

POLITECHNIKA WROCŁAWSKA

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NADCIEKŁOŚĆ ^3He W CIENKICH WARSTWACH

PRACA DOKTORSKA

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STRESZCZENIE WYNIKÓW

A1, A2: Wyznaczono zależność przejścia fazowego ze stanu nadciekłego do stanu bezszczelinowej nadciekłości od nierówności podłoża i grubości warstwy ^3He . (rozdz.5)

A3: Zbadano wpływ nierówności powierzchni cienkiej warstwy fermionów na rodzaj przejścia fazowego w polu magnetycznym ze stanu nadciekłego typu s do stanu normalnego. (rozdz.5)

A4, A6: Zbadano wpływ potencjału podłoża na temperaturę krytyczną w cienkiej warstwie. (rozdz.2)

A5, A6: Określono dwuwymiarowe stany nadciekłe ^3He i wykazano energetyczną stabilizację dwuwymiarowej fazy A w obecności nadciekłego prądu masy. Wyznaczono nadciekły prąd krytyczny w zależności od temperatury i grubości warstwy. Zbadano wpływ potencjału podłoża na prąd krytyczny. (rozdz.3)

A7: Określono wpływ nierówności powierzchni podłoża na prąd i temperaturę krytyczną. Znaleziono zależność nadciekłego prądu krytycznego od temperatury. (rozdz.4)

WSTĘP

Cienka warstwa nadciekłego ^3He jest układem, w którym istotne znaczenie, oprócz wyróżnionego kierunku w przestrzeni, ma oddziaływanie kwazicząstek z podłożem. Hel jest adsorbowany na powierzchniach metalicznych (miedź, srebro, stal), a także na powierzchniach materiałów o strukturze grafitu (ang. grafoil). W obu przypadkach powierzchnia podłoża jest układem niejednorodnym i od jej lokalnego ukształtowania zależy grubość warstwy [1-3]. Ta geometryczna struktura podłoża ma znaczny wpływ na zjawiska zachodzące w cienkiej warstwie. Małe nierówności powierzchni, czyli małe wahania grubości warstwy, powodują dyfuzyjne rozpraszanie cząstek ^3He na granicy ośrodka i adsorbentu. Szczegółowe określenie przyjętego warunku brzegowego odpowiadającego temu procesowi zostanie opisane w części 1 rozprawy.

Znaczny rozwój technik doświadczalnych w ciągu ostatnich lat umożliwił badania nadciekłości w warstwach o grubości rzędu stałej koherencji $\xi_0 \approx 60\text{nm}$ (dla $P=0$ bar), czyli długości fali de Broglie'a pary Coopera w ^3He [1-8]. Harrison et al. [1-3] przeprowadzili pomiary prądów nadciekłych j_{sc} i temperatur T_c^F przejścia fazowego : stan nadciekły - stan normalny dla szeregu grubości warstwy ^3He . Powtórzono te same doświadczenia gdy pomiędzy ^3He a podłożem znajdowała się warstwa ^4He i otrzymano znacznie większe

wartości krytycznego prądu nadciekłego j_{sc} i temperatury krytycznej T_c^F [1-3]. Efekt ten, zaobserwowany także przez Richardsona et al. [5,6], świadczy o dużym wpływie podłoża na stany koherentne w cienkiej warstwie.

Richardson et al. [5,6] zasugerowali, że izolująca warstwa ^4He wygładza nierówności powierzchni i powoduje zmianę rozpraszania cząstek ^3He w tym obszarze z dyfuzyjnego na zwierciadlane. Zwierciadlane warunki brzegowe nie powodują tak dużego tłumienia nadciekłości jak warunki dyfuzyjne i tym samym prąd j_{sc} oraz temperatura T_c^F osiągają większe wartości.

Warstwa ^4He zmienia także potencjał typu van der Waalsa, w którym znajduje się ^3He [9,10] i również w ten sposób wpływa na wielkości mierzone [A4-A6].

Mimo obszernego zbioru wyników doświadczalnych nie było dotąd teoretycznego opracowania zjawiska nadciekłego przepływu masy w cienkiej warstwie ^3He . Nie wytłumaczone są bardzo małe wartości prądów nadciekłych j_{sc} przy jednocześnie względnie dużych wartościach temperatur T_c^F [1-6]. Istnieją jedynie fenomenologiczne modele Jacobsena i Smitha [11] oraz Fettera i Ullaha [12] dla obszaru Ginzburga-Landaua tzn. temperatur $T \approx T_c^F$, ale nie są one odpowiednie do porównania z wynikami najnowszych pomiarów Xu i Crookera [4], które zostały przeprowadzone w innym zakresie temperatur. Wymagało to ogólniejszego, mikroskopowego opisu nadciekłości ^3He w cienkiej warstwie, czego próbą jest ta rozprawa.

W rozdziale 1 jest przedstawiony formalizm stosowanych

przez nas modeli warstw. Wpływ izolującej warstwy ${}^4\text{He}$ na temperaturę krytyczną T_C^F jest analizowany w rozdziale 2, a rozdziały 3 i 4 są poświęcone zagadnieniu nadciekłego prądu masy ${}^3\text{He}$ w cienkiej warstwie. W rozdziale 5 przedstawiamy pewne konsekwencje niejednorodności podłoża na przejścia fazowe w tych układach.

W rozprawie opisane są cienkie warstwy o grubości d ograniczonej przez ξ_0 i odległość międzyatomową p_0^{-1} :

$$p_0^{-1} \ll d \ll \xi_0 ,$$

gdzie p_0 jest pędem Fermiego.

Przyjęto jednostki atomowe $\hbar = k_B = 1$.

1. MODELE CIENKICH WARSTW

Cienka warstwa o zwierciadlanych powierzchniach w polu van der Waalsa

Warstwa o grubości d i osi OZ prostopadłej do jej powierzchni jest określona potencjałem :

$$V_1(z) = \begin{cases} 0 & \text{dla } 0 < z < d \\ \infty & \text{dla } z \leq 0 \text{ lub } z \geq d \end{cases} . \quad (1.1)$$

Zwykle oddziaływanie atomów z podłożem jest określane przez potencjał Lennarda-Jonesa lub Azisa [9, 10]. Metody

wariacyjne zakładające te potencjały jako początkowe prowadzą do potencjału efektywnego o silnej części odpychającej i wolnozmiennnej, dalekozasięgowej części przyciągającej [9,10]. Nieregularny potencjał efektywny może być przybliżony potencjałem schodkowym :

$$V_2(z) = \begin{cases} V & \text{dla } 0 \leq z \leq a \\ 0 & \text{dla } z < 0 \text{ lub } z > a \end{cases}, \quad (1.2)$$

gdzie V jest amplitudą jądra odpychającego o zasięgu oddziaływania a . Zwierciadlana warstwa z potencjałem typu van der Waalsa jest określona przez :

$$V(z) = V_1(z) + V_2(z). \quad (1.3)$$

Wyznaczone przez Krotscheka [9] potencjały efektywne mają nieregularny, oscylacyjny przebieg, co prowadzi do powstania kilku stanów powierzchniowych [9,10]. Ponieważ rozpatrujemy warstwy o grubościach znacznie przewyższających skalę odległości międzyatomowych i liczbie stanów dużo większej od jedności, możemy zaniedbać niewielką liczbę stanów powierzchniowych w opisie makroskopowego efektu nadciekłości.

Widmo energetyczne ϵ_p potencjału $V(z)$ zależy jedynie od dwóch parametrów : d/a i V/μ , gdzie μ jest potencjałem chemicznym układu. Zajmiemy się systemami

o ustalonym potencjale chemicznym i wybierzemy dwa bezwymiarowe parametry charakteryzujące cienką warstwę $V(z)$, jako d/a i Va^2 . Liczba kwantowa ν , numerująca poziomy energetyczne ϵ_ν , przyjmuje wartości ze zbioru $\{1, \dots, \nu_c\}$, a ν_c jest określone zależnością [A4-A6] :

$$\nu_c = \max\{ \nu: \epsilon_\nu \leq \mu \} . \quad (1.4)$$

Przyjmujemy, że cienka warstwa jest jednorodna w płaszczyźnie OXY, więc energia kwazicząstek wynosi :

$$\epsilon_\nu(p) = \frac{p^2}{2m} + \epsilon_\nu , \quad (1.5)$$

gdzie $p=(p_x, p_y)$ jest dwuwymiarowym pędem, a m - masą atomową ${}^3\text{He}$. Po nałożeniu warunków brzegowych (1.3) nastąpiło skwantowanie sfery Fermiego i jest ona zbiorem okręgów Fermiego indeksowanych liczbami ν . Pęd Fermiego $p_F=(p_{Fx}, p_{Fy})$ kwazidwuwymiarowego układu jest związany z pędem Fermiego $p_0=(3\pi^2 n)^{1/3}$ układu trójwymiarowego o tej samej gęstości cząstek n zależnością [A5] :

$$p_F = p_0 \left[2p_0 d \left(3\pi \sum_{\nu=1}^{\nu_c} \sin^2 \vartheta_\nu \right)^{-1} \right]^{1/2} , \quad (1.6)$$

gdzie $\sin \vartheta_\nu = (1 - \epsilon_\nu/\mu)^{1/2}$, a $\mu = p_0^2/2m$ jest potencjałem chemicznym cienkiej warstwy ${}^3\text{He}$.

Warunek brzegowy na swobodną powierzchnię warstwy w potencjale (1.3) jest dużym uproszczeniem, pominięto napięcie powierzchniowe cieczy [13,14] i proces powstawania pary nasyconej na granicy ciecz-gaz. Uwzględnienie tych efektów wymagałoby znacznej komplikacji teorii, dlatego zakładamy najprostszy warunek (1.3).

Zmiana wartości parametrów V i a oddziaływania $V_2(z)$ powoduje zmianę wartości zbioru energii ϵ_ν , liczby ν_c (1.4), a także pędu Fermiego p_F (1.6). W dalszej części rozprawy (rozdz.2 i 3) przedstawimy fizyczną interpretację regulacji potencjału $V_2(z)$ i zbadamy jej wpływ na mierzalne wielkości : krytyczny prąd nadciekły j_{sc} i temperaturę przejścia fazowego T_c^F .

Znalezione widmo kwazicząstkowe $\epsilon_\nu(p)$ (1.5) służy do wprowadzenia, przez przedstawienie Lehmana [15], funkcji Greena-Matsubary dla nie oddziałujących fermionów :

$$\mathcal{G}_{0\nu}(i\omega_n, \mathbf{p}) = \left[i\omega_n - \epsilon_\nu(\mathbf{p}) + \mu \right]^{-1}, \quad (1.9)$$

gdzie $\omega_n = (2n+1)\pi T$ jest fermionową częstotliwością Matsubary. Nadciekłość jest analizowana przy użyciu formalizmu funkcji Greena dla skończonych temperatur [15,16] przez rozwiązanie układu równań Abrikosowa-Gorkowa :

$$\mathcal{G}_{\nu}(i\omega_n, \mathbf{p}) = \mathcal{G}_{0\nu}(i\omega_n, \mathbf{p}) + \mathcal{G}_{0\nu}(i\omega_n, \mathbf{p}) \Delta_{\nu}(\mathbf{p}) \mathcal{F}_{\nu}^{+}(i\omega_n, \mathbf{p}) , \quad (1.10)$$

$$\mathcal{F}_{\nu}^{+}(i\omega_n, \mathbf{p}) = - \mathcal{G}_{0\nu}^{-}(i\omega_n, \mathbf{p}) \Delta_{\nu}^{+}(\mathbf{p}) \mathcal{G}_{\nu}(i\omega_n, \mathbf{p}) , \quad (1.11)$$

uzupełnionego samouzgodnionym równaniem szczeliny energetycznej :

$$\Delta_{\nu}^{+}(\mathbf{p}) = -T \sum_{\mathbf{p}_1, \nu_1, n} V_{\nu\nu_1} \left[\hat{\mathbf{p}}, \hat{\mathbf{p}}_1 \right] \mathcal{F}_{\nu_1}^{+}(i\omega_n, \mathbf{p}_1) , \quad (1.12)$$

gdzie górny indeks "-" w (1.11) oznacza operację odbicia czasu, a $\mathcal{G}_{\nu}(i\omega_n, \mathbf{p})$ i $\mathcal{F}_{\nu}(i\omega_n, \mathbf{p})$ są funkcjami Greena dla układu nadciekłego. Potencjał $V_{\nu\nu_1}(\mathbf{p}\mathbf{p}_1)$, powodujący niestabilność układu na tworzenie stanów par cząstek, jest uogólnieniem tego typu oddziaływania z układu trójwymiarowego dla systemu z dyskretnymi liczbami kwantowymi ν [17, 18] :

$$V_{\nu\nu_1} \left[\hat{\mathbf{p}}, \hat{\mathbf{p}}_1 \right]_{\alpha\beta\gamma\delta} = \frac{3}{2}g \left[\hat{p}\hat{p}_1 \sin\vartheta_{\nu} \sin\vartheta_{\nu_1} + \cos\vartheta_{\nu} \cos\vartheta_{\nu_1} \right] \times \\ \left[\delta_{\alpha\gamma} \delta_{\beta\delta} + \delta_{\alpha\delta} \delta_{\beta\gamma} \right] , \quad (1.13)$$

gdzie $\cos\vartheta_{\nu} = \varepsilon_{\nu}/\mu$.

