilon \subset A \times P(\mathbf{R}^{2l})$ is a correspondence which assigns to every consumer $a \in A$ a preference relation $\preccurlyeq_a \in \mathrm{P}$, restricted to the consumption set $x(a)$, $p \in \mathbf{R}^l$ is a price system, $\beta \subset A \times P_0(\mathbf{R}^l)$ is a correspondence of budget sets, $\varphi \subset A \times P_0(\mathbf{R}^l)$ is a demand correspondence.

A private ownership economy E_p is a combination of a production system P and a consumption system C such that the consumers share in the producers' profits (the shares are measured by a mapping θ), and some fixed (initial) total resource ω of the economy E_p is the consumers' property. It means that a budget set correspondence $\beta \subset A \times P_0(\mathbf{R}^l)$ is modified for the economy E_p, so that the expenditures of a-th consumer do not exceed the value $w_a = pe(a) + \sum_{b \in B} \theta_{ab}\pi_b(p)$. Then $\beta(a) := \beta_{(p,e(a))}(a) := \{x \in x(a) : px \le w_a\}$.

Thus the private ownership economy E_p can be concisely described in the form: $E_p = (\mathbf{R}^l, P, C, \theta, \omega)$, where $\mathbf{R}^l$ is a commodity price/space, P is a production system, C is a consumption system, $\theta \subset (A \times B) \times \mathbf{R}_+$ is a function describing the consumers' shares in the producers' profits, i.e., for every $(a, b) \in A \times B$ the number $\theta_{ab} := \theta(a,b) \in [0,1]$ measures the consumer a's share in the producer b's profit and there is, for every $b \in B$, $\sum_{a \in A} \theta_{ab} = 1$, $\omega := \sum_{a \in A} e(a)$, so that $\omega \in \mathbf{R}^l$.

Based on the above premises, it was projected [6; 8] to build a homogeneous general mathematical model coherently describing the circular flow and economic development. However, they turned out to be there mutually incomparable like in the original Schumpeter's work. In this context, the previous definitions of a cumulative and an innovative extension have been modified [2] so that these concepts, being comparable now, can model the rules of the circular flow and economic development respectively. They will be employed below in this latter form.

So, let two production systems: $P = (B, \mathbf{R}^l, Ch_P)$ and $P' = (B', \mathbf{R}^{l'}, Ch_{P'})$ be given. To compare their characteristics we employ the formal concept of projection of the respective elements of "new" characteristic $Ch_{P'}$ on the "old" commodity/price space $\mathbf{R}^l$, as follows.

Definition 3.1. A production system $P' = (B', \mathbf{R}^{l'}, Ch_{P'})$ is a cumulative extension of a production system $P = (B, \mathbf{R}^l, Ch_P)$, shortly $P \subset_c P'$, if:

(1) $l \leq l'$ and $B \subset B'$,
(2) $p \leq \text{proj } p' / \mathbf{R}^l$,
(3) $\forall\, b \in B$,
(3.1) $Y_b \subset \text{proj } Y_b' / \mathbf{R}^l$,
(3.2) $\eta_b(p) \subset \text{proj } \eta_b'(p') / \mathbf{R}^l$,
(3.3) $\pi_b(p) \leq \pi_b'(p')$.

According to the definition, in a cumulative extension of a production system new firms or commodities may appear, which has been ruled out in its previous version [6-8]. Then, in particular, when $l = l'$, $B = B'$, and consequently, the respective projections are identity mappings, no new firms or commodities appear, nor are the old ones eliminated from the production process. Then all old technologies are still being used (Condition (3.1)) and, with non-decreasing prices (Condition (2)), the firms' optimal production plans remain optimum (Condition (3.2)) and do not generate less profit (Condition (3.3)). All this means that the "old" core of the production sector is preserved implying that the idea of a cumulative extension can be interpreted as one modelling the circular flow of the production sphere as it follows the rules ascribed by J.A. Schumpeter to precisely this sphere of economic life. It is also evident that the producers play a purely passive, managerial role adopting actions to the given prices and technologies.

Let us now consider a different form of a production system extension called an innovative extension. So, let two production systems, $P = (B, \mathbf{R}^l, Ch_P)$ and $P' = (B', \mathbf{R}^{l'}, Ch_{P'})$ be given.

