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MATHEMATICAL MODEL+ OF A LONG-DISTANCE CONVEYOR

Oleh M. PIHNASTYI[1]*, Valery D. KHODUSOV[2], Anna V. KOTOVA[2]

[1] National Technical University (Kharkiv Polytechnic Institute), Kharkov
[2] V.N. Karazin Kharkiv National University, Kharkov

Abstract: The model of the multi-conveyor transport system of a conveyor type is given in the article. The conveyor is considered as a complex dynamic distributed system. The analytical expression which allows calculating linear density and material flow at any point of the transport route for a specific instant in time is obtained. The conveyor belt speed and the material flow from the accumulative bunker to the input of the conveyor are represented as given time functions. The decision analysis for the steady and transient periods of the transport system operation is accomplished. The estimated duration of the transient process is given. The model is of interest for the design of highly efficient flow control systems for long-ranged multi-conveyor transport systems of a conveyor type.

Keywords: *long-ranged conveyor, production line, flow line, PDE-conveyor model, production control system, optimal control, accumulative bunker, distributive system, transport system*

1. INTRODUCTION

The conveyor method is a common way to organize the in-line production in different fields of industries. The conveyor is used for both synchronization of technological operations and as a storage device. The conveyor transport systems are widespread in the mining industry. A characteristic feature of the mining industry is that the input flow of material coming in the conveyor transport system is a non-stationary stochastic flow of material (Jeftenić et al. 2010; Kawalec et al. 2021; Bardzinski 2018). This leads to a non-uniform distribution of the material along the transport route (DIN

* Corresponding author: pihnastyi@gmail.com (O.M. Pihnastyi)

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22101:2002-08, 2002). At the same time, specific energy consumption for rock transportation can significantly increase due to a decrease in the loading factor of the conveyor with material (Semenchenko et al. 2016; Kiriia et al. 2019). The low linear density of some conveyors cannot be compensated by an increase in linear density on other sides due to the maximum ultimate specific load limitations on the belt and the power of the electrical drives. That is why, nowadays it is of high importance to develop highly effective algorithms to control the flow parameters of a conveyor, which will allow the conveyor system to operate in a mode close to the nominal one, and at the same time contribute to the minimum energy consumption for transporting the unit-mass rock (DIN 22101:2002-08, 2002; Korniienko et al. 2018). The success of solving this problem is determined by the construction of new models that take into account the uneven distribution of material along the transport route. The belt speed control makes it possible to increase the loading of the conveyor systems and, as a result, to reduce the specific costs of electricity. However, due to the result of switching speed mode, longitudinal oscillations occur in the conveyor belt, due to which the risk of failure of the conveyor elements increases (Alspaugh 2004; Sanjay et al. 2019; Burduk et al. 2020). The problem of the elimination of non-uniform rock distribution along the transport route is particularly important for long distance conveyor systems (Mathaba and Xia 2015). A common solution when designing the long distance conveyor is the division of the total transport route into conveyors (Table 1) (Pihnastyi and Khodusov 2017). The division allows localizing the control of non-uniform rock distribution within a specific conveyor, which greatly simplifies the solution of the problem.

Table 1. Characteristics of long-ranged conveyor transport systems (Pihnastyi and Khodusov 2017)

	Length [km]	Conveyors	Power [kW]	Speed [m/sec]	Capacity [t/h]
Neyveli Lignite Corp., India (2007), (Siemens, 2023)	14	8	2520	5.4	–
Çöllolar Lignite Open Pit Mine, Turkey (2011), (Siemens, 2023)	17.4	26	46 300	–	9350
Coarse ore conveyor system Minera Los Pelambres, Chile (1998), (Siemens, 2023)	12.7	3	25 000	–	8700
Open Cast Mine Reichwalde, Germany (2010), (Siemens, 2023)	13.5	6	19 350	5.5	6000
From a mine in India to a cement plant in Bangladesh (2005), (Conveyorbeltguide, 2023)	16.5	1	–	6.5	2400
Sasol’s Impumelelo project in South Africa (2015), (Conveyorbeltguide, 2023)	27.5	1	4900	6.5	2400
From the Bu Craa mine to the coast at El Aaiún, Western Sahara, (Conveyorbeltguide, 2023)	128.7	11	–	–	2000
The Henderson Coarse Ore Conveying System, the North American Continental Divide (2000), (Alspaugh, 2004), (Kung, 2004)	24	3	12 700	4.5	2270

This control can be accomplished in two ways: a) conveyor belt speed control (Pihnastyi 2018; Lauhff 2005); b) control of inflow amount with the use of the accumulative bunker (Hartford et al. 2013; Kramadzhyan et al. 2015; Marais and Pelzer, 2008; Wolstenholm 2008).