Jest to tak zwana nadciekłość typu p , którą będziemy też nazywać sparowaniem typu p . Parametr porządku $\Delta_{\nu}(\mathbf{p})$ dla tego typu oddziaływania jest zdefiniowany jako [19, 20] :

$$\Delta_{\nu}(\mathbf{p}) = \Delta[\mathbf{d}(\hat{\mathbf{p}}, \nu) \hat{\sigma}] i\sigma_y, \quad (1.14)$$

gdzie $\hat{\sigma} = [\sigma_x, \sigma_y, \sigma_z]$ są macierzami Pauliego.

Wektor $\mathbf{d}(\hat{\mathbf{p}}, \nu)$ jest liniowy względem pędu $\mathbf{p}$ i jest reprezentowany tensorem d_{ij} [19,20] :

$$d_i(\hat{\mathbf{p}}, \nu) = d_{ij} \hat{p}_j \sin\theta_{\nu}. \quad (1.15)$$

Ze względu na znacznie większą grubość warstwy od średniej odległości międzyatomowej $d \gg p_0^{-1}$ przyjmujemy, że stała g (1.13) jest wielkością z układu trójwymiarowego, to znaczy, że nie zostały zmienione własności oddziaływań międzycząstkowych [17,18].

Prąd w układzie jest uwzględniony przez dokonanie transformacji Galileusza [21] :

$$\mathbf{p} \longrightarrow \mathbf{p} + m\mathbf{v}, \quad (1.16)$$

gdzie $\mathbf{v}$ jest prędkością przepływu nadciekłego. Przy założeniu jednorodności przestrzeni OXY i małości prędkości nadciekłej $v \ll p_F/m$, transformacja (1.16) jest równoważna, z dokładnością do wyrażen liniowych względem $\mathbf{v}$, przekształceniu :

$$i\omega_n \longrightarrow i\omega_n - \mathbf{v}\mathbf{p} \quad (1.17)$$

w równaniach (1.9)-(1.12).

Cienka warstwa na niejednorodnej powierzchni

Jest to model zaproponowany przez Tešanović'a i Vallsa [22] w oparciu o warunki brzegowe Czaplika i Entina [23] dla jednocząstkowej funkcji falowej $\Psi(x, y, z)$:

$$\Psi(x, y, d) + u(x, y) \frac{\partial}{\partial z} \Psi(x, y, z) \Big|_{z=d} = 0 . \quad (1.18)$$

Równanie (1.18) opisuje z dokładnością do wyrazów liniowych względem $u(x, y)$ znikanie funkcji falowej Ψ na nieregularnej powierzchni $z = d + u(x, y)$ tzn. $\Psi(x, y, d+u(x, y)) = 0$. Funkcja losowa $u(x, y)$ określa małe zmiany grubości cienkiej warstwy, spowodowane nierównościami powierzchni podłoża. Zmienna losowa $u(x, y)$ jest nieskorelowaną zmienną gaussowską o dwóch pierwszych momentach :

$$\langle u(x, y) \rangle = 0 , \quad (1.19)$$

$$\langle u(x, y) u(x', y') \rangle = w^2 \delta(x-x') \delta(y-y') , \quad (1.20)$$

Parametr $w \ll d$ jest bezwzględną, średnią wielkością nierówności podłoża. Równania (1.18)-(1.20) opisują powierzchnię z nieskorelowanymi, punktowymi niejednorodnościami. Model ten nie uwzględnia defektów liniowych takich jak rysy, a także dużych wypukłości i wgłębień powierzchni. Jego adekwatność będzie jeszcze dyskutowana w rozdziale 4.

Na swobodną powierzchnię cienkiej warstwy ^3He jest nałożony warunek znikania jednocząstkowej funkcji falowej :

$$\Psi(x,y,0) = 0 . \quad (1.21)$$

Rozwiązanie równania Schrödingera przy użyciu rachunku zaburzeń zbliżonego do technik diagramowych [23] daje dodatkową energię oddziaływania kwazicząstek z nierównościami podłoża $\Sigma_{\nu}(i\omega_n)$, określoną zgodnie z teorią Abrikosowa-Gorkowa [24,25] domieszkowanych metali jako [22,A1] :

$$\Sigma_{\nu}(i\omega_n) = \frac{\omega^2}{2m^2d^2} \left(\frac{\nu\pi}{d} \right)^2 \frac{1}{S} \sum_{\mathbf{p}, \nu'} \left(\frac{\nu'\pi}{d} \right)^2 \mathcal{G}_{\nu'}(i\omega_n, \mathbf{p}), \quad (1.22)$$

gdzie S oznacza powierzchnię próbki.

Ponieważ $\Sigma_{\nu}(i\omega_n)$ jest prawie stałą funkcją częstości Matsubary ω_n , jak to wykazują nasze obliczenia numeryczne, energię wzbudzeń jednocząstkowych można zapisać jako :

$$\epsilon_{\nu}(\mathbf{p}) = \frac{\mathbf{p}^2}{2m} + \frac{1}{2m} \left(\frac{\nu\pi}{d} \right)^2 + \Sigma_{\nu} , \quad (1.23)$$

gdzie $\Sigma_{\nu} = \Sigma_{\nu}(i\omega_n)$. Wielkość $\Sigma_{\nu}(i\omega_n)$ jest liczbą urojoną i zgodnie z równaniem (1.23) $i\Sigma_{\nu}^{-1}$ ma interpretację skończonego czasu życia kwazicząstek. Równanie (1.22) jest warunkiem na część urojoną energii oddziaływania kwazicząstek

z niejednorodnościami podłoża, jej część rzeczywista zmienia jedynie potencjał chemiczny μ .

Wprowadzając liczbę ν_0 przez : $\frac{1}{2m} \left(\frac{\nu_0 \pi}{d} \right)^2 = \mu$ i oznaczając $\epsilon_\nu = \frac{1}{2m} \left(\frac{\nu \pi}{d} \right)^2$ zgodnie z (1.4) otrzymujemy zbiór wartości liczby kwantowej $\nu = 1, \dots, \nu_c$, gdzie ν_c jest częścią całkowitą ν_0 . Kwantowanie jest takie, jak w nieskończonej studni potencjału.

Ze względu na częste stosowanie w dalszej części rozprawy, wygodnie jest wprowadzić parametr :

$$\Gamma = \frac{\pi^4}{2md^2} \frac{\omega^2}{d^4} \nu_0^5, \quad (1.24)$$

oraz wielkości Δ_0 i T_{c0} będące odpowiednio : przerwą energetyczną w temperaturze $T=0$ i temperaturą krytyczną T_c^F dla warstwy o gładkich powierzchniach ($\Gamma=0$). Krytyczna wartość Γ , dla której układ staje się normalny , jest oznaczona przez Γ_c .

Funkcja Greena dla układu w stanie normalnym jest określona przez (1.9) przy uwzględnieniu (1.23). Równania (1.9)-(1.13) i (1.22)-(1.23) opisują układ nadciekły na porowatej powierzchni.

W obu modelach, ze względu na duży wpływ podłoża, zaniedbujemy oddziaływania kwazicząstkowe.

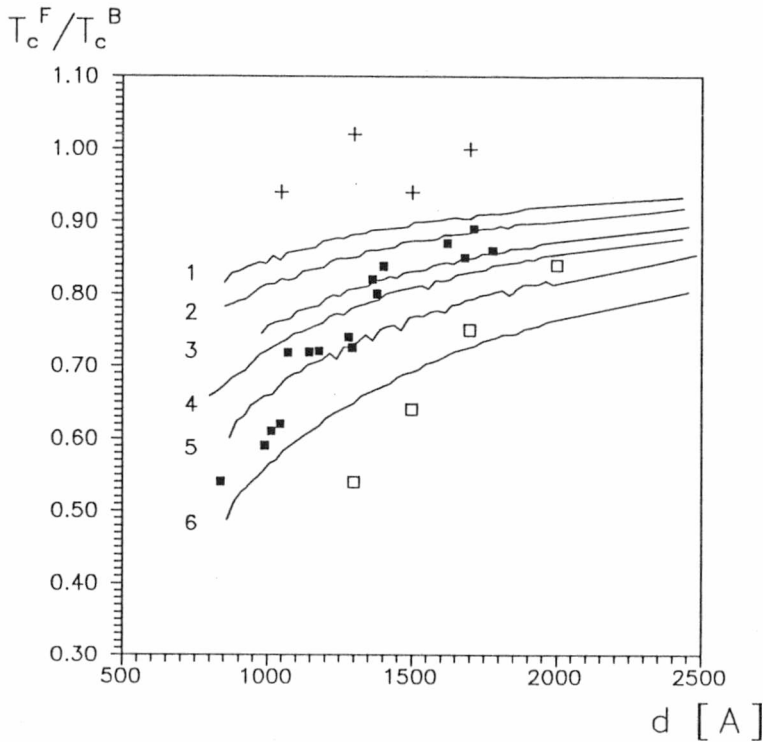
2. WPŁYW ODDZIAŁYWANIA VAN DER WAALSA NA TEMPERATURĘ KRYTYCZNĄ W CIENKIEJ WARSTWIE

Temperaturę przejścia fazowego ze stanu nadciekłego do stanu normalnego T_C^F w cienkiej warstwie ^3He wyznaczamy na podstawie równań (1.10)-(1.12) z warunkiem $\Delta=0$ [A4, A6] :

$$\frac{T_C^F}{T_C^B} = \left[\frac{2\omega_c e^\gamma}{\pi T_C^B} \right] \left[1 - 2p_0 d \left[3\pi \sum_{\nu=1}^{\nu_c} \sin^2 \vartheta_\nu \right]^{-1} \right]^{-1} \quad (2.1)$$

gdzie T_C^B oznacza temperaturę przejścia fazowego w układzie trójwymiarowym, a ω_c jest energetycznym parametrem obciążenia. W obliczeniach przyjęto $\omega_c/T_C^B=100$, zgodnie z odpowiednimi relacjami tych wielkości w teorii BCS. Parametr ω_c powinien być interpretowany jako graniczna wartość energii, dla której są określone kwazicząstki.

Krotschek et al. [9] pokazali, że warstwa ^4He zmniejsza oddziaływanie ^3He z podłożem - gęstszym pokryciem powierzchni przez ^4He odpowiadają mniejsze wartości efektywnego potencjału działającego na atomy ^3He . W naszej pracy rolę tego potencjału spełnia $V_2(z)$ (1.2). Badamy wpływ zmian parametrów V i a (1.2) na wartości temperatury krytycznej (2.1). Zależność T_C^F od grubości warstwy d , dla kilku wartości potencjału $V_2(z)$ jest przedstawiona na rys.1.