Definition 3.2. A production system $P' = (B', \mathbf{R}^{l'}, Ch_{P'})$ is an innovative extension of a production system $P = (B, \mathbf{R}^l, Ch_P)$, shortly $P \subset_i P'$, if:
(1) $l \leq l'$,
(2) $\exists\, b' \in B'\ \forall\, b \in B$,
(2.1) $\text{proj } Y_{b'}' / \mathbf{R}^l \not\subset Y_b$,
(2.2) $\text{proj } p' / \mathbf{R}^l = p$,
(2.3) $\text{proj } \eta_{b'}'(p') / \mathbf{R}^l \not\subset \eta_b(p)$,
(2.4) $\pi_b(p) < \pi_{b'}'(p')$.

According to the definition, there may appear in the production system P' at least one new product or commodity, which can be interpreted as a better way of meeting the needs present earlier in the system P. At the same time, the definition does not specify whether new products are brought out by brand new firms or by the ones already existing but modernized. All that is demanded here is that in the production system P' there is at least one producer b' whose technological abilities go beyond the abilities of all producers acting within the production system P. Hence, the optimal (i.e., maximizing the profit) production plans of the producer b' cannot be reduced to the analogous plans being implemented by the producers in the production system P. Moreover, although the prices of "old" products do not change, the given producer's maximum profit is greater than the one that any of the producers in the system P can make. Thus, according to the Schumpeterian terminology, the producer b' can be treated as an innovator and entrepreneur who makes the profit resulting from his or her initiative and who should be given a leading role in the production system P'. Let us recall Schumpeter's words [13, p. 134]: "The innovation is hazardous, impossible for most producers. But if someone establishes a business ..., and everything goes well, then he can produce a unit of product more cheaply, while at first the existing prices substantially continue to exist. He then makes a profit. ... Again he is an entrepreneur, his profit entrepreneurial profit. And again the latter, and also the entrepreneurial function as such, perish in the vortex of the competition which streams after them".

It is easy to see that in the previous version, i.e., in particular, when $l < l'$, Definition 3.2 covers at least four cases of five internal changes characterized by Schumpeter as development [13, p. 66]:

1) the introduction of a new good – Condition (1),
2) the introduction of a new method of production – Condition (2.1),
3) the opening of a new market – Condition (1),
4) the carrying out of the new organization of any industry – Condition (2) as a whole.

The above may also imply eliminating less efficient technologies as well as less attractive goods what is the core of the Schumpeterian principle of creative destruction.

We also assume that the extensions defined above are called:

1) weak, if $l = l'$, in short: $P \subset_{cw} P'$, $P \subset_{iw} P'$ respectively,
2) technological, if $B = B'$, in short: $P \subset_{ct} P'$, $P \subset_{it} P'$ respectively.

Consequently, the combinations of these cases are denoted by $P \subset_{cwt} P'$, $P \subset_{iwt} P'$. As mentioned before, for $l = l'$ the projections used in Definitions 3.1 and 3.2 can be reduced to identity mappings. For $l < l'$ in Definition 3.2 we get its previous version (cf. [6-8]).

It seems to be clear that changes in the production sphere defined above determine changes in the private ownership economy E_p. This fact is described by J. Schumpeter in the following way [13, p. 65]: "It is, however, the producer who as a rule initiates economic change, and consumers are educated by him if necessary". This leads us to the definitions of an innovative extension of a whole economic system E_p as well as cumulative one of a consumption system.

Definition 3.3. An economic system $E_p' = (\mathbf{R}^{l'}, P', C', \theta', \omega')$ is called an innovative extension of a system $E_p = (\mathbf{R}^l, P, C, \theta, \omega)$, shortly $E_p \subset_i$ Ep$'$, if $P \subset_i P'$.

The weak and technological versions of this extension are defined as for the production system, and denoted: $E_p \subset_{iw} E_p'$, $E_p \subset_{it} E_p'$.

Now, it is worth emphasizing that innovative changes in the production system do not exclude the possibility of cumulative changes for producers who are not innovators. In this context let us now define a specific kind of innovative extension in which non-innovators preserve the rules of a cumulative extension. Let us denote a set of innovators by $\hat{B}$.