The division of the transport route of the long distance transport system will allow us to establish the optimal flow parameter control mode for each conveyor, as well as to create conditions for the long distance conveyor to operate in the mode close to a nominal one, which is determined by the amount of rock inflow at the entry of a separate conveyor of the long distance conveyor. The accumulative bunker can be used to reduce the dispersion of the flow rate which enters a separate conveyor. In a conveyor system, the flow rate of the conveyed material changes due to a change in the speed of the conveyor or a fluctuation in the input material flow. Using an accumulative bunker for temporary storage and accumulation of material allows us to compensate for these fluctuations and maintain a more constant material flow value. The belt speed control provides switching to the nominal mode determined by the average value of the inflow amount to the entry of a separate conveyor for a characteristic control period (He et al. 2016; Lauhoff 2005; Pihnastyi and Khodusov 2017; Batrec 2023). Here arises the problem of matching the average value of the inflow which enters a separate conveyor with the belt speed mode. The problem is complicated by the fact that the choice of the belt speed control mode and the value of the flow rate at the entry of the conveyor are interconnected with the flow parameters of the neighbouring conveyors. The flow of the previous conveyor is the inflow to the accumulative bunker, and it depends both on the belt speed of the previous conveyor and on the amount of flow coming to its entry from the accumulative bunker. In addition, the complexity of the dynamic analysis of the flow parameters of the conveyor is superimposed (Kung 2004). This is manifested in the fact that the transport system of a separate conveyor with a moving rock along the transport route is a distributed system with a number of limitations, among which the limitation on the maximum separate linear load on the conveyor belt and the limitation on the maximum volume of the transported mass are important.

The mentioned unresolved issues related to the description of multi-conveyor transport system have determined the goal of this paper, which is formulated as follows: improvement of existing models used for the description of long distance conveyors. According to the goal, the main objectives of the research are: a) the development of a distributed model of the long distance transport system, each conveyor of which is equipped with an inflow accumulative bunker;
b) the definition of the dependencies between the flow parameters of conveyors of long distance transport system; c) time estimate of transient modes of the main line operation; d) the evaluation of the directions for future research in the field of conveyors modeling.

2. THE MODEL OF THE CONVEYOR

Conveyor systems are a kind of production systems with a flow method of production organization. The model of the flow line in the one-moment approximation can be represented in the form (Pihnastyi 2018):

$$\frac{\partial[\chi]_0(t,S)}{\partial t}+\frac{\partial[\chi]_1(t,S)}{\partial S}=0, \qquad [\chi]_0(0,S)=\Psi(S), \tag{1}$$

$$[\chi]_1(t,S)=[\chi]_{1\psi}(t,S),, \qquad [\chi]_1(t,0)=\lambda(t), \tag{2}$$

where S_d is a coordinate for the final technological position (the length of the flow line in a phase space) (Pihnastyi 2017), $[\chi]_0(t,S)$, $[\chi]_1(t,S)$ are linear density and the flow of the material at time t for a technological position $S\in[0,S_d]$, $\Psi(S)$ is a linear density of the initial distribution of workpieces, material or semi-finished material along the technological route, $[\chi]_{1\psi}(t,S)$ is a given nominal rate of product processing at the technological positions, $\lambda(t)-$ is flow of material incoming the conveyor. The review of flow line models by means of partial diffrential equation (PDE-models) is given in (Pihnastyi 2018). The conveyor as a variant of the production line has the following feature: products or material that are in diffrent places of the same conveyor move with the same speed, which is equal to the belt speed. In this regard, the flow of the material $[\chi]_1(t,S)$ is determined through the linear density $[\chi]_0(t,S)$ of the material distribution along the conveyor and the belt speed $a = a(t)$. The conveyor belt speed can be either constant or have continuous or stepped control over time (Halepoto et al. 2016; Lauhoff 2005; Pihnastyi and Khodusov 2017). This makes it possible to write down a closed system of Eqs. (1), in the form:

$$\frac{\partial[\chi]_0(t,S)}{\partial t}+\frac{\partial[\chi]_1(t,S)}{\partial S}=\delta(S)\lambda(t),, \quad [\chi]_0(0,S)=H(S)\Psi(S), \quad [\chi]_1(t,0)=\lambda(t), \tag{3}$$

$$[\chi]_1(t,S)=a(t)[\chi]_0(t,S), \quad H(S)=\begin{cases}0, & S<0,\\ 1, & S\geq 0,\end{cases} \quad \int_{-\infty}^{\infty}\delta(S)\,dS=1. \tag{4}$$

The system of Eqs. (3) and (4) is used to model the rock flow which moves along the transport route (Pihnastyi and Khodusov 2017). The addend $\delta(S)\lambda(t)$ presupposes the input of the material to the technological operation $S = 0$ with the intensity $\lambda(t)$. The material which is distributed along the transport route with the linear density $[\chi]_0(0,S)$ (t/m), is on the conveyor belt at the initial time $t = 0$. The system of equation is closed with respect to flow parameters $[\chi]_0(t,S)$ and $[\chi]_1(t,S_d)$. The closure condition of Eqs. (4) allows setting up the exact solution for the system of Eqs. (3) and (4) with respect to the flow parameters $[\chi]_0(t,S)$ and $[\chi]_1(t,S)$. The solution of the system of Eqs. (3) and (4) determines the distribution of material along the transportation route for an arbitrary point in time *t*. A schematic diagram of the conveyor with an accumulating bunker at the entry is shown in Fig. 1 (Conveyorbeltguide 2023).