Rys.1: Temperatura krytyczna T_c^F jako funkcja grubości warstwy d dla potencjałów o parametrach :

1. $d/a=50$, $2mVa^2=2500\pi^2$, 2. $d/a=40$, $2mVa^2=10000\pi^2$
3. $d/a=30$, $2mVa^2=90000\pi^2$, 4. $d/a=50$, $2mVa^2=100\pi^2$
5. $d/a=20$, $2mVa^2=100\pi^2$, 6. $d/a=30$, $2mVa^2=100\pi^2$.

Są tu również zaznaczone doświadczalne wyniki Harrisona et al. [2-3] dla cienkiej warstwy ^3He znajdującej się bezpośrednio na powierzchni adsorbentu (puste kwadraty) i izolowanej od podłoża warstwą ^4He (krzyżyki). Pokrycie powierzchni przez ^4He znacznie zwiększa wartości temperatur T_c^F - są one prawie równe temperaturze krytycznej T_c^B . Zmniejszenie potencjału $V_2(z)$ prowadzi także do tego

samemu efektowi, świadczą o tym krzywe otrzymane teoretycznie (rys.1). Jednakże, w przedstawionym modelu niemożliwe jest osiągnięcie mierzonej zależności T_c^F od grubości warstwy. Na rysunku 1 przedstawiliśmy również rezultaty pomiarów Xu i Crookera [4], są one zaznaczone pełnymi kwadratami.

Tešanović i Valls znaleźli temperaturę T_c^F jako funkcję grubości warstwy na nieregularnym podłożu [22]. Także i ta, teoretycznie otrzymana zależność nie odzwierciedla wyników doświadczalnych [1,22,26]. Istnieje fenomenologiczny model KKR [27], który daje rezultat bardzo bliski do wyników Xu i Crookera [4], jednak odbiega od zależności otrzymanych przez Harrisona et al. [1-3]. Charakterystyczna jest duża zależność mierzonej temperatury przejścia fazowego od rodzaju przygotowanego podłoża [1-4]. Komplikuje to opracowanie teoretycznego modelu, nie zawierającego zbyt wielu parametrów.

3. NADCIĘŁY PRĄD MASY W CIENKIEJ WARSTWIE ^3He PRZY UWZGLĘDNIENIU ODDZIAŁYWAŃ VAN DER WAALSA

Parametr porządku

Para kwazicząstek utworzona przez wzajemne oddziaływanie typu p ma moment pędu $L=1$ i spin $S=1$ [19,20]. Parametr porządku jest reprezentowany przez tensory sferyczne $|J,m\rangle$, gdzie $J=L+S$ jest całkowitym momentem pędu pary Coopera, a m - jego rzutem na oś OZ. Dwuwymiarowość pędu sprawia, że rzut momentu pędu m_L na oś prostopadłą do powierzchni warstwy (oś OZ) przyjmuje tylko dwie wartości $m_L = \pm 1$. Dlatego parametr porządku $\Delta_{\nu}(\mathbf{p})$ jest reprezentowany przez sześć tensorów sferycznych $|J,m\rangle$:

$$\Delta_{\nu}(\mathbf{p}) = \Delta \sin\vartheta_{\nu} \left[a_{0,0}|0,0\rangle + a_{1,0}|1,0\rangle + a_{1,1}|1,1\rangle + \right. \\ \left. a_{1,-1}|1,-1\rangle + a_{2,2}|2,2\rangle + a_{2,-2}|2,-2\rangle \right], \quad (3.1)$$

co na mocy (1.15) prowadzi do reprezentacji tensora d_{ij} :

$$d_{ij} = \frac{1}{\sqrt{3}}a_{0,0}\delta_{ij} + \frac{1}{\sqrt{2}}a_{1,0}\left[\delta_{iy}\delta_{jx} - \delta_{ix}\delta_{jy} \right] + \\ \frac{1}{2}ia_{1,1}\delta_{iz}\left[\delta_{jx} + i\delta_{jy} \right] + \frac{1}{2}ia_{1,-1}\delta_{iz}\left[\delta_{jx} - i\delta_{jy} \right] +$$

$$\begin{aligned}
& + \frac{1}{2} a_{2,2} \left[\delta_{ix} + i\delta_{iy} \right] \left[\delta_{jx} + i\delta_{jy} \right] + \\
& - \frac{1}{2} a_{2,-2} \left[\delta_{ix} - i\delta_{iy} \right] \left[\delta_{jx} - i\delta_{jy} \right] . \quad (3.2)
\end{aligned}$$

Równanie na szczelinę (1.12) może być zapisane w uproszczonej postaci przy użyciu tensora d_{ij} :

$$d_{ij} \hat{p}_j \sigma_i = T_{jl}^{ik} d_{kl} \hat{p}_l \sigma_k , \quad (3.3)$$

gdzie T jest operatorem działającym po prawej stronie równania (1.12), górne indeksy są indeksami spinowymi, a dolne odpowiadają indeksom orbitalnym. Gdy układ nie znajduje się w zewnętrznym polu magnetycznym, operator T nie zależy od zmiennych spinowych :

$$T_{jl}^{ik} = T_{jl} \delta_{ik} \quad (3.4)$$

i równanie na szczelinę (3.3) redukuje się do :

$$d_{ij} \hat{p}_j = T_{jl} d_{il} \hat{p}_l . \quad (3.5)$$

Znaleźliśmy dwa tensory d_{ij} postaci (3.2) będące niezmiennikami równania (3.5). Odpowiadają one dwóm dwuwymiarowym fazom nadciekłego ^3He , dwuwymiarowej fazie A :

$$d = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \Delta_1 & i\Delta_2 \end{bmatrix}, \quad (3.6)$$

$$\Delta_\nu(\mathbf{p}) = \sqrt{1.5} \sin\vartheta_\nu \left[\Delta_1 \hat{p}_x + i\Delta_2 \hat{p}_y \right] \sigma_z i\sigma_y \quad (3.7)$$

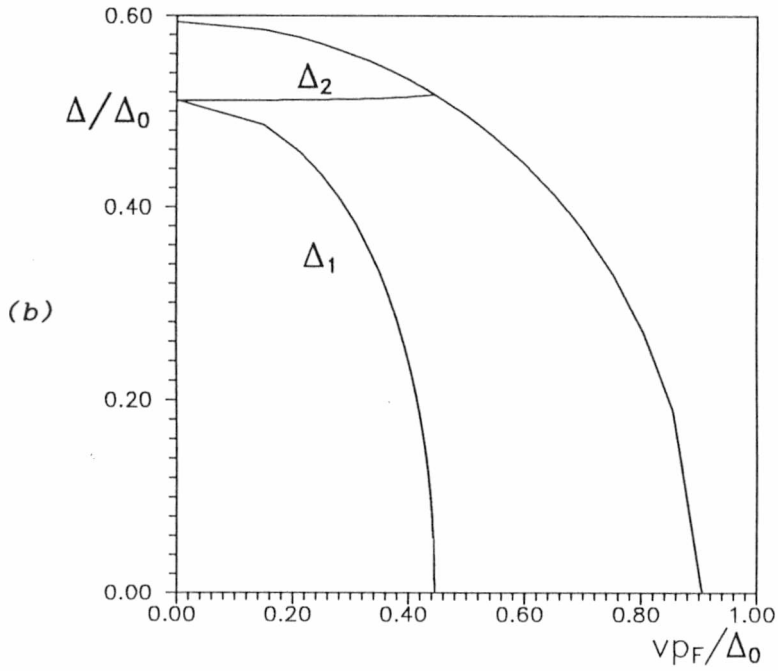
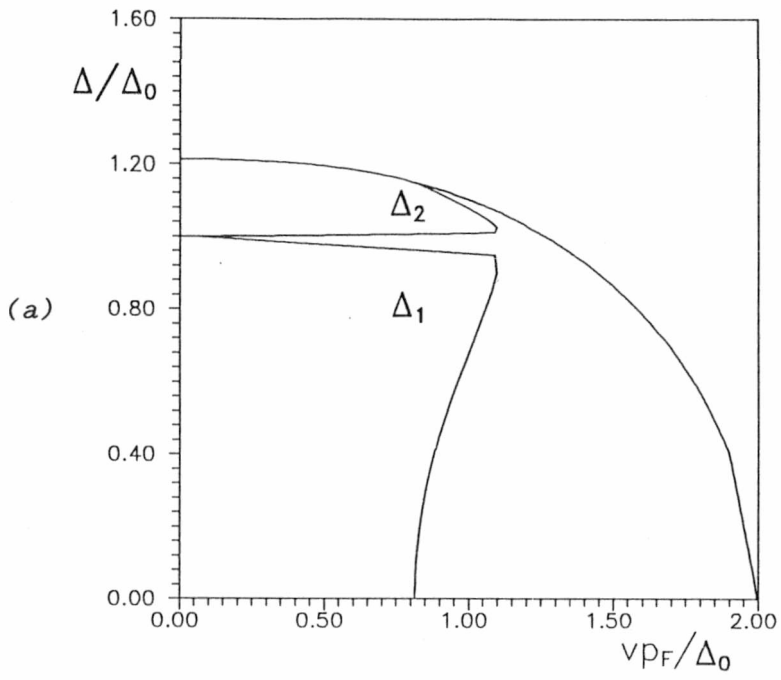
i dwuwymiarowej fazie B :

$$d = \begin{bmatrix} \Delta_1 & -\Delta_2 \\ \Delta_2 & \Delta_1 \\ 0 & 0 \end{bmatrix}, \quad (3.8)$$

$$\Delta_\nu(\mathbf{p}) = \sqrt{1.5} \sin\vartheta_\nu \left[\left[\Delta_1 \hat{p}_x - \Delta_2 \hat{p}_y \right] \sigma_x + \left[\Delta_2 \hat{p}_x + \Delta_1 \hat{p}_y \right] \sigma_y \right] i\sigma_y. \quad (3.9)$$

Parametry Δ_1, Δ_2 są określone przez równanie na szczelinę (3.5) i dla każdej fazy mogą przyjmować inne wartości.

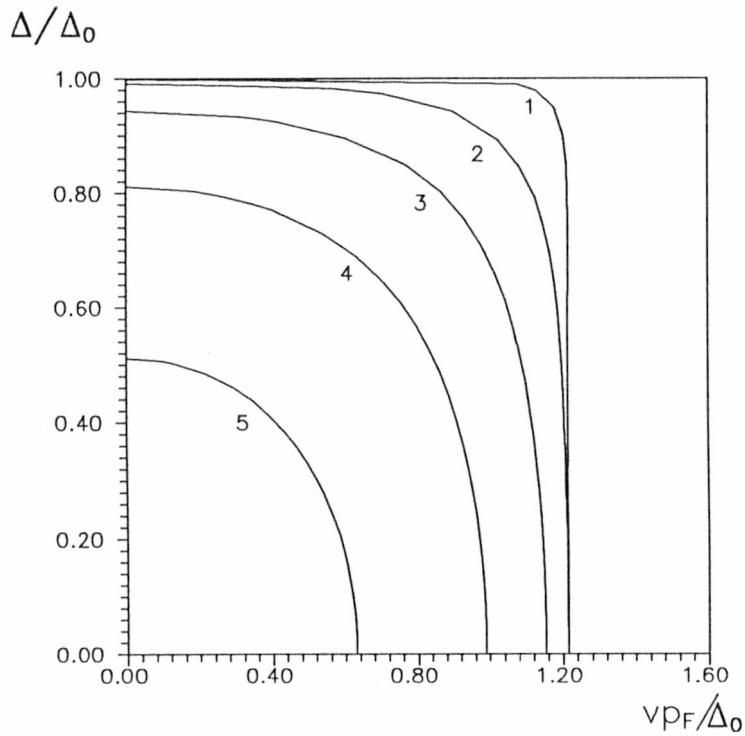
Rozpatrywane stany nadciekłe (3.6)-(3.9) są uogólnieniem dwuwymiarowych faz otrzymanych przez Briusowa i Popowa [28] na układ bez symetrii obrotowej. Prąd w płaszczyźnie warstwy łamie symetrię względem obrotów wokół osi prostopadłej do powierzchni warstwy. Mimo to, każdej fazie odpowiada inna liczba kwantowa m rzutu całkowitego momentu pędu na oś OZ, tak jak w przypadku układu zachowującego symetrię obrotową względem osi OZ. Na rysunkach 2a-b jest przedstawiony parametr porządku dwuwymiarowej fazy A (3.7), jako funkcja prędkości prądu nadciekłego $\mathbf{v}$.