Definition 3.4. An innovative extension of a production system such that $B \subset B'$ preserves a cumulative extension in a set of non-innovators $B - \hat{B}$ (shortly, $P \subset_{ic} P'$), if:

(1) $P \subset_i P'$,

(2) $\hat{P} \subset_c \hat{P}'$, where $\hat{P} = (B - \hat{B}, \mathbf{R}^l, Ch_{\hat{P}})$, $\hat{P}' = (B' - \hat{B}', \mathbf{R}^{l'}, Ch_{\hat{P}'})$.

The idea of a cumulative extension can be related to the consumption sector of an economy as well as to an economy as a whole.

Definition 3.5. A consumption system $C' = (A', \mathbf{R}^{l'}, \mathrm{P}', Ch_{C'})$ is a cumulative extension of a consumption system $C = (A, \mathbf{R}^l, \mathrm{P}, Ch_C)$, in short: $C \subset_c C'$, if:

(1) $A \subset A'$ and $l \leq l'$,

(2) $p \leq \text{proj}\, p' / \mathbf{R}^l$,

(3) $\forall\, a \in A$:

(3.1) $X_a \subset \text{proj}\, X_a' / \mathbf{R}^l$ such that $(x_{1a}, x_{2a}, \ldots, x_{la}, 0, \ldots, 0) \in X_a'$ for each $(x_{1a}, x_{2a}, \ldots, x_{la}) \in X_a$,

(3.2) $e_a \leq \text{proj}\, e_a' / \mathbf{R}^l$,

(3.3) $\varepsilon_a \subset \text{proj}\varepsilon_a' / \mathbf{R}^l \Leftrightarrow\ \preccurlyeq_{a'} \big|_{X_a^2} = \preccurlyeq_a$,

(3.4) $\beta_a(p, e_a) \subset \text{proj}\, \beta_a'(p', e_a') / \mathbf{R}^l$,

(3.5) $\varphi_a(\varepsilon_a, p, e_a) \subset \text{proj}\varphi_a'(\varepsilon_a', p', e_a') / \mathbf{R}^l \Leftrightarrow \forall\ x_a^* \in \varphi_a(\varepsilon_a, p, e_a)\ \forall\ x_a^{*'} \in \varphi_a'(\varepsilon_a', p', e_a')$

$x_a^* \preccurlyeq_a \text{proj}\, x_a^{*'} / \mathbf{R}^l$.

According to the definition, the psycho-physical structure of individuals $a \in A$ does not grow worse and each consumer is able to ignore new goods (Condition (3.1)), the initial resources and prices do not decrease (Conditions (3.2) and (2)), the budget constraints of individuals are relaxed (Condition (3.4)), and their wants are satisfied at least at the same level of utility (Condition (3.5)). Consequently, each consumer is not worse off in transition from a consumption system C to its cumulative extension C', which implies that Pareto-optimal states of an economy E_p do not get worse in the process examined.

Moreover, the cumulative extensions of production and consumption systems define such extension of a Debreu economy E_p, in short $E_p \subset_c E_p'$.

Now it is possible to formulate the sufficient conditions to guarantee the cumulative extension of a consumption system in terms of innovativeness [2].

Theorem 3.1. If an economic system $E_p' = (A, B, \mathbf{R}^{l'}, \mathrm{P}, Ch_{P'}, Ch_{C'}, \theta, \omega')$ is a technological innovative extension of a system $E_p = (A, B, \mathbf{R}^{l}, \mathrm{P'}, Ch_P, Ch_C, \theta, \omega)$, an innovative extension of a production system preserves a cumulative extension in a set of producers $B - \hat{B}$ *and* $\forall\, a \in A$

(1) $X_a \subset \mathrm{proj}\, X_a' / \mathbf{R}^l$,

(2) $e_a \leq \mathrm{proj}\, e_a' / \mathbf{R}^l$,

(3) $\varepsilon_a \subset \mathrm{proj}\varepsilon_a' / \mathbf{R}^l \Leftrightarrow \preccurlyeq_a' \big|X_a^2 = \preccurlyeq_a$,

(4) $\sum_{k=l+1}^{l'} p_k e_{ak} \geq 0$,

then $C \subset_c C'$.