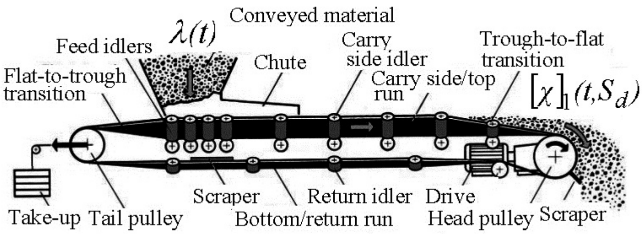

Fig. 1. Schematic diagram of the conveyor

The flow of rock enters the conveyor from the accumulative bunker with the uploading which provides required cargo flow in the output. Let us add to the system of Eqs. (3) and (4) the equation modeling the accumulative bunker operation:

$$\frac{dN(t)}{dt} = \lambda_{in}(t) - \lambda(t), \quad N(0) = N_0, \quad 0 \le N \le N_{\max}, \quad 0 \le \lambda(t) \le \lambda_{\max}, \tag{5}$$

where $N(t)$ − is the current amount of the material in the bunker with the $N_{\max}$ capacity. The material flow which enters the accumulative bunker $\lambda(t)$ is a given value. Let us set out the system of Eqs. (3)–(5) in its dimensionless form. To accomplish this, let us apply dimensionless parameters:

$$\tau = t/T_d, \quad \xi = S/S_d, \quad H(S_d\xi) = H(S), \quad \delta(\xi) = S_d\delta(S), \tag{6}$$

$$\psi(\xi) = \frac{\Psi(S)}{\Theta}, \quad \gamma(\tau) = \lambda(t)\frac{T_d}{S_d\Theta}, \quad \gamma_{in}(\tau) = \lambda_{in}(t)\frac{T_d}{S_d\Theta}, \quad g(\tau) = a(t)\frac{T_d}{S_d}, \quad a(t) \ne 0,$$

$$\theta_0(\tau,\xi) = \frac{[\chi]_0(t,S)}{\Theta}, \quad \Theta = \max\left\{\Psi(S), \frac{\lambda(t)}{a(t)}\right\}, \quad n(\tau) = \frac{N(t)}{S_d\Theta}, \quad n_0 = \frac{N_0}{S_d\Theta}.$$

Load per unit of length on the conveyor belt must not exceed overload capacity

$$\lambda(t)/a(t) = [\chi]_0(t,0) \le [\chi]_{0\max}.$$

When $n(\tau)=1.0$ and $\Theta=[\chi]_{0\max}$, the amount of the material in the accumulative bunker is $N(t)= S_d[\chi]_{0\max}$, which is required to fill the conveyor with the maximum allowable rock linear $[\chi]_0(t,S)=[\chi]_{0\max}$ through the full length of the conveyor. Taking into account the denominations (6), the balance equation of flow parameters of the conveyor can be presented in its dimensionless form:

$$\frac{\partial\theta_0(\tau,\xi)}{\partial\tau}+g(\tau)\frac{\partial\theta_0(\tau,\xi)}{\partial\xi}=\delta(\xi)\gamma(\tau),\quad \frac{dn(t)}{dt}=\gamma_{in}(\tau)-\gamma(\tau), \tag{7}$$

$$\theta_0(0,\xi)= H(\xi)\psi(\xi),\quad n(0)=n_0,\quad 0\le n(\tau)\le n_{\max},\quad 0\le\gamma(\tau)\le\gamma_{\max}.$$

For a constant conveyor belt speed $g(\tau)=g_0=\mathrm{const}$

$$\theta_0(\tau,\xi)=\left[H(\xi)-H(\xi-g_0\tau)\right]\frac{\gamma(\tau-\xi/g_0)}{g(\tau-\xi/g_0)}+H(\xi-g_0\tau)\psi(\xi-g_0\tau). \tag{8}$$

When $\tau>\xi/g_0$, the transport system operates in a steady state mode. The value of the flow parameters does not depend on the initial distribution of the material along the transport route

$$\theta_0(\tau,\xi)=\gamma(\tau-\xi/g_0)/g_0,\quad \theta_1(\tau,\xi)=\gamma(\tau-\xi/g_0),\quad \tau>\xi/g_0.$$

The stationary mode is of greatest interest for research, since here the transport system operates for much of that time. To define value of linear density $\theta_0(\tau,\xi)$ or the flow of the material $\theta_1(\tau,\xi)$ at the time τ in any part ξ of the route, we really need to know the value of inflow to the conveyor at time $\tau_\xi=\tau-\xi/g_0$. The association of linear density values $\theta_0(\tau,\xi)$ at any points of the transport route ξ_1 and ξ_2 at a constant conveyor belt speed is researched in (Antoniak 2010). The value of the output material flow of conveyor $\theta_1(\tau,1)$ is an important feature of the transport system and when the belt speed is constant, it is defined by the product of g_0 and $\theta_0(\tau,1)$

$$\theta_1(\tau,1) = g_0\theta_0(\tau,1)=\gamma(\tau-1/g_0).$$

3. MODEL OF MULTI-CONVEYOR TRANSPORT SYSTEM

The long distance conveyor is made as a set of some conveyors (Siemens 2023; Antoniak 2010; Xia and Zhang 2015), Fig. 2. The *m*-th accumulative bunker is at the output from the *m*-th conveyor at the technological position with the coordinate S_m. Bunkers provide the accumulation of the material between the conveyors. The flow of the raw material from the *m*-th conveyor enters the *m*-th bunker. The value of the flow which enters the bunker is equal to the product of the material linear density $[\chi]_0(t, S_m)$ at the output from the conveyor and the belt speed of the *m*-th conveyor $a_m(t)$. The material from the *m*-th bunker enters (m + 1)-th conveyor at the $\lambda_m(t)$ rate. The location of accumulative bunkers is set pointed as S_m. The system of equations of the multi-conveyor system with non-limiting capacity bunkers is written as:

$$\frac{\partial[\chi]_0(t,S)}{\partial t}+\frac{\partial\left(a(t,S)[\chi]_0(t,S)\right)}{\partial S}=\delta(S)\lambda_b(t,S), \quad [\chi]_1(t,S)=a(t)[\chi]_0(t,S), \tag{9}$$

$$\lambda(t,S)=\sum_{m=0}^{M}\delta(S-S_m)\lambda_m(t), \quad Pi(S,S_m)=H(S-S_{m-1})-H(S-S_m),$$

$$\lambda_b(t,S)=\lambda(t,S)-\sum_{m=0}^{M}\delta(S-S_m)\left(\lambda_m(t)-a_m(t)[\chi]_0(t,S)\right), \quad S\in[S_0,S_d],$$

$$a(t,S)=\sum_{m=1}^{M}Pi(S,S_m)a_m(t), \quad [\chi]_0(0,S)=\sum_{m=1}^{M}Pi(S,S_m)\Psi_m(S).$$

where $\Psi_m(S)$ is the rock linear density along the *m*-th conveyor at initial time.

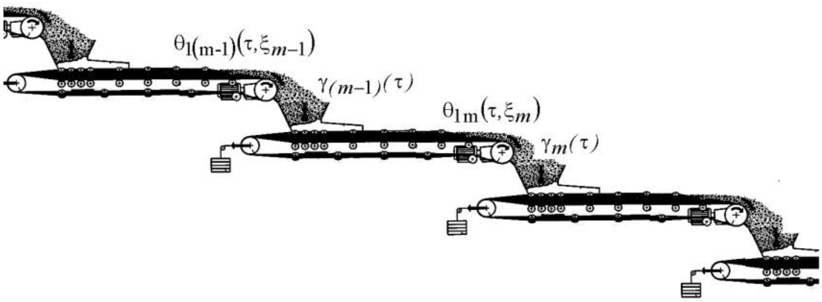

Fig. 2. The scheme of the multi-conveyor system

Let us set that the input device, through which the material enters the main line at the rate $\lambda_{in\,0}(t)$, is at the technological position, which is determined by the coordinate S_0 = 0. In a general case, the number and location of input points, providing the inflow of the material, are arbitrary. The material leaves the long distance transport system through the accumulative bunker at the output of the last conveyor. There are no other alternatives of the material outflow. Applying dimensionless variables:

$$\tau = t / T_d, \quad \xi = S / S_d, \quad H(\xi) = H(S / S_d), \quad \delta(\xi - \xi_m) = S_d \delta(S - S_m), \tag{10}$$

$$\gamma_m(\tau) = \lambda_m(t) T_d / \left(S_d \Theta\right), \quad \gamma_{in\, m}(\tau) = \lambda_{in\, m}(t) T_d / \left(S_d \Theta\right), \quad \xi_m = S_m / S_d,$$

$$\psi_m(\xi) = \Psi_m(S) / \Theta, \quad \Theta = \max\left\{\Psi_m(S), \lambda_m(t) / a_m(t)\right\}, \quad \theta_0(\tau, \xi) = \left[\chi\right]_0 (t, S) / \Theta, \quad a_m(t) \neq 0,$$

$$n_m(\tau) = N_m(t) \; / \left(S_d \Theta\right), \quad n_{0m}(\tau) = N_{0m}(t) / \left(S_d \Theta\right), \quad g_m(\tau) = a_m(t) T_d \; / S_d,$$

let us write the expression for the speed $g(\tau, \xi)$, linear density at initial time $\theta_0(0, \xi) = \psi(\xi)$ and $\gamma_m(t)$ as follows:

$$g(\tau, \xi) = \sum_{m=1}^{M} Pi(\xi, \xi_m) g_m(\tau), \quad \psi(\xi) = \sum_{m=1}^{M} Pi(\xi, \xi_m) \psi_m(\xi), \quad \gamma(\tau, \xi) = \sum_{m=0}^{M} \delta(\xi - \xi_m) \gamma_m(\tau),$$

$$g(\tau, \xi_m) = g_{m+1}(\tau), \quad \psi(\xi_m) = \psi_{m+1}.$$

Taking into account that

$$\theta_0(\tau, \xi) \frac{\partial g(\tau, \xi)}{\partial \xi} = \theta_0(\tau, \xi) g(\tau, \xi) \sum_{m=0}^{M-1} \delta(\xi - \xi_m) - \theta_{1\; in\; b}(\tau, \xi),$$

the equation for the main line is

$$\frac{\partial \theta_0(\tau, \xi)}{\partial \tau} + g(\tau, \xi) \frac{\partial \theta_0(\tau, \xi)}{\partial \xi} = \gamma(\tau, \xi) - \theta_0(\tau, \xi) \sum_{m=1}^{M} \delta(\xi - \xi_m) g(\tau, \xi_{m-1}). \tag{11}$$

Let us write the system of characteristic equations for the Eq. (11):

$$\frac{d\xi}{d\tau} = g(\tau, \xi), \quad \xi(0) = \rho, \quad \rho \in \left[\xi_{k-1}, \xi_k\right[, \tag{12}$$

$$\frac{d\theta_0(\tau, \xi)}{d\xi} = \frac{\gamma(\tau, \xi)}{g(\tau, \xi)} - \theta_0(\tau, \xi) \sum_{m=1}^{M} \delta(\xi - \xi_m), \quad \psi(\rho) = \sum_{m=1}^{M} Pi(\rho, \xi_m) \psi_m(\rho). \tag{13}$$