Rys.2: Parametr porządku (Δ_1, Δ_2) dwuwymiarowej fazy A w funkcji prędkości przepływu nadciekłego v dla :
 (a) $T=0.1T_c^F$, (b) $T=0.9T_c^F$ w warstwie o grubości $p_0 d=100$ i potencjale zadanym parametrami $d/a=100$, $2mVa^2=\pi^2$.

Obserwujemy pojawienie się dwuwymiarowej fazy polarnej (linia przerywana) gdy parametr $\Delta_1=0$. Przejście fazowe z dwuwymiarowej fazy A ($\Delta_1 \neq 0$, $\Delta_2 \neq 0$) do dwuwymiarowej fazy polarnej ($\Delta_1=0$) jest przejściem fazowym pierwszego rodzaju dla niskich temperatur (rys.2a), a drugiego rodzaju dla wysokich temperatur (rys.2b).

Zależność przerwy energetycznej w dwuwymiarowym stanie nadciekłym B (3.9) od prędkości v jest przedstawiona na rys.3.



Rys.3 Zależność parametru porządku Δ dwuwymiarowej fazy B od prędkości przepływu nadciekłego v dla temperatur :

1. $T=0.1T_C^F$, 2. $T=0.3T_C^F$, 3. $T=0.5T_C^F$, 4. $T=0.7T_C^F$, 5. $T=0.9T_C^F$
w warstwie z rys.2

Dwuwymiarowe fazy A i B są stabilne i zdegenerowane, gdy w układzie nie ma prądu [28]. W A5 wykazaliśmy, że prąd nadciekły stabilizuje dwuwymiarową fazę A i dwuwymiarową fazę polarną.

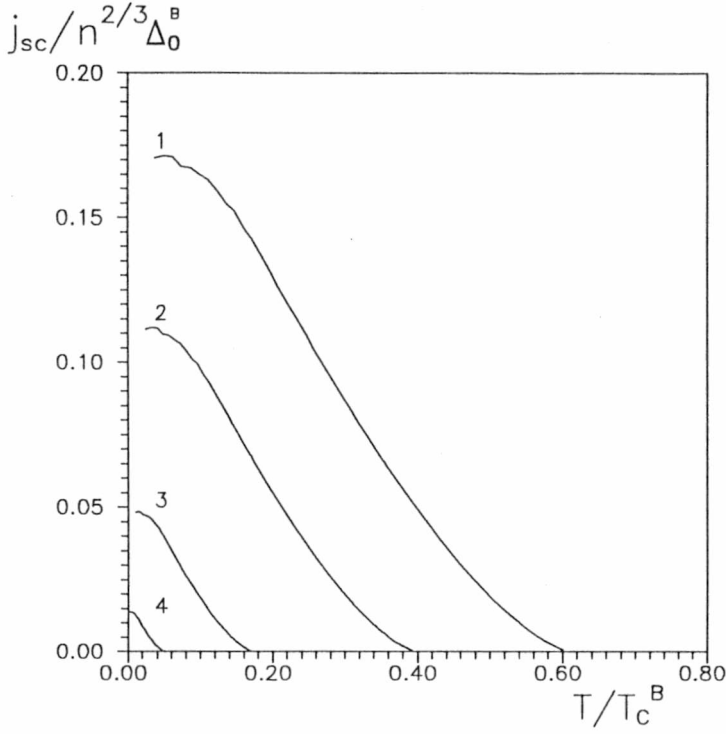
Nadciekły prąd masy

Nadciekły prąd masy j_s jest określony jako [21]:

$$j_s = \frac{1}{\omega} T \sum_{\mathbf{p}, \nu, n} \frac{1}{m} (\mathbf{p} + m\mathbf{v}) \mathcal{E}_{\nu}(i\omega_n, \mathbf{p}) , \quad (3.10)$$

gdzie ω jest objętością próbki.

W pracy A5 wykazano, że maksymalny prąd nadciekły j_{sc} , nazywany prądem krytycznym, jest osiągany w dwuwymiarowej fazie A. Zgodnie z naszą hipotezą z rozdz.2, pokrywanie podłoża warstwami ^4He zmniejsza wielkość oddziaływania typu van der Waalsa (1.2) atomów ^3He z powierzchnią adsorbentu. Prowadzi to do większych wartości prądów krytycznych, które przedstawiamy na rys.4. Krzywym od 1 do 4 odpowiadają rosnące wielkości potencjału $V_2(z)$.



Rys.4 Krytyczny prąd nadciekły w funkcji temperatury dla $p_0 d = 100$ i 1. $d/a=10$, $2mVa^2=100\pi^2$, 2. $d/a=5$, $2mVa^2=400\pi^2$, 3. $d/a=2.9$, $2mVa^2=1225\pi^2$, 4. $d/a=2$, $2mVa^2=2500\pi^2$; Δ_0^B jest przerwą energetyczną układu trójwymiarowego w $T=0$.

W pracy A6 otrzymaliśmy nadciekły prąd krytyczny, jako funkcję grubości warstwy ^3He dla $T=0$:

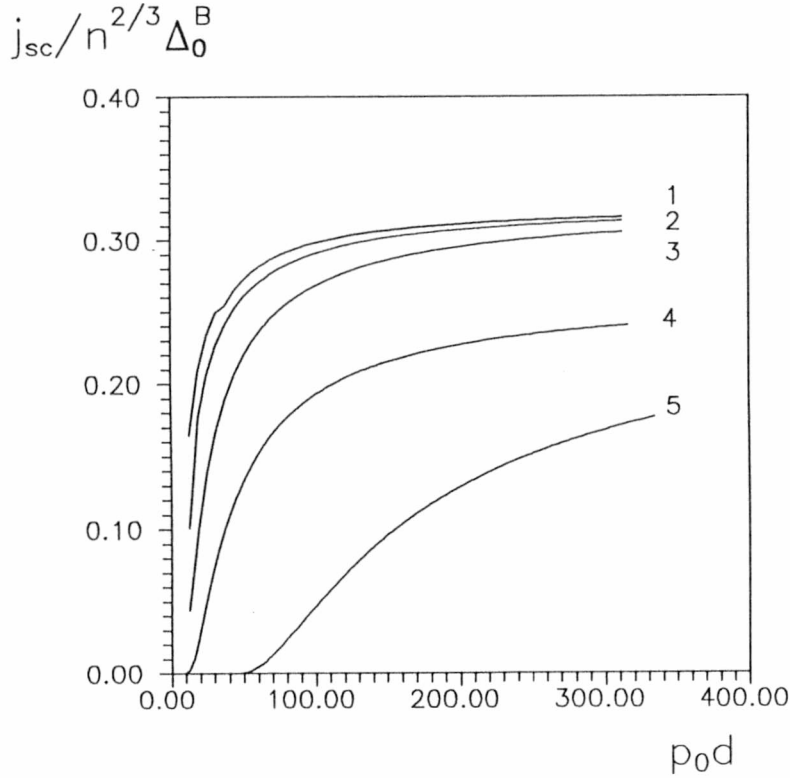
$$j_{sc} = (3/8\pi)^{1/6} \left[\frac{1}{p_0 d} \sum_{\nu=1}^{\nu_c} \sin^2 \vartheta_{\nu} \right]^{1/2} \Delta_0 n^{2/3}, \quad (3.11)$$

gdzie $\Delta_0 = \Delta(\nu=0, T=0)$ jest parametrem porządku cienkiej warstwy nadpłynnego ^3He bez nadciekłego prądu masy w $T=0$ i jest związany z przerwą energetyczną Δ_0^B z układu trójwymiarowego dla $T=0$:

$$\frac{\Delta_0}{\Delta_0^B} = \left[\frac{2}{3} \right]^{1/2} \exp[-\langle \ln(\sin \vartheta_\nu) \rangle] \left[\frac{2\omega_c e^\gamma}{\pi T_c^B} \right]^{1 - 2p_0 d \left[3\pi \sum_{\nu=1}^{\nu_c} \sin^2 \vartheta_\nu \right]^{-1}}, \quad (3.12)$$

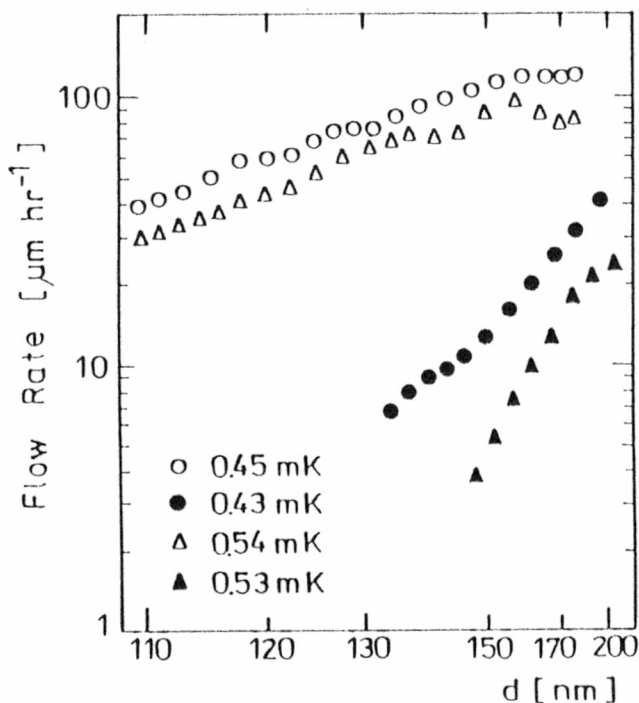
$$a \quad \langle \ln(\sin \vartheta_\nu) \rangle \equiv \left[\sum_{\nu=1}^{\nu_c} \sin^2 \vartheta_\nu \right]^{-1} \sum_{\nu=1}^{\nu_c} \sin^2 \vartheta_\nu \ln(\sin \vartheta_\nu).$$

Zależność (3.11) dla kilku wartości potencjału $V_2(z)$ jest przedstawiona na rys.5.



Rys.5 Krytyczny prąd nadciekły w $T=0$, jako funkcja grubości warstwy dla potencjałów : 1. $d/a=100$, $2mVa^2=0.1\pi^2$, 2. $d/a=100$, $2mVa^2=0.5\pi^2$, 3. $d/a=100$, $2mVa^2=10\pi^2$, 4. $d/a=50$, $2mVa^2=50\pi^2$, 5. $d/a=10$, $2mVa^2=50\pi^2$

Porównanie rys.5 z rys.6, prezentującym wyniki doświadczalne [3], uwidacznia podobny wpływ potencjału $V_2(z)$ i pokryć podłoża przez ^4He na zmianę j_{sc} z grubością warstwy.