Proof: see [2].

3.2. The dynamic analysis

A dynamic analysis of the economic systems P, C and E_p defined above is now made possible by the mathematical idea of a (quasi)-semidynamical system (cf. [19]). In this meaning, a correspondence $f : X \times \mathbf{R}_+ \rightarrow \mathrm{P}_0(X)$ is a (quasi)--semidynamical system if:

(1) $f(x, 0) = \{x\}$ for every $x \in X$,

(2) $f(f(x, t_1),t_2) = f(x, t_1+t_2)$ for every $x \in X$ and $t_1, t_2 \in \mathbf{R}_+$,

where for any $A \subset X$ and $K \subset \mathbf{R}_+$ the set $f(A, K)$ is defined by $f(A, K) = \bigcup_{x\in \mathrm{A}, t\in \mathrm{K}} \mathrm{f}(x,t)$.

The symbol $\mathrm{P}_0(X)$ stands for the family of all compact subsets of the space X.

To apply the above, let us shortly denote a space **P** of all production systems such that both ranges B, $\mathbf{R}^l$ and characteristic Ch_P may change, as follows: $\mathbf{P} := \{P : P = (B, \mathbf{R}^l, Ch_p)$ where $Ch_p = (y, p, \eta, \pi)$ is the characteristic of the production system P, $l \in \mathbf{N}\}$.

Definition 3.6.

(1) A quasi-semidynamical production system f_P: $\mathbf{P} \times \mathbf{R}_+ \rightarrow \mathrm{P}_0(\mathbf{P})$ is called:

– single-valued if every value of f_P is a one-element set.

(2) A single-valued quasi-semidynamical production system is called:

– cumulative if $t_1 < t_2 \Rightarrow f_P(P, t_1) \subset_c f_P(P, t_2)$;

– innovative if $t_1 < t_2 \Rightarrow f_P(P, t_1) \subset_i f_P(P, t_2)$;

– innovative preserving a cumulative extension *if* $t_1 < t_2 \Rightarrow f_P(P, t_1) \subset_{ic} f_P(P, t_2)$.

Similarly, let us shortly denote a space **C** of all consumption systems, as follows:

$\mathbf{C} := \mathbf{C}(A, \mathbf{R}^l, \mathrm{P}) := \{C : C = (A, \mathbf{R}^l, \mathrm{P}, Ch_C)$, where $Ch_C = (x, e, \varepsilon, p, \beta, \varphi)$, is the characteristic of the consumption system C, $l \in \mathbf{N}\}$.

Definition 3.7. A single-valued quasi-semidynamical consumption system f_C: $\mathbf{C} \times \mathbf{R}_+ \to \mathrm{P}_0(\mathbf{C})$ is called cumulative if $t_1 < t_2 \Rightarrow f_C(C, t_1) \subset_c f_C(C, t_2)$.

In the same way we may denote the space $\mathbf{E_p}$ of all Debreu economies:

$\mathbf{E_p} := \{E_p : E_p = (B, A, \mathbf{R}^l, \mathrm{P}, \theta, Ch_{E_p})$ where $Ch_{E_p} = (y, p, \eta, \pi, x, e, \varepsilon, \beta, \varphi)$ is a Debreu economy characteristic, $l \in \mathbf{N}\}$.

Definition 3.8. A single-valued quasi-semidynamical economic system f_{Ep}: $\mathbf{E_p} \times \mathbf{R}_+ \to \mathrm{P}_0(\mathbf{E_p})$ $t_1 < t_2 \Rightarrow f_{Ep}(E_p, t_1) \subset_i f_{Ep}(E_p, t_2)$.

The further analysis of a (quasi)-semidynamical economic system f_{Ep} is based on the general premise that it is possible to decompose it into component systems in the following way:

$f_{Ep} = (f_P, f_C, f_\theta, f_\omega)$, where $f_P = (f_B, f_{\mathrm{R}^l}, f_{Ch_P})$, $f_C = (f_A, f_P, f_{Ch_C})$,

$f_{E_p} = (f_A, f_B, f_{\mathrm{R}^l}, f_P, f_{Ch_P}, f_{Ch_C}, f_\theta, f_\omega)$,

where $f_{Ch_P} = (f_y, f_p, f_\eta, f_\pi)$, $f_{Ch_C} = (f_x, f_e, f_\varepsilon, f_p, f_\beta, f_\varphi)$.