We present the solution of the Eq. (12) as follows:

$$\xi = \sum_{m=1}^{M} Pi(\tau, \tau_m) \left(G_m(\tau) + \rho_m\right), \quad G_m(\tau) = \int_0^{\tau} g_m\left(\alpha\right) d\alpha, \quad \rho_m = \xi_{m-1} - G_m(\tau_{m-1}), \tag{14}$$

where:

$$\tau_\xi = G_k^{-1}\left(\xi_k - \rho\right), \quad \rho \in \left[\xi_{k-1}, \xi_k\right[, \tag{15}$$

$$\tau_m = G_m^{-1}\left(\xi_m - \xi_{m-1} + G_m\left(\tau_{m-1}\right)\right), \quad \xi_m \geq \xi_k, \tag{16}$$

$$\tau_m = G_{m+1}^{-1}\left(\xi_m - \xi_{m+1} + G_{m+1}\left(\tau_{m+1}\right)\right), \quad \xi_m < \xi_k \tag{17}$$

and G^{-1} is an inverse function. As the belt speed of the *m*-th conveyor can be only positive (we assume that the conveyor does not stop during the period under consideration), only one $\xi(\tau)$ function value corresponds to each time value τ. Using Eq. (14) we will express the variables τ and $\tau_{\xi m}$:

$$\tau = G_m^{-1}\left(\xi - \rho_m\right), \quad \tau_{\xi(m-1)} = G_m^{-1}\left(\xi_{m-1} - \xi + G_m(\tau)\right), \quad \xi \in \left[\xi_{m-1}, \xi_m\right[.$$

On integrating the second characteristic equation we have:

$$\theta_0(\tau,\xi) = \sum_{m=0}^{M} H(\xi - \xi_m)\frac{\gamma_m\left(\tau_{\xi m}\right)}{g_{m+1}\left(\tau_{\xi m}\right)} - \sum_{m=1}^{M} H(\xi - \xi_m)\theta_0\left(\tau_{\xi m}, \xi_m\right) + C_\xi. \tag{18}$$

For this purpose let us evaluate the integrals

$$\int \frac{\gamma(\tau,\xi)}{g(\tau,\xi)}d\xi = \sum_{m=0}^{M-1} H(\xi - \xi_m)\frac{\gamma_m\left(\tau_{\xi m}\right)}{g_{m+1}\left(\tau_{\xi m}\right)},$$

$$\theta_0(0,\rho_m) = \sum_{m=0}^{M-1} H(\rho_m - \xi_m)\frac{\gamma_m\left(\tau_{\xi m}\right)}{g_{m+1}\left(\tau_{\xi m}\right)} - \sum_{m=1}^{M} H(\rho_m - \xi_m)\theta_0\left(\tau_{\xi m}, \xi_m\right) + C_\xi = \sum_{m=1}^{M} Pi(\rho_m, \xi_m)\psi_m(\rho_m),$$

where

$$C_\xi = \sum_{m=1}^{M} H(\rho_m - \xi_m)\theta_0\left(\tau_{\xi m}, \xi_m\right) + \sum_{m=1}^{M} Pi(\rho_m, \xi_m)\psi_m(\rho_m) - \sum_{m=0}^{M-1} H(\rho_m - \xi_m)\frac{\gamma_m\left(\tau_{\xi m}\right)}{g_{m+1}\left(\tau_{\xi m}\right)}.$$

Let us apply the integration constant C_ξ to (18) making use of the preceding equation, and we will get

$$\theta_0(\tau,\xi) = \sum_{m=0}^{M-1}\left(H(\xi - \xi_m) - H(\rho_m - \xi_m)\right)\frac{\gamma_m\left(\tau_{\xi m}\right)}{g_{m+1}\left(\tau_{\xi m}\right)}$$

$$-\sum_{m=1}^{M}\left(H(\xi - \xi_m) - H(\rho_m - \xi_m)\right)\theta_0\left(\tau_{\xi m}, \xi_m\right) + \sum_{m=1}^{M} Pi(\rho_m, \xi_m)\psi_m(\rho_m). \tag{19}$$

The expression (19) is the solution for the system of Eq. (11), and it determines the state of flow parameters of the multi-conveyor transport system with non-limiting capacity bunkers. The value of the function $\theta_0(\tau_{\xi m}, \xi_m)$ can be calculated by means of accumulative bunkers parameters $\gamma_{m-1}(\tau_{\xi m-1})$ and initial conditions $\psi_m(\rho)$

$$\theta_0\left(\tau_{\xi m}, \xi_m\right) = Pi(\rho_m, \xi_m)\psi_m(\rho_m) + \left(H(\xi - \xi_{m-1}) - H(\rho_m - \xi_{m-1})\right)\frac{\gamma_{m-1}\left(\tau_{\xi m-1}\right)}{g_m\left(\tau_{\xi m-1}\right)}.$$

Let us apply the expression for the function $\theta_0\left(\tau_{\xi m}, \xi_m\right)$ in (19) and write the solution in the form of

$$\theta_0(\tau, \xi) = \sum_{m=1}^{M} Pi(\xi, \xi_m) Pi(\rho_m, \xi_m)\psi_m(\rho_m)$$

$$+\sum_{m=0}^{M-1} Pi(\xi, \xi_m)\left(\left(H(\xi - \xi_m) - H(\rho_m - \xi_m)\right)\frac{\gamma_m\left(\tau_{\xi m}\right)}{g_{m+1}\left(\tau_{\xi m}\right)}\right). \quad (20)$$