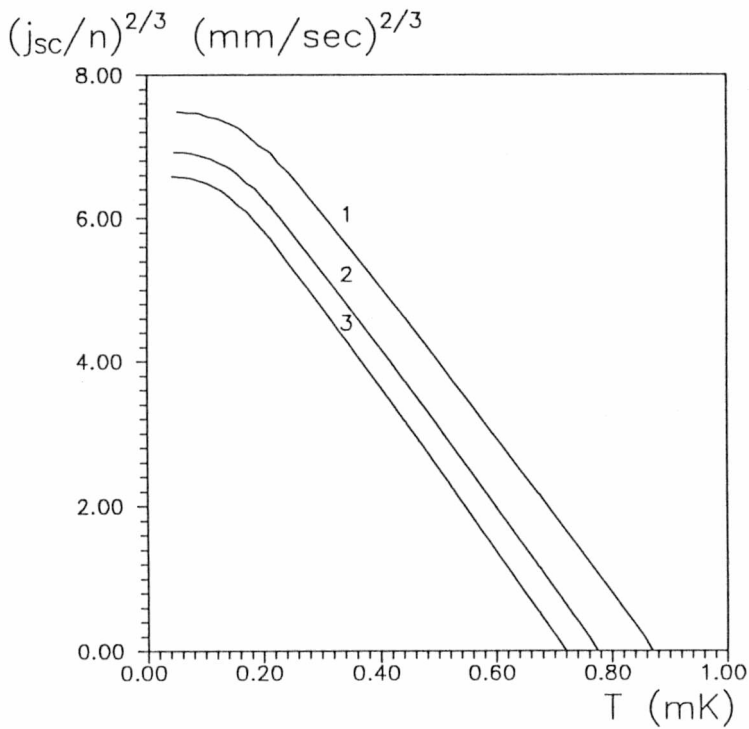


Rys.6 Nadciekły przepływ ^3He w zależności od grubości warstwy dla czystego ^3He (czarne symbole) i ^3He izolowanego atomową warstwą ^4He od powierzchni podłoża (białe symbole) [3].

Próbie odtworzenia wyników doświadczalnych Harrisona et al. [1,2] oraz Xu i Crookera [4] przedstawiamy na rys.7 i 8. Wyniki teoretyczne są o rząd wielkości większe niż rezultaty doświadczalne. W rozległym obszarze temperatur (od $T \approx 0.2T_C^F$ do $T = T_C^F$) nadciekły prąd krytyczny j_{sc} można przybliżyć zależnością [A6] (rys.7) :

$$j_{sc} \propto \left[1 - \frac{T}{T_C^F} \right]^{3/2}, \quad (3.13)$$

jest ona zgodna z wynikami pomiarów [1,2,4]. Ta sama funkcja potęgowa została otrzymana w obszarze Ginzburga-Landaua przez Jacobsena i Smitha [11].



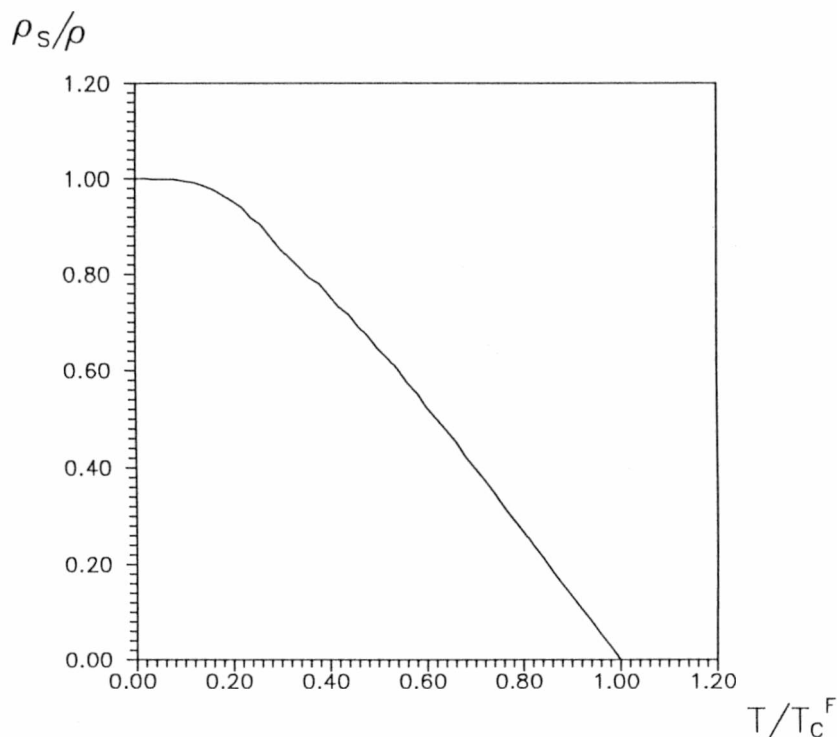
Rys.7 Krytyczny prąd nadciekły w funkcji temperatury dla warstw o grubości d i temperaturze krytycznej T_C^F :

1. $d=1200 \text{ \AA}$, $T_C^F/T_C^B = 0.94$,
2. $d=1100 \text{ \AA}$, $T_C^F/T_C^B = 0.84$,
3. $d=1000 \text{ \AA}$, $T_C^F/T_C^B = 0.78$.

Gęstość fazy nadciekłej ρ_{sc} , zdefiniowana przez :

$$j_{sc} = v \rho_{sc} , \quad (3.16)$$

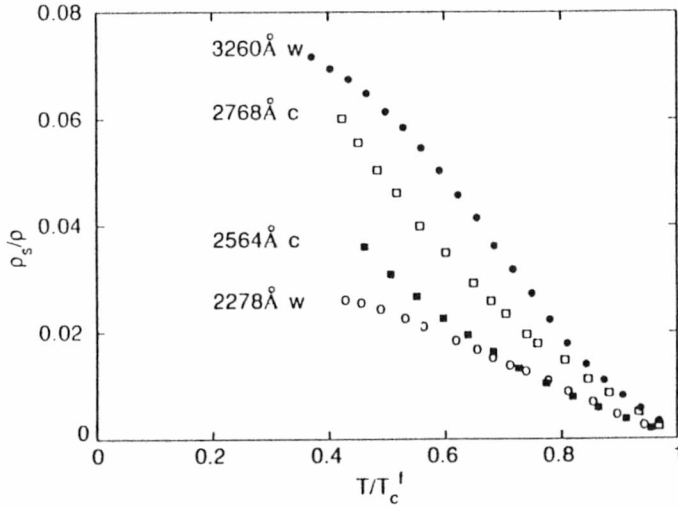
jest wykreślona na rys.8. W jednostkach przyjętych na rysunku otrzymujemy uniwersalną krzywą, niezależną od wyboru potencjału $V_2(z)$. Ponieważ oddziaływanie z potencjałem podłoża zmienia jedynie wartości poziomów energetycznych ε_v i nie jest efektem zaburzającym koherencję par Coopera, w temperaturze $T=0$ cały układ jest nadciekły - $\rho_{sc}=\rho$ (rys.8).



Rys.8 Gęstość fazy nadciekłej w zależności od temperatury.

W pomiarach Xu i Crookera [4] każdej warstwie, o grubości

i temperaturze T_c^F takiej jak w warstwach z rys.8, odpowiada inna krzywa zależności nadciekłego prądu krytycznego j_{sc} od temperatury (rys.9). Świadczy to o innym efekcie powodującym znaczne obniżenie wartości prądu nadciekłego.



Rys.9 Mierzona gęstość fazy nadciekłej [4].

W A5 przyjęliśmy uproszczony model potencjału $V_2(z)$ (1.2), ale nawet potencjały bardziej zbliżone do rzeczywistego oddziaływania z podłożem dadzą prąd krytyczny tego samego rzędu. Wynika to z ogólności zastosowanej metody - rezultaty zależą tylko od spektrum ϵ_ν (1.5), (1.9). Prądy krytyczne zostały wyznaczone dla ściśle określonych temperatur T_c^F (rys.7 i 8), a więc dla ustalonych wartości poziomów energetycznych ϵ_ν . Dowolny potencjał, aby odtworzyć zadaną temperaturę T_c^F (2.1) musi dawać widmo ϵ_ν zbliżone do otrzymanego przy użyciu potencjału $V_2(z)$, a w konsekwencji prawie ten sam prąd krytyczny j_{sc} .

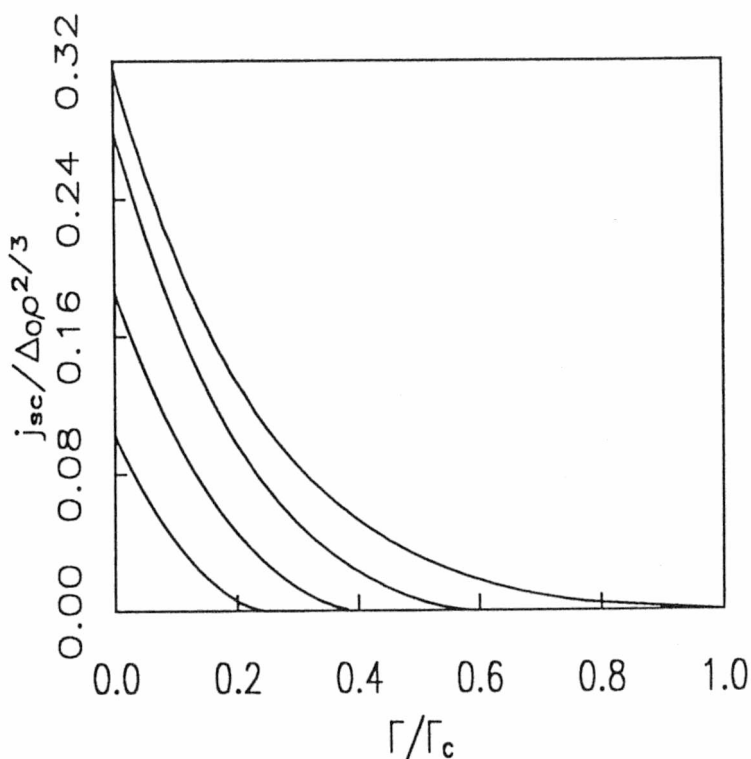
4. NADCIĘKŁY PRĄD MASY ^3He W CIENKIEJ WARSTWIE NA NIEJEDNORODNYM PODŁOŻU

Nieregularności podłoża prowadzą do skończonego czasu życia kwazicząstek (rozdz.1). Jest to efekt powodujący rozrywanie par Coopera.

W pracy A7 przeprowadziliśmy analizę wpływu oddziaływania kwazicząstek ^3He z niejednorodnościami podłoża na nadciekłość w cienkiej warstwie. Do zbadania własności faz opisanych w rozdz.3 wykorzystaliśmy model przedstawiony w rozdz.1. Metoda postępowania przy obliczaniu prądów nadciekłych, została przedstawiona w części 3 rozprawy. W przypadku układu o porowatej powierzchni występuje jedynie ta różnica, że ze względu na istnienie wielkości $\sum_{\nu} (i\omega_n)$ (1.22)-(1.23) niemożliwe jest analityczne wykonanie sumy po częstościach Matsubary w równ.(1.12) i (3.10). Z tego względu analitycznie całkujemy po pędach, a sumy po częstościach ω_n wykonujemy numerycznie [A1-A3, A7].