In particular, the component dynamics are called cumulative if they are involved as components in the cumulative dynamics of the respective system as a whole.

It is now easily seen that the (quasi)-semidynamical system of private ownership economy f_{Ep} is innovative if and only if a production system f_P is innovative (Definitions 3.3, 3.8).

Now we may apply the idea of a quasi-semidynamical system to construct the dynamical versions of Theorem 3.1.

Theorem 3.2. Let be given a single-valued technological quasi-semidynamical innovative Debreu system. If

(1) f_P is an innovative production system preserving a cumulative extension,

(2) f_X, f_e, f_ε are cumulative,

(3) $\sum_{k=l+1}^{l'} p_k^{(t)} e_{ak}^{(t)} \geq 0$ for each $t \in \mathbf{R}_+$, where $f_p(p, t) = p^{(t)}$, $f_e(e, t) = e^{(t)}$,

then f_C is a cumulative quasi-semidynamical consumption system.

The proof is based on the fact that all adequate components of an innovative system of private ownership economy fulfil the assumptions of Theorem 3.1. Indeed, let $t_1 < t_2$. Then by Definition 3.6 $E_p^{t1} \subset_i E_p^{t2}$, where $E_p^{t1} = f_{Ep}(E_p, t_1)$, $E_p^{t2} = f_{Ep}(E_p, t_2)$. Similarly, by Definition 3.4 $P^{t1} \subset_{ic} P^{t2}$, where $P^{t1} = f_P(P, t_1)$,

$P^{t_2} = f_P(P, t_2)$, and by the Assumption (2) $X_a^{t_1} \subset X_a^{t_2}$, $e_a^{t_1} \le e_a^{t_2}$, $\varepsilon_a^{t_1} \subset \varepsilon_a^{t_2}$ for each $a \in A$, where $X_a^{t_i} = f_X(X_a, t_i)$, $e_a^{t_i} = f_e(e_a, t_i)$, $\varepsilon_a^{t_i} = f_\varepsilon(\varepsilon_a, t_i)$ for $i = 1, 2$. This means that all assumptions of Theorem 3.1 are satisfied. Therefore $C^{t_1} \subset_c C^{t_2}$, where $C^{t_i} = f_C(C, t_i)$ for $i = 1,2$.

4. Distributive justice in the context of the Schumpeterian evolution

4.1. The static analysis

In our framework, as mentioned in Section 2.2, the rules of distributive justice will be referred to the social position of consumers who are given various quantities of commodities available in the budget sets. It means that we apply the non-preference-based approach [20] to the problem of ranking opportunity sets in terms of freedom of choice, where the bigger set of opportunities, the wider range of freedom. To be coherent with the Rawlsian set-up, it is assumed that in the worst position is a consumer having the smallest number of commodities available – this is dimension of non-zero volume of his budget set. If two or more individuals have the same number of commodities available, then volumes of the sets are taken into account. It means, formally, that a relation $<_v$ is defined that measures their ranges of freedom of choice and determines the social position of the consumers (cf., e.g., [10; 20]). Thus we can use the following definitions [10, pp. 321-324].

Definition 4.1. In comparison with the consumer a_j the consumer a_i is said to be:
(1) worse-off (symbolically: $a_i < a_j$) if $\beta(a_i) <_v \beta(a_j)$,
(2) in the same position (symbolically: $a_i = a_j$) if $\beta(a_i) =_v \beta(a_j)$,
(3) not better-off (symbolically: $a_i \le a_j$) if $\beta(a_i) \le_v \beta(a_j)$.
In other words, the above conditions mean that the consumer a_i has less, the same amount of or not more freedom of choice than the consumer a_j, respectively. We can now refer this relationship to economies E_p.