In the outlined solution (20) it is assumed that accumulative bunkers are adjusted and material flow time law $\gamma_m(\tau)$ is given for them. If the bunker is transferred (the bunker serves only to direct material flow from one conveyor to another, but it does not alter the intensity of current), then the outflow from the bunker is equal to the inflow, and consequently in this case we have:

$$\frac{\gamma_m\left(\tau_{\xi m}\right)}{g_{m+1}\left(\tau_{\xi m}\right)} = \theta_0\left(\tau_{\xi m}, \xi_m\right)\frac{g_m\left(\tau_{\xi m}\right)}{g_{m+1}\left(\tau_{\xi m}\right)} = \frac{\gamma_{m-1}\left(\tau_{\xi m-1}\right)}{g_m\left(\tau_{\xi m-1}\right)}\frac{g_m\left(\tau_{\xi m}\right)}{g_{m+1}\left(\tau_{\xi m}\right)}. \quad (21)$$

4. DECISION ANALYSIS

The expression (20) which determines the linear density of the material distribution for main multi-conveyor can be written for one conveyor transport system as:

$$\theta_0(\tau, \xi) = \left(H(\xi) - H(\rho_0)\right)\frac{\gamma_0\left(\tau_{\xi 0}\right)}{g_1\left(\tau_{\xi 0}\right)} + H(\rho_0)\psi_1(\rho_0). \quad (22)$$

The solution $\theta_0(\tau, \xi)$ in the form (22) has been deduced in [10] when developing the distributive dynamic model of one conveyor with the belt speed control. Let us consider the model of the long distance conveyor which consists of 5 conveyors with different speed modes of the belt Fig. 3. The transport route is divided into conveyors with lengths $\Delta\xi_m = \xi_m - \xi_{m-1} = \left\{0.2; 0.3, 0.2; 0.1; 0.2\right\}$, each of which has the entering bunker and a device to control the belt speed.

Accumulative buckers are located at positions $\xi_m = \left\{0.0; 0.2, 0.5; 0.7; 0.8; 1.0\right\}$ and their values of flow intensity are $\lambda_m = \left\{1.0; 0.4, 1.2; 0.1; 0.5; 1.5\right\}$. The motion trajectories for a single element of the material located in different places of the transport route at a constant conveyor speed within each *m*-conveyor is shown in Fig. 4: $g_m(\tau)=\{1.0; 0.5, 1.5; 0.5; 2.0\}$. A set of curves which defines the linear density of the material along the transport route at times $\tau_i=\Delta\tau\cdot i$, $\Delta\tau=0.025$, $i=1\div 23$ Fig. 5 corresponds to this set of characteristics. The initial linear density of the material along the transport route is given by the expression $\theta_0(0,\xi)=0.5\sin(2\pi\xi)$. At times $\tau_i=\Delta\tau\cdot i$ there is the right shift of the function $\theta_0(0,\xi)$ within limits of each *m*-conveyor with a fixed lead $\Delta\xi_{g\,m}(\tau)=g_m(\tau)\Delta\tau$.

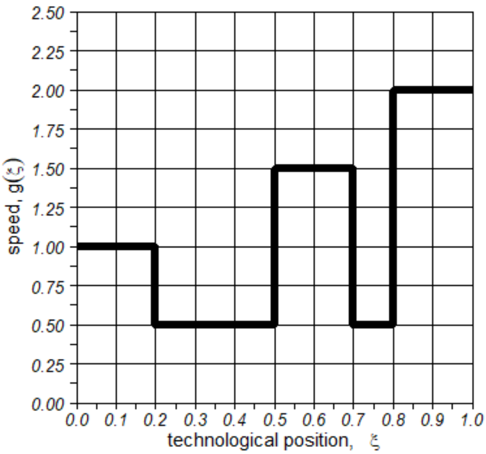

Fig. 3. The belt speed
of the long distance transport system
depending on the conveyor

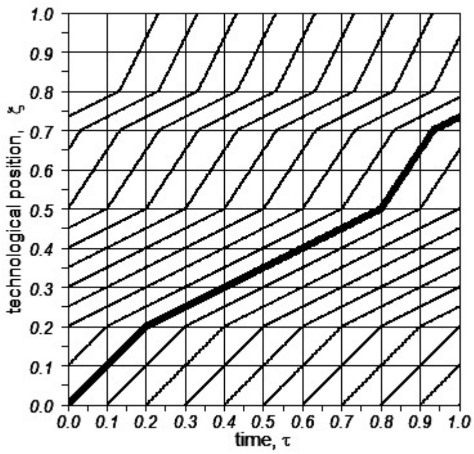

Fig. 4. The motion trajectories
for a single material element located in different
conveyors of the transport route

The angular slope of the motion trajectories for a single material element is proportional to the belt speed of an *m*-conveyor and it is a constant value when the constant speed is $g_m(\tau)$ correspondingly. Within limits of every conveyor the point $\xi=\xi_{m-1}+i\cdot g_m(\tau)\Delta\tau$ divides the conveyor into two parts: linear density along the route for the left side is determined by the ratio of the value of intensity of the material arrival from the accumulative bunker at the conveyor entry to the belt speed; initial distribution of linear density $\theta_0(0,\xi)=\psi(\xi)$ is applied to the right side. This mode of transport system operation is transient (unsteady) (Kung 2004). In general cases a transient period is marked by the maximum resource spend of the transport system. This is due to the fact that in most cases initial linear density $\psi(\xi)$ sufficiently deviates from the optimal value or is filled with the material of another sort, which must be unloaded from the transport route.