Na rys.10 przedstawiono wpływ wielkości niejednorodności podłoża (parametr Γ) na krytyczny prąd nadciekły. Parametr Γ (1.24) jest proporcjonalny do bezwzględnej wartości nierówności powierzchni w (1.20), czyli mniejszym wartościom Γ odpowiadają gładze powierzchnie. W rzeczywistości podłoże jest wygładzane przez pokrywanie warstwami ^4He . Wraz ze wzrostem gęstości powierzchniowej ^4He zapełniane są wgłębienia powierzchni i tworzona jest jednorodna struktura

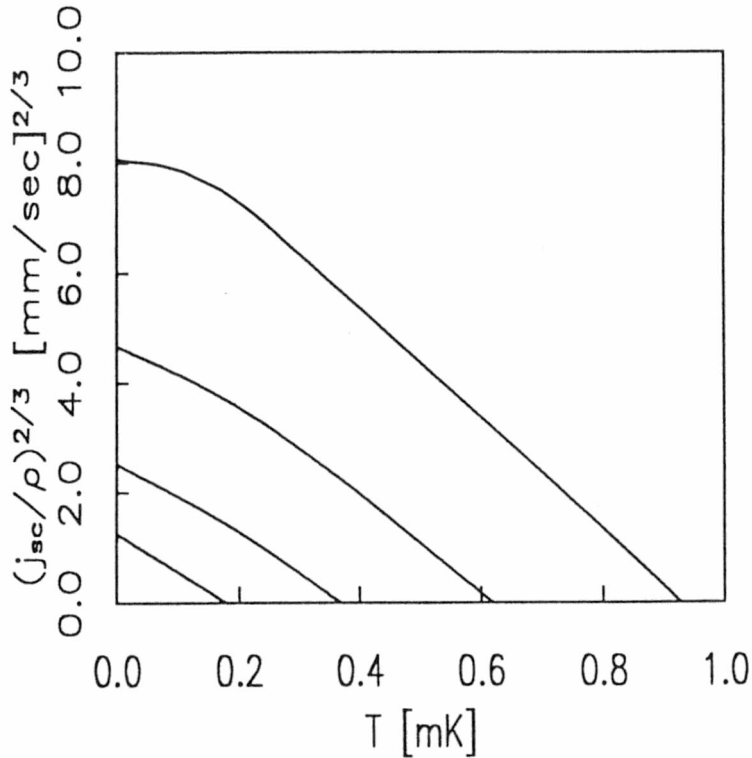
podłoża [2,3,5,6]. Jest to drugie zjawisko, oprócz zmiany potencjału adsorbentu (rozdz.2 i 3), wywołane wprowadzeniem izolującej warstwy ^4He . Zmiana wartości parametru Γ odpowiada efektowi pokrywania podłoża przez ^4He .



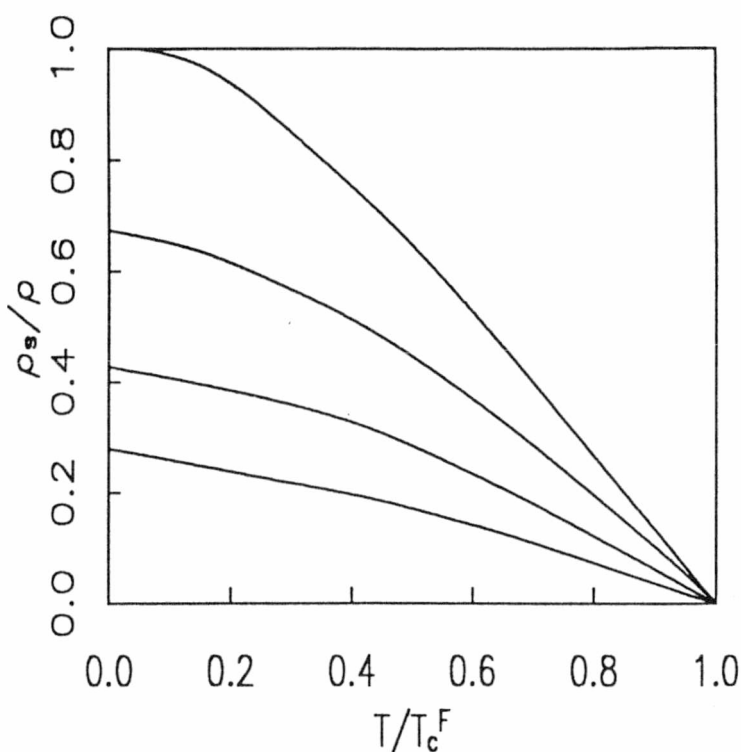
Rys.10 Krytyczny prąd nadciekły w funkcji niejednorodności powierzchni podłoża Γ dla temperatur $T = 0, 0.2T_c^F, 0.4T_c^F, 0.6T_c^F$ i grubości warstwy $p_0 d = 100$.

Na rys.11 i 12 prezentujemy wyniki teoretyczne otrzymane w tym formalizmie dla warstw doświadczalnie badanych przez Harrisona et al.[1-3] (rys.11) oraz Xu i Crookera [4] (rys.12). Tak, jak w przypadku modelu zawierającego

oddziaływanie typu van der Waalsa (rozdz.3) otrzymujemy wyniki o rząd wartości większe od mierzonych. Zachowana jest ta sama przybliżona zależność j_{sc} od temperatury T (3.13).



Rys.11 Krytyczny prąd nadciekły j_{sc} w funkcji temperatury dla warstw o grubościach d i temperaturach krytycznych T_c^F określonych na rys.7 oraz $d=900\text{\AA}$ i $T_c^F/T_c^B=0.68$.



Rys.12 Gęstość fazy nadciekłej ρ_{sc} w zależności od temperatury dla warstw o grubościach i temperaturach krytycznych zgodnych z rys.9 .

W odróżnieniu od uniwersalnej zależności z rys.8, każdej warstwie z niejednorodnościami powierzchni odpowiada inna krzywa na rys.12, tak jak w wynikach doświadczalnych [4] (rys.9). Jednak efekt rozpraszania na małych niejednorodnościach podłoża nie powoduje tak znacznego, jak obserwowane, obniżenia wartości prądów nadciekłych [1-6]. Pomiaru dają duże wartości temperatur przejść fazowych T_c^F (rys.1), przy jednocześnie małych wartościach prądów [1-6] (rys.9). Sugeruje to istnienie innego, oprócz rozpraszania na małych niejednorodnościach podłoża, zjawiska dużego oporu cienkiej warstwy na przepływ nadciekłego prądu masy. Może to być związane z dużymi, skorelowanymi niejednorodnościami, powodującymi lokalne zwężenia warstwy ^3He . Takie obszary

o małej grubości mogłyby obniżać przepływ prądu nadciekłego , a ze względu na małą objętość zawartej w nich cieczy, nie miałyby prawie wpływu na stan nadciekły układu, czyli na T_c^F . Nieregularność podłoża może również powodować powstawanie wirów, których dynamika może wpłynąć na wartości prądów nadciekłych [1,6,29].

Wszystkie doświadczenia [1-7], o których pisaliśmy, dotyczą warstw o grubościach rzędu stałej koherencji zależnej od temperatury $\xi(T)$ i większych od zakresu grubości rozpatrywanych w przyjętych modelach dwuwymiarowej nadciekłości (rozdz.1). Jednak ze względu na silne oddziaływanie układu nadciekłego z podłożem [30,31], parametr porządku może być przybliżony parametrem dwuwymiarowym i nasze teoretyczne rozważania mogą być porównywane z wynikami doświadczalnymi, jak to zostało zrobione w rozdz.2-4.

5. WPLYW NIEJEDNORODNOŚCI PODŁOŻA NA PRZEJŚCIA FAZOWE W CIENKICH WARSTWACH NADCIEKŁEGO ^3He

Bezszczelinowa nadciekłość dwuwymiarowej fazy A

W 1986 roku Tešanović i Valls [22] zauważyli, że w cienkich warstwach nadciekłego ^3He na porowatym podłożu możliwe jest zjawisko bezszczelinowej nadciekłości. Stan ten charakteryzuje brak przerwy energetycznej w widmie wzbudzeń jednocząstkowych, przy jednoczesnym istnieniu stanów koherentnych par kwazicząstek, czyli par Coopera. Jest to zjawisko znane z domieszkowanych układów nadprzewodzących, dla których zostało opracowane teoretycznie przez Abrikosowa i Gorkowa [24,25]. W przypadku nadciekłości typu p, anomalna funkcja Greena $\mathcal{F}$ jest funkcją nieparzystą pędu i tym samym parametr porządku Δ nie jest zmieniany bezpośrednio przez oddziaływanie z centrami rozpraszającymi, ale tylko poprzez renormalizację wzbudzeń jednocząstkowych (1.23) [22,32,33]. Oddziaływanie z nierównościami podłoża prowadzi do niejednakowej renormalizacji częstości Matsubary i parametru porządku Δ [22,A1], analogicznie do układów ze złamaną symetrią odbicia czasu [34]. Konsekwencją tego jest bezszczelinowa nadciekłość. W pracach A1 i A3 postulujemy także możliwość bezszczelinowej nadciekłości typu s (5.4) w cienkich warstwach. Taki typ nadciekłości ^3He może wystąpić dla pewnych koncentracji ^3He w mieszaninach z ^4He [35].

Rozpraszanie na niejednorodnościach podłoża zostało wprowadzone przez warunki brzegowe (1.18) i (1.21), które mają wpływ jedynie na dynamikę trójwymiarowych swobodnych fermionów. Przyjmujemy, jako konsekwencję nierówności $w \ll d \leq \xi_0$ (rozdz.1), że nadciekły kondensat jest ściśle dwuwymiarowy i nie nakładamy na niego żadnych dodatkowych warunków brzegowych. To znaczy, że anomalna energia własna Δ nie jest bezpośrednio zmieniona przez rozpraszanie na granicy warstwy. Prowadzi to do niejednakowej renormalizacji częstości ω_n i parametru porządku Δ , czyli do bezszczelinowości widma wzbudzeń jednocząstkowych.

^3He jest układem, którego nie można zanieczyścić i w ten sposób otrzymać bezszczelinowej nadciekłości. Dlatego cienkie warstwy stwarzają unikalną możliwość obserwacji tego zjawiska w ^3He . Warunki przejścia fazowego do stanu bezszczelinowego są określone przez grubość warstwy d i wielkość nierówności podłoża - parametr w (1.20). Dlatego istotnym jest zbadanie wpływu tych dwóch parametrów na występowanie fazy bezszczelinowej nadciekłości. Warunkiem bezszczelinowości widma wzbudzeń jednocząstkowych jest dodatniość funkcji gęstości stanów $N(\omega)$ na powierzchni Fermiego:

$$N(\omega = 0) > 0 \quad . \quad (5.1)$$

Z (5.1) otrzymujemy warunek na bezszczelinową nadciekłość [A1] typu p :

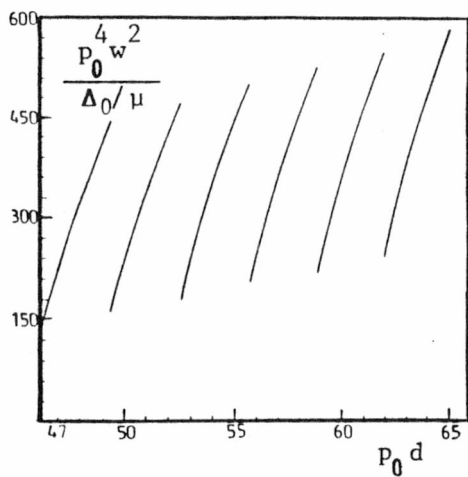
$$\left\{ \begin{array}{l} 1 = \frac{\zeta}{\nu_0} \sum_{\nu=1}^{\nu_c} \cos^4 \vartheta_{\nu} \left[\sin^2 \vartheta_{\nu} + \zeta^2 U_2^2 \cos^4 \vartheta_{\nu} \right]^{-1/2} \\ U_2 > 0 \end{array} \right. \quad (5.2)$$

i typu s:

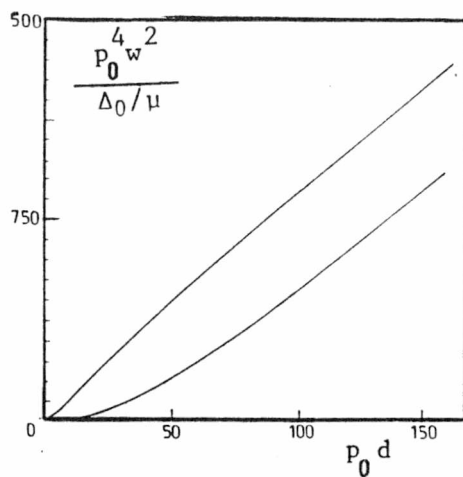
$$\left\{ \begin{array}{l} 1 = \frac{\zeta}{\nu_0} \sum_{\nu=1}^{\nu_c} \cos^4 \vartheta_{\nu} \left[1 + \zeta^2 U_2^2 \cos^4 \vartheta_{\nu} \right]^{-1/2} \\ U_2 > 0 \end{array} \right. \quad (5.3)$$

Parametr $\zeta = \Gamma/\Delta$ opisuje nieregularność podłoża. Minimalna wartość ζ spełniająca warunki (5.2), (5.3) określa krytyczną wartość nierówności podłoża u (1.20), przy której układ przechodzi w stan bezszczelinowej nadciekłości. Otrzymane numerycznie wartości u w zależności od grubości warstwy d są przedstawione na rys.13(a)-(b) (dla sparowania typu p) i rys.14(a)-(b) (dla sparowania typu s). Oscylacyjny charakter krzywych (rys.13(a) i 14(a)) wynika z dyskretności powierzchni Fermiego w cienkiej warstwie (rozdz.1). Jest to odpowiednik efektu de Haasa-van Alphen, przy zastąpieniu kwantującego działania pola magnetycznego skończoną grubością układu. Dla nadciekłości typu p istnieje pewna nieokreśloność

wyznaczenia dolnej krzywej z rys.13(b) [A1].

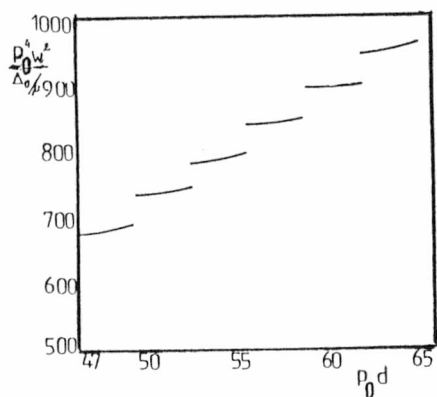


(a)

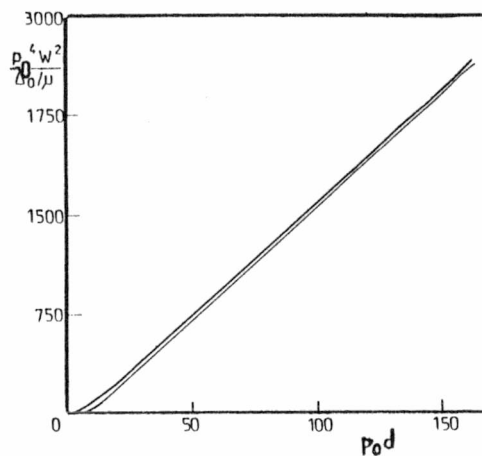


(b)

Rys.13 (a) Oscylacje krytycznej niejednorodności powierzchni podłoża ze zmianą grubości warstwy, dla temperatury $T=0.01T_C^B$ (nadciekłość typu p). (b) Obwiednia funkcji z rys.13(a).



(a)



(b)

Rys.14 (a) Krytyczna niejednorodność powierzchni podłoża dla nadciekłości typu s w temperaturze $T=0.01T_C^B$. (b) Obwiednia krzywej z rys.14(a).

Cienka warstwa nadciekłego gazu fermionów w polu magnetycznym

Pole magnetyczne, poprzez efekt paramagnetyzmu Pauliego, zmienia rodzaj przejścia fazowego ze stanu nadprzewodzącego do stanu normalnego [36,37]. Maki i Tsuneto zwrócili uwagę na zależność rzędu przejścia fazowego od koncentracji domieszek w układzie nadprzewodzącym [36,37]. Jacak i Krzyżanowski [38] zbadali to zjawisko jako funkcję wielkości oddziaływań kwazicząstkowych dla nadciekłego ^3He .

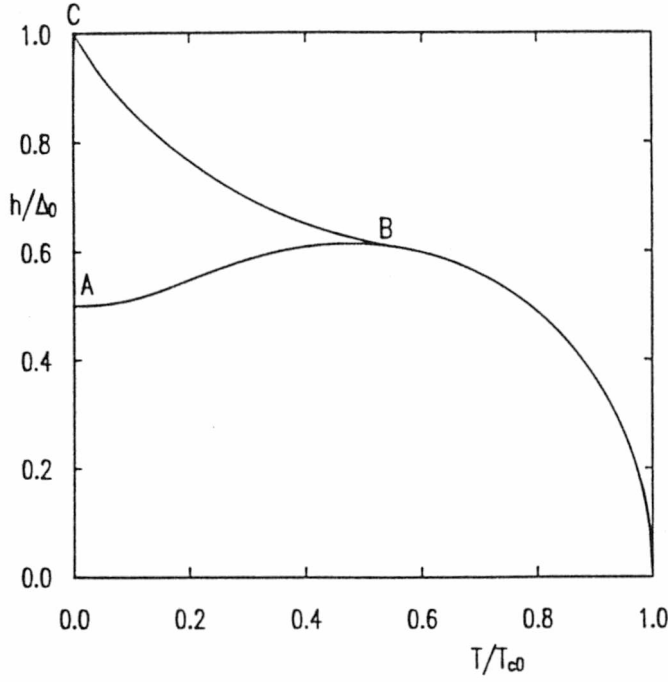
W pracy A3 badamy wpływ nierówności podłoża cienkiej warstwy nadciekłego gazu fermionów na efekt zmiany rodzaju przejścia fazowego w polu magnetycznym. Rozpatrujemy nadciekłość typu s :

$$V_{\alpha\beta\gamma\delta} = \frac{1}{2} g_0 \left[\delta_{\alpha\gamma} \delta_{\beta\delta} - \delta_{\alpha\delta} \delta_{\beta\gamma} \right] . \quad (5.4)$$

Jest to rodzaj nadciekłości ^3He , dotąd nie potwierdzonej doświadczalnie, mogącej występować w mieszaninach ^3He i ^4He [35].

Typowy diagram fazowy, otrzymany przez nas dla warstwy o idealnie gładkich powierzchniach tzn. $\Gamma=0$ (1.24), jest przedstawiony na rys.15. Linia AB przedstawia pole magnetycznego "przechłodzenia", a BC - pole magnetycznego "przegrzania" fazy nadciekłej. W obszarze ABC parametr porządku Δ jest dwuwartościową funkcją pola magnetycznego

i przejście fazowe jest pierwszego rodzaju. Pozostała część krzywej przedstawia przejście fazowe drugiego rodzaju. Współrzędne wspólnego punktu tych dwóch obszarów oznaczono (T_c^*, h_c^*) , są to współrzędne punktu zmiany rodzaju przejścia fazowego.

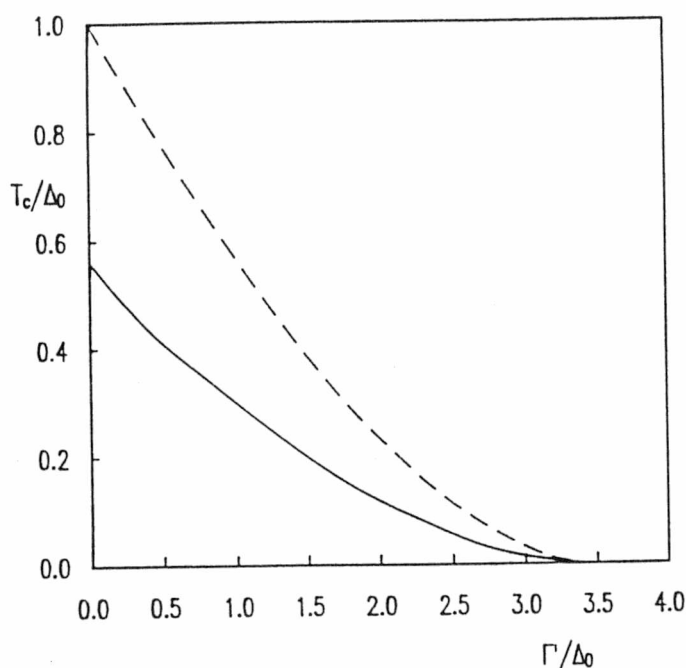


Rys.15 Diagram fazowy dla $\Gamma'=0$ i grubości warstwy $p_0 d=100$.

Znaleźliśmy wpływ nierówności podłoża, zdefiniowanych jako :

$$\frac{\Gamma'}{\Delta_0} = \frac{p_F^4 \omega^2}{\Delta_0 / \mu} \frac{\pi^4 \nu_0 \nu_c (\nu_c + 1) (\nu_c + 0.5)}{3(p_0 d)^6} \quad (5.5)$$

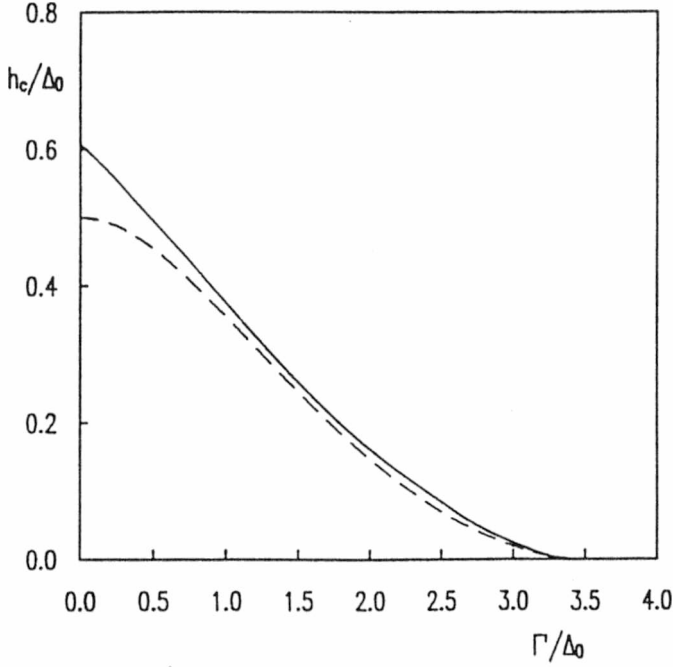
na T_c^* i h_c^* (rys.16 i 17). Na obu wykresach przerywane linie oznaczają : temperaturę krytyczną w polu magnetycznym $h=0$ (rys.16) i pole krytyczne przejścia fazowego drugiego rodzaju w temperaturze $T=0$ (rys.17).



Rys.16 Temperatura krytyczna (linia przerywana) i T_c^* (linia ciągła) w funkcji niejednorodności podłoża Γ' .

Numerycznie wykazano [A3] istnienie fazy nadciekłej dla nierówności podłoża nie przekraczających $\Gamma'/\Delta_0 \approx 3.4$ (rys.16-17). Dopóki układ jest nadciekły, zachodzi zmiana rodzaju przejścia fazowego, to znaczy, punkt (T_c^*, h_c^*) znajduje się na wykresie fazowym (rys.16-17). W przeciwieństwie do domieszkowanych nadprzewodników [36,37],

nie można usunąć obszaru przejścia fazowego pierwszego rodzaju przez zwiększenie nierówności podłoża.



Rys.17 Pole magnetyczne przemiany fazowej drugiego rzędu w $T=0$ (linia przerywana) i h_c^* w zależności od Γ' .

Brak ładunku elektrycznego w układzie sprawia, że przeprowadzone rozważania są słuszne dla warstw o dowolnej grubości, zgodnej z ograniczeniami modelu ($d \leq \xi_0 \approx 60\text{nm}$), w przeciwieństwie do nadprzewodników, gdzie ze względu na silny diamagnetyzm, obserwacje są możliwe tylko w bardzo cienkich warstwach ($d \approx 10\text{\AA}$) [36,37].

W czystym ^3He pole magnetyczne orientuje parametr porządku [19,39] i zjawisko zmiany rodzaju przejścia fazowego może

wystąpić jedynie w niskich polach $h \leq 20G$ i temperaturach dużo niższych od temperatury krytycznej T_c^F [19,40]. Lepszym układem do obserwacji tego zjawiska jest cienka warstwa ^3He z nadciekłym prądem masy [A7].

ZAKOŃCZENIE

Opisane w rozprawie zagadnienia dotyczą wpływu podłoża na zjawisko nadciekłości ^3He w cienkich warstwach. Zastosowane modele uwzględniają dwa typy oddziaływań kwazicząstek ^3He z podłożem :

- oddziaływanie z indukowanym przez podłoże potencjałem typu van der Waalsa,
- rozpraszanie na losowych, punktowych niejednorodnościach powierzchni.

Rozpatrywaliśmy układy, których grubość jest rzędu skali odległości ξ_0 charakteryzującej stan nadciekły. Daje to możliwość obserwacji, przy zmianie grubości warstwy, przejścia od trójwymiarowej do dwuwymiarowej dynamiki kondensatu. Jednak procesy związane z nadciekłością w cienkich warstwach są znacznie zaburzone przez oddziaływanie z podłożem, którego obecności nie można uniknąć. Z tego względu niezmiernie ważnym jest poznanie zjawisk zachodzących w obszarze styczności badanego układu z ciałem stałym. Porównanie otrzymanych wyników z rezultatami doświadczalnymi wskazuje na konieczność dokładniejszego przybliżenia skomplikowanej struktury powierzchni. Trudno jest jednak uwzględnić tak złożony element bez wprowadzania do teorii dużej liczby parametrów. Naturalnym, ze względu na losowość ukształtowania powierzchni podłoża, wydaje się zastosowanie metod probabilistycznych. Powinny one brać pod

uwagę występowanie kilku skal długości opisujących
niejednorodność podłoża [1].

Mamy nadzieję, w przyszłości kontynuować poznawanie
zjawisk zachodzących w bezpośredniej bliskości powierzchni
ciał stałych, nie ograniczając się jedynie do problemów
nadciekłości ^3He .

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TEKSTY PRAC OPUBLIKOWANYCH I WYSŁANYCH DO DRUKU

ON THE GAPLESS SUPERFLUIDITY IN ^3He FILMS

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The transition from the superfluid to the gapless superfluid state in ^3He films is studied. The equations for the energy gap are derived and solved numerically. The influence of surface roughness and film thickness on this transition is discussed. Phase transition curves are found.

1. Introduction

Recently, considerable progress in experimental research on confined geometries of ^3He has been made [1–9]. Especially interesting is the discovery of the superfluid transition in a simple geometry of an adsorbed liquid film [6]. The interface liquid–substrate in such a system is of basic importance [8–13], and the surface irregularities should be included in the theoretical description.

This problem was originally studied by Z. Tešanović and O.T. Valls [14]. They indicated the possibility of existence of a gapless superfluid phase for sufficiently rough substrates. This phenomenon is analogous to the nonergodic case in the ordinary “dirty” superconductors [15, 16].

Regarding the recent experimental advances with flat geometries of superfluid ^3He , it is quite probable that we will have the unique opportunity to control the transition superfluid phase–gapless superfluid phase by the film thickness and the substrate roughness. Hence, it is interesting to investigate this problem more thoroughly in order to find the transition curve as a function of these two parameters.

2. Formalism

In this paper we study the usual p-paired ^3He film and the s-paired system. Note, that the latter could be treated as a model for a hypothetical superfluidity of a ^3He subsystem in a film consisting of a mixture of ^3He and ^4He , where the bosonic medium of ^4He atoms makes the triplet pairing inconvenient.

We consider the thickness regime $p_F^{-1} \ll d \leq \xi_0$, where $p_F = (3\pi^2 n)^{1/3}$ is the bulk Fermi momentum, n is the bulk density and $\xi_0 \sim 400 \text{ \AA}$ (40 nm) is the zero-temperature coherence length, i.e., the de Broglie length of the Cooper pair. For this thickness range we deal with the three-dimensional dynamics of unpaired fermions (^3He atoms) affected by the boundary conditions and the two-dimensional behavior of the superfluid component.

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In the limit of perfectly smooth surfaces the energy spectrum of normal fermions is [17]:

$$\varepsilon_\nu(p) = \frac{p^2}{2m} + \frac{1}{2m} \left(\frac{\nu\pi}{d} \right)^2, \quad (2.1)$$

where p is the two-dimensional momentum and $\nu = 1, 2, \dots, \nu_c$ indicate the quantum numbers associated with the boundary condition of an infinite potential square well, $\nu_c = [\nu_0]$, i.e., ν_c is the nearest integer less than ν_0 . $\nu_0 = [2md\mu/\pi^2]^{1/2}$ and μ is the chemical potential [17–20]. Hence, instead of the usual Fermi sphere, we have a set of parallel Fermi circles, similar to the de Haas – van Alphen effect. The influence of substrate irregularities is taken into account here, as in the work of A. Chaplik and M. Entin [21, 14]. Their method is based on an assumption of a diffusive type of reflection from the porous surface and is given by the boundary condition for the single-particle wave function [21]:

$$\Psi(x, y, d) + u(x, y) \frac{\partial}{\partial z} \Psi(x, y, z)_{z=d} = 0. \quad (2.2)$$

$u(x, y)$ describes the irregular thickness of the film and is modeled by the Gaussian random function [14]

$$\langle u(x, y) \rangle = 0, \quad (2.3)$$

$$\langle u(x, y)u(x', y') \rangle = w^2 \delta(x - x') \delta(y - y'). \quad (2.4)$$

These equations express the assumption of totally uncorrelated irregularities. This simple distribution simulates an amorphous substrate, e.g. a grafoil surface.

The free liquid surface is an infinite potential well:

$$\Psi(x, y, 0) = 0. \quad (2.5)$$

This condition for the free surface of the film is a great simplification and represents an ideal liquid–solid boundary rather than an interface liquid–gas. In principle, one should consider here a free surface with surface tension and vaporization as well. However, it does not look simple to include surface tension through a local modification of the boundary condition for Ψ . Therefore, we use a simpler condition (2.5).

The boundary conditions (2.2) and (2.5) lead to an imaginary term in the self-energy of quasiparticles. It corresponds to the finite lifetime of single-quasiparticle excitations. Although the physical picture of this effect differs from that of bulk systems with impurities, the formal structure of obtained equations turns out to be equivalent to the well known Abrikosov–Gorkov theory for alloys [22].

For the superfluid ^3He film we apply the conditions (2.2) and (2.5) to the normal (unpaired) component. As a consequence of the inequality $w \ll \xi_0$ the superfluid counterpart does not directly interact with substrate irregularities. The roughness influences the Cooper's condensate in some indirect way via the unpaired fermions. In the case of the p-pairing this argument is not necessary, because the anomalous Green's function F is an odd function with respect to $\mathbf{p}$ and the scattering renormalization of Δ is excluded for general symmetry reasons [25].

For the ABM phase of superfluid ^3He which is stable in the thin film [7], the two-dimensional gap parameter has the following form:

$$\Delta_\nu = \Delta_0(d(\hat{p}, \nu)\boldsymbol{\sigma})(i\sigma_y), \quad (2.6)$$

where $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are Pauli matrices.

In the proximity of the boundary (distance less than ξ_0) the angular momentum $\mathbf{l}$ of the Cooper pair is oriented perpendicularly to the surface [23, 24] and we have

$$d_\alpha(\hat{p}, \nu) = \left(\frac{3}{2}\right)^{1/2} d_\alpha(\hat{p}_x + i\hat{p}_y) \sin \theta_\nu, \quad (2.6)$$

$$|d(\hat{p}, \nu)| = \left(\frac{3}{2}\right)^{1/2} \sin \theta_\nu. \quad (2.7)$$

Here, $\sin \theta_\nu = (1 - (\nu/\nu_0)^2)^{1/2}$ and $\mathbf{d}$ is the unit vector in the spin space.

Let us note that considering here a free film of thickness of the order ξ_0 (correlation length) we assume that the angular momentum vector of the Cooper pair is perpendicular to the surface. This is reflected in eqs. (2.7) and (2.8). It is an approximation, since the finite thickness of the film with a porous substrate is certainly causing some deviation of $\mathbf{l}$ from the direction normal to the boundaries. In principle, one should treat this problem 3-dimensionally with the restriction of the thin film geometry which implies dealing with difficult problems of textures in ^3He . In our approach we introduce some smoothing out in the orientation of $\mathbf{l}$ by imposing a lower limit on $\sin \theta_\nu$ in (2.7), namely

$$\sin \theta_\nu \geq \sin \theta_c. \quad (2.8)$$

This condition means that we remove zeroes in the definition of the gap parameter d_α (2.7). This in turn implies that sums over ν in the equations for normal (2.13) and anomalous (2.14) self-energies are limited by the inequalities $\nu \leq \nu_0 (1 - \sin^2 \theta_c)^{1/2}$ instead of $\nu \leq \nu_0$. Since in numerical calculations we need a specific value of $\sin \theta_c$ we take $\sin \theta_c = 0.1$, which is equivalent to $\nu \leq \sqrt{0.99} \nu_0$. This effective smoothing out of the order parameter allows to avoid a non-physical behavior of some of the results. If we let $\sin \theta_c = 0$ we will find, e.g. a gapless superfluidity for certain thicknesses of the film without any irregularities on the boundary, i.e., without a scattering mechanism. This is demonstrated in fig. 1 where the dashed line refers to $\sin \theta_c = 0$ and the solid line represents $\sin \theta_c = 0.1$. In other words, zeroes of the order parameter (2.7) would result in that the oscillations of the critical parameter ζ would reach zero for some values of the film thickness. To avoid this non-physical, as it seems, behavior we smooth out the order parameter, as discussed above. We address this problem again in our discussion of numerical results.

For the s-paired superfluid thin film the order parameter does not depend on space dimensionality and equals

$$\Delta = \Delta_0(i\sigma_y). \quad (2.9)$$

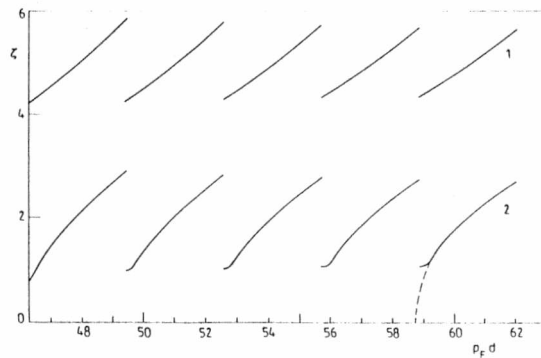


Fig. 1. ζ critical as a function of layer thickness; 1, singlet paired state and 2, triplet paired state.

We use the Abrikosov–Gorkov equations for the Green's functions $G_\nu(i\omega_n, \mathbf{p})$ and $F_\nu(i\omega_n, \mathbf{p})$:

$$G_\nu(i\omega_n, \mathbf{p}) = G_{0\nu}(i\omega_n, \mathbf{p}) + G_{0\nu}(i\omega_n, \mathbf{p})\Delta_\nu(\mathbf{p})F_\nu^+(i\omega_n, \mathbf{p}), \quad (2.11)$$

$$F_\nu^+(i\omega_n, \mathbf{p}) = -G_{0\nu}^-(i\omega_n, \mathbf{p})\Delta_\nu^+(\mathbf{p})G_\nu(i\omega_n, \mathbf{p}), \quad (2.12)$$

completed by two following self-consistent equations for the normal $\Sigma_\nu(i\omega_n)$ and anomalous Δ_ν parts of the self-energy (spin indices are suppressed):

$$\Sigma_\nu(i\omega_n) = \frac{w^2}{2m^2d^2} \left(\frac{\nu\pi}{d} \right)^2 \frac{1}{s} \sum_{\mathbf{p}, \nu'} \left(\frac{\nu'\pi}{d} \right)^2 G_{\nu'}(i\omega_n, \mathbf{p}), \quad (2.13)$$

$$\Delta_\nu^+ = -T \sum_{\mathbf{p}', \nu', n} V_{\nu\nu'}(\hat{\mathbf{p}}, \hat{\mathbf{p}}') F_{\nu'}^+(i\omega_n, \mathbf{p}'), \quad (2.14)$$

where

$$G_{0\nu}(i\omega_n, \mathbf{p}) = [i\omega_n - \xi_p - \lambda_\nu - \Sigma_