Definition 4.2. An economy E_p offers at most, the same amount of, or less freedom of choice as an economy E_p' if, respectively:
(1) $E_p \le_v E_p' \Leftrightarrow \forall a \in A: a \le a'$,
(2) $E_p =_v E_p' \Leftrightarrow \forall a \in A: a = a'$,
(3) $E_p <_v E_p' \Leftrightarrow E_p \le_v E_p' \wedge \exists\, a_0 \in A: a_0 < a_0'$.
The above considerations enable us to study the core of the Rawlsian theory of distributive justice [11]. As based on the maximin rule or, in other words, on the difference principle, it can be interpreted [12, p. 282] as calling for maximizing the

advantage of the worst-off. In our formulation the difference principle can be presented in two versions, standard [5] and weak ones. They refer to the situation, where an economy E_p' guarantees the worst-off more (not less) freedom of choice than the E_p, respectively. So, we assume the definition.

Definition 4.3.

(1) $E_p <_{dp} E_p' \Leftrightarrow \forall\, a \in A\ \ a_0 \leq a \ \Rightarrow \forall\, a \in A\ \ a_0 < a,$

(2) $E_p \leq_{dp} E_p' \Leftrightarrow \forall\, a \in A\ \ a_0 \leq a \ \Rightarrow \forall\, a \in A\ \ a_0 \leq a.$

Here, $a_0 \in A$ is a consumer who is worst-off in the economy E_p and the relations $<_{dp}$, ($\leq_{dp}$) mean that each consumer $a \in A$ is better-off (not worse-off) in the economy E_p' than a_0 in the economy E_p as far as freedom of choice is concerned. In this context the economy E_p is also called less (not more) just than the economy E_p' in the difference principle meaning. Clearly, the difference principle (Condition (1)) implies the weak difference principle (Condition (2)).

The Rawlsian conception of distributive justice as based on the difference principle can be now strengthened and generalized to the concept of leximin [15]. Lexicographical ordering, in brief leximin or lexical difference principle involving interpersonal comparisons, locates the least advantaged person, deserving better treatment. Next, consumers deserving special treatment are defined as less advantaged persons, and so on.

To define which of two economies offers more freedom of choice in the leximin's sense, we start with comparing the least advantaged in both economies. So firstly, we compare the position of a consumer in the worst situation in E_p with the position of a consumer in the worst situation in economy E_p'. The economy with a better situation of the least advantaged is more just. If the position (dimension and volume) is equal, the next consumers having number 2 are compared, and so on. The idea outlined above is presented formally in the definition.

Definition 4.4.

(1) $E_p <_l E_p' \Leftrightarrow \exists k \geq 1\ a^i = a'^i$ for $i = 1, ..., k-1$ and $a^k < a'^k$,

(2) $E_p =_l E_p' \Leftrightarrow\ a^i = a'^i$ for $i = 1, ..., m$,

(3) $E_p \leq_l E_p' \Leftrightarrow\ E_p <_l E_p'$ or $E_p =_l E_p'$.

In the first case an economy E_p' is said to offer more freedom in the leximin's sense than an economy E_p. In the second one, both economies offer the same amount of freedom in the leximin's sense, and in the third case, an economy E_p is said to offer at most freedom as an economy E_p' in the leximin sense. In this context the economy E_p is also called less (equally, not more) just than the economy E_p' in the leximin meaning.

The relationship between the concepts of difference principle, leximin and v-relation studied above are established by the following lemmas.

Lemma 4.1. $E_p <_{dp} E_p' \Rightarrow E_p <_l E_p'$.

Lemma 4.2. $E_p <_v E_p' \Rightarrow E_p <_l E_p'$.

Now, in keeping with the more general approach to the basic changes defining the Schumpeterian evolution in Section 3, the stronger versions of theorems proved in [10] can be formulated. Indeed, the following theorems are true.

Theorem 4.1. If $E_p \subset_c E_p'$, then $E_p \leq_v E_p'$.

Theorem 4.2. If $E_p \subset_c E_p'$, then $E_p \leq_{dp} E_p'$.

Proofs. Cumulative extension of a Debreu economy makes volume of budget sets not less.

However, the cumulative extension of a Debreu economy does not guarantee that the relation $<_l$ is preserved. To increase the level of freedom of choice offered by economy, the strict version of a cumulative extension of an economy E_p with respect, e.g., to maximum profit, denoted by $\subset_\pi$ is needed, meaning that $E_p \subset_{c\pi} E_p' \Leftrightarrow E_p \subset_c E_p'$ and $\exists\ b \in B$: $\pi_b(p) < \pi_b'(p')$. Thus, we obtain:

Theorem 4.3. If $E_p \subset_{c\pi} E_p'$, then $E_p <_l E_p'$.

Proof. If the profit of a producer is growing, so there is at least one consumer whose wealth is bigger. The budget set of this consumer has bigger volume because of shifting the hyperplane of a budget constraint.

However, in our more general framework presented here, the rules of distributive justice turned out to be preserved within innovative changes of an economy.

Theorem 4.4. If the assumptions of Theorem 3.1 hold, then

(i) $E_p \leq_v E_p'$,
(ii) $E_p \leq_{dp} E_p'$.

Proof. By Theorem 3.1 we get $C \subset_c C'$. Consequently, by Theorems 4.1, 4.2 theses (i) and (ii) hold.

4.2. A dynamic analysis

The relationships examined above can be now studied in a dynamic way as presented in Section 3.2. Consequently, we obtain the theorems analogous to Theorems 5-7 in [10]. Indeed, by Theorem 4.1 we get:

Theorem 4.5. For a single-valued, cumulative quasi-semidynamical Debreu economic system f_{E_p}: $\boldsymbol{E}_p \times \boldsymbol{R}_+ \rightarrow \boldsymbol{E}_p$ the range of freedom of choice does not diminish in time, i.e,. if $t_1 < t_2$, and $E_p^{t_1} \subset_c E_p^{t_2}$, then $E_p^{t_1} \leq_v E_p^{t_2}$, where $E_p^t = f(E_p, t)$ for $t = t_1, t_2$.

Analogically, by Theorems 4.2 and 4.3 we obtain:

Theorem 4.6. For a single-valued, cumulative quasi-semidynamical Debreu economic system f_{E_p}: $\boldsymbol{E}_p \times \boldsymbol{R}_+ \rightarrow \boldsymbol{E}_p$ the range of freedom of choice in the sense of the difference principle relation $\leq_{dp}$ does not diminish in time, i.e. if $t_1 < t_2$, and $E_p^{t_1} \subset_c E_p^{t_2}$, then $E_p^{t_1} \leq_{dp} E_p^{t_2}$.

Theorem 4.7. A single-valued, strictly cumulative with respect to maximum profit quasi-semidynamical Debreu economic system f_{E_p}: $\boldsymbol{E}_p \times \boldsymbol{R}_+ \rightarrow \boldsymbol{E}_p$ preserves strict $<_l$ relation of freedom of choice in time, i.e., $t_1 < t_2$, $E_p^{t_1} \subset_{c\pi} E_p^{t_2}$, then $E_p^{t_1} <_l E_p^{t_2}$.

Finally, Theorems 3.2 and 4.4 imply:

Theorem 4.8. Let be given a single-valued technological (quasi)--semidynamical innovative Debreu system such that the assumptions 1-3 of Theorem 3.2 hold, then the theses of Theorems 4.5 and 4.6 are satisfied.

In other words, not only the cumulative dynamics involved in Theorems 4.5--4.7 preserves the given level of justice in the difference principle sense and improves it in the leximin meaning, respectively, but, what is more, Theorem 4.8 suggests that under additional assumptions the innovative one makes the same. Thus the Schumpeterian innovative dynamics does not destroy the rules of social cohesion in the area of distributive justice.

References

[1] Arrow K.J., *Social Choice and Individual Values*, Wiley, New York 1963.

[2] Ciałowicz B., Malawski A., *Credit as a source of innovations – an axiomatic Schumpeterian approach*, [in:] *Proceedings of the 3rd International Conference on Business, Management and Economics 2007*, Yasar University, Cesme – Izmir, Turkey, 2007.

[3] *Concerted Development of Social Cohesion Indicators. Methodological Guide* (2005), Council of Europe, Belgium.

[4] Debreu, G., *Theory of Value*, Wiley, New York 1959.

[5] Hammond P.J., *Equity, Arrow's condition, and Rawls' difference principle*, "Econometrica" 1976, vol. 44, pp. 793-804.

[6] Malawski A., *A dynamical system approach to the Arrow-Debreu theory of general equilibrium*, [in:] *The 9th World Multi-Conference on Systemics, Cybernetics and Informatics, Proceedings 2005*, July 10-13, Orlando, Florida, USA, vol. VII, pp. 434-439.

[7] Malawski A., *Beyond Schumpeterian illusions: from general equilibrium to evolutionary economics*, the paper presented at the International Schumpeter Society Meeting, Milan, 9-11 June 2004, Italy, 2004.

[8] Malawski A., *Metoda aksjomatyczna w ekonomii*, Ossolineum, Wrocław 1999.
[9] Malawski A., Woerter M., *Diversity Structure of the Schumpeterian Evolution. An Axiomatic Approach*, Arbeitspapiere/Working Papers of the Swiss Institute for Business Cycle Research, No 153, Zurich 2006.
[10] Mrówka J., Malawski A., *Axiomatic analysis of distributive justice and freedom of choice in the context of the quality of life improvement*, [in:] *Towards Quality of Life Improvement*, ed. W. Ostasiewicz, Publishing House of the Wrocław University of Economics, Wrocław 2006, pp. 314-329.
[11] Rawls J., *A Theory of Justice*, Oxford University Press, Oxford 1971.
[12] Roemer J.E., *Theories of Distributive Justice*, Harvard University Press, Cambridge, Mass., 1996.
[13] Schumpeter J.A., *The Theory of Economic Development*, Harvard University Press, Cambridge, Mass., 1934/1961.
[14] Schumpeter J.A., *Capitalism, Socialism and Democracy*, Harper, New York 1950.
[15] Sen A.K., *Collective Choice and Social Welfare*, Holden Day, San Francisco 1970.
[16] Sen A.K., *Development as Freedom*, Knopf, New York 1999.
[17] Sen A.K., *Markets and freedoms: Achievements and limitations of the market mechanism in promoting individual freedoms*, Oxford Economic Papers 1993, vol. 45, pp. 519-541.
[18] Sen A.K., *Social choice theory*, [in:] *Handbook of Mathematical Economics* 1986, Vol. III, pp. 1073-1181.
[19] Sibirskij K.S., Szube A.S., *Semidynamical Systems* (*Topological Theory*), Sztinca, Kishyniev 1987 [in Russian].
[20] Xu Y., *On ranking linear budget sets in terms of freedom of choice*, “Social Choice and Welfare” 2004, vol. 22, pp. 281-289.
[21] Zajac E.E., *Political Economy of Fairness*, MIT Press, Cambridge 1995.

SPRAWIEDLIWOŚĆ DYSTRYBUTYWNA I ROZWÓJ INNOWACYJNY U SCHUMPETERA – UJĘCIE AKSJOMATYCZNE W KONTEKŚCIE SPÓJNOŚCI SPOŁECZNEJ

Streszczenie

Celem pracy jest aksjomatyczna analiza zasad sprawiedliwości dystrybutywnej jako składowej szerokiej idei spójności społecznej w kontekście koncepcji ewolucji Schumpetera. Metodologia badania Schumpeterowskiej ewolucji w dynamicznym ujęciu Arrowa–Debreu jest tu rozwinięta w dwóch kierunkach. Po pierwsze, zanalizowano związek między ruchem okrężnym a rozwojem gospodarczym w rozumieniu Schumpetera, wątek dotąd pomijany. Po drugie, reguły sprawiedliwości dystrybutywnej, stanowiące pewien składnik koncepcji spójności społecznej w ujęciu Rady Europy, są analizowane w sposób ścisły. W szczególności dowodzi się, że rozwój innowacyjny w sensie Schumpetera oparty na zasadzie twórczej destrukcji nie niszczy struktury społecznej zbudowanej na zasadach sprawiedliwości dystrybutywnej Rawlsa. W konsekwencji konstruuje się proces ewolucyjny utrzymujący dany poziom sprawiedliwości dystrybutywnej i wzmacniający spójność społeczną w tym znaczeniu.