When the belt speed $g_m(\tau) = \text{const}$ is constant, the time term when the conveyor operates in a transient mode can be determined for each conveyor

$$\Delta\tau_{tr\ m} = \left(\xi_m - \xi_{m-1}\right) / g_m(\tau), \quad m = 1...M.$$

For the transport conveyor system per se the time of the transient period can be written as follows: $\Delta\tau_{tr} = \max\left\{\Delta\tau_{tr\ m}\right\}$. This has led to two very important and obvious results: a) the distribution of the transport system allows reducing the duration of the transient period proportionately to the number of conveyors; b) when $\tau \geq \Delta\tau_{tr}$, the system reaches a steady-state operation mode, which does not depend on the initial material distribution along the transport route, that allows presenting the obtained solution (20) in a simple form:

$$\theta_0(\tau,\xi) = \sum_{m=0}^{M-1} Pi(\xi,\xi_m) \frac{\gamma_m\left(\tau_{\xi m}\right)}{g_{m+1}\left(\tau_{\xi m}\right)}. \tag{23}$$

If the capacity of the bunkers is quite significant and contains a sufficient amount of the material to provide uninterrupted functioning of the *m*-th conveyor, then it can be assumed that within the given period every conveyor of the transport system operates as an independent system with it's the inflow material $\gamma_m\left(\tau\right)$ and the belt speed $g_{m+1}\left(\tau\right)$. When constant values of the material inflow is $\gamma_m\left(\tau\right)$ and the belt speed is $g_{m+1}\left(\tau\right)$, the steady mode is fixed, which is clearly demonstrated in Fig. 5.

Figure 5 shows the linear density $\theta_0(\tau,\xi)$ of the material on the belt along the transport route at times $\tau = 0.025i$ (20). For definiteness, we will assume that at the moment of time $\tau = 0.0$ the linear density of the material is determined by the initial distribution

$$\theta_0(0,\xi) = \sum_{m=1}^{M} Pi(\xi,\xi_m) Pi(\rho_m,\xi_m) \psi_m(\rho_m) = 0.5\left(1+\sin(2\pi\xi)\right).$$

We shall mention that the constant value of the linear density at a given point can also be provided when we have a constant values $\gamma_m\left(\tau\right) = \gamma_{m0}$ = const, $g_{(m+1)}\left(\tau\right) = g_{(m+1),0} = \text{const}$. At the next moment of time, the initial linear density $\theta_0(0,\xi)$ shifts at a speed $g_m(\tau)$. The movement occurs on each conveyor. The conveyor belt is filled with a material with a linear density $\theta_0(\tau,\xi)$, the value of which is determined by expression (23). Since $\gamma_m\left(\tau\right)$, $g_{(m+1)}\left(\tau\right)$ are constant, $\lambda_m = \left\{1.0; 0.4, 1.2; 0.1; 0.5; 1.5\right\}$, $g_m(\tau) =$ $\left\{1.0; 0.5, 1.5; 0.5; 2.0\right\}$, but different for each conveyor ξ_m = {0.0; 0.2, 0.5; 0.7; 0.8, 1.0}, then the value of the linear density is also constant, which is shown in Fig. 5.

After a period of time $\Delta\tau_{tr} \sim 0.6$ the transport route is filled with material with a constant linear density (23). The filling time $\Delta\tau_{tr}$ is determined by the maximum value of $\Delta\xi_m / g_{m0}$. This time is less than 1, $\Delta\tau_{tr} < 1$, due to the fact that the transportation route is divided by conveyors.

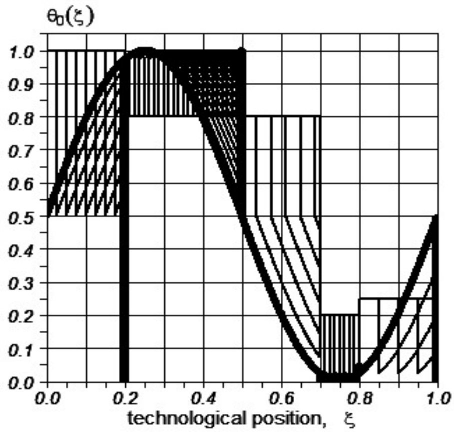

Fig. 5. The linear density of the rock along the route at times $\tau = 0.025i$, $i = 1\ldots23$, $a = 0$

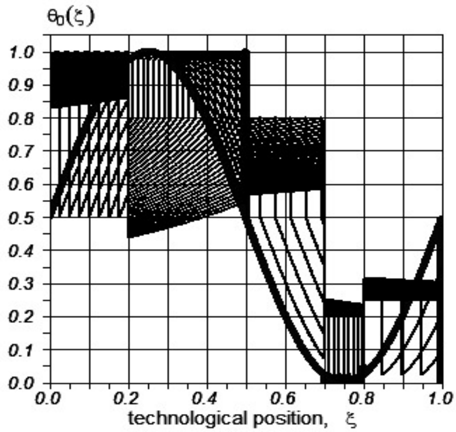

Fig. 6. The linear density of the rock along the route at times $\tau = 0.025i$, $i = 1\ldots23$, $a = 0{,}25$

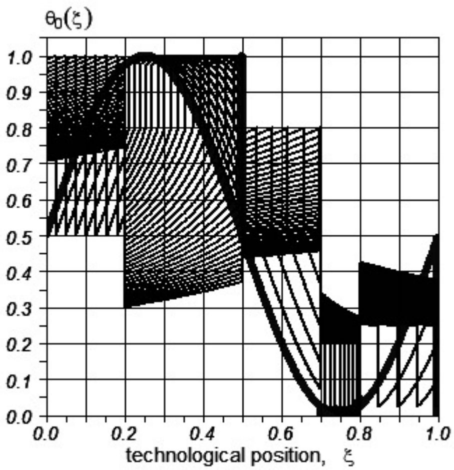

Fig. 7. The linear density of the rock along the route at times $\tau = 0.025i$, $i = 1\ldots23$, $a = 0.5$

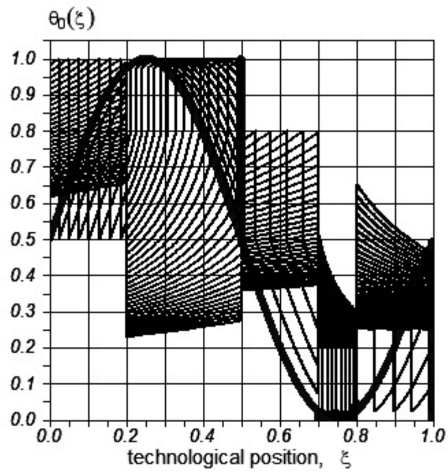

Fig. 8: The linear density of the rock along the route at times $\tau = 0.025i$, $i = 1\ldots23$, $a = 0.75$

Let us consider the transport route with a timevarying belt speed of the m-th conveyor (Figs. 6–8) $g_m(\tau) = g_{m0} + g_{m1}\tau$, $g_{m0} = \{1.0; 0.5, 1.5; 0.5; 2.0\}$, $g_{m1} = \{a; 2a, 3a; –a/2; –2a\}$, $a = 0.25$ (Fig. 6), $a = 0.5$ (Fig. 7), $a = 0.75$ (Fig. 8).

The variation with the time of the belt speed $g\ m(\tau)$ of the m-th conveyor leads to inversely proportional varying of the linear density of the material $\theta_0(\tau, \xi)$. The duration of the transient period $\Delta\tau_{tr\ m}$ for the m-th conveyor is determined by the equation (12):

$$\xi_m - \xi_{m-1} = g_{m0}\Delta\tau_{tr\ m} + g_{m1}\frac{\Delta\tau_{tr\ m}^2}{2}, \quad \Delta\tau_{tr\ m} = \max\{\Delta\tau_{tr\ m}\}.$$

The process of the conveyor belt slow downing (mode $g_{m1} < 0$) is of practical interest for the research. The deceleration leads to the hyperbolical growth of the material linear density at the entry to the conveyor (Figs. 6–8, conveyors 4 and 5), which is the cause of highly uneven loading of the belt with the material, and as a consequence leads to the breakdown of the conveyor belt in case if load per unit area exceeds admissible load.

5. CONCLUSION

A distributed dynamic model of a multi-conveyor transport system with intermediate accumulative bunkers is presented in the article. As a rule the efficiency of the transport system of a conveyor type is achieved by means of conveyor belt speed control and the intensity of the material flow, which enters a conveyor. Control algorithms development which is used to eliminate the non-uniform distribution of rock along the transport route is of particular importance for long-ranged conveyor systems. The division of the transport route into conveyors decreases the specific cost of the energy for transporting the rock and increases the operational life of long-ranged conveyor systems. However, as it is shown in this article, it greatly complicates the modeling of a transport system of a conveyor-type. The interaction of the flow parameters of the distributed system both within a separate conveyor and between the flow parameters of different conveyors should be taken into account. For the first time an analytical model with partial differential equations for a single-conveyor transport system was presented in (Pihnastyi and Khodusov 2017). The significance of the result obtained in (Pihnastyi and Khodusov 2017) lies in the fact that a model of a distributed transport system is developed and the dependencies between the flow parameters of the system are presented in an analytical form. This provided new opportunities to design energy-efficient systems for optimal control of conveyor transport by means of belt speed control and the intensity of the material which enters a conveyor. However, the problem of the development of long-ranged multi-conveyor systems with intermediate accumulative bunkers and variable belt speed hasn't been solved yet. The topicality of this problem is due to the ever-increasing number of operating main multi-conveyor systems, which require highly efficient optimal control algorithms.

An important result of this article is the further development of a single-conveyor model of a distributed transport system and the development of a model of a long-ranged multi-conveyor system with accumulative bunkers. The solution which determines the value of the linear density of the material in any place of the transport route as a time function for a given law of variation of belt speed and the material flow to the entry of the conveyor from the accumulative bunker is given. The decision analysis for transient and steady modes of the conveyor operation is presented. The analytical dependencies of the material flow and the linear density of the material for technological positions within limits of one and different conveyors are presented. The evaluation of the duration of the main line unsteady mode operations is accomplished. It is shown that the division of the transport route into conveyors leads to a significant decrease in the duration of unsteady modes, which increases the efficiency of the system as a whole. The presented solution opens up new opportunities to develop optimal algorithms for the flow parameters control of a long--ranged multi-conveyor system.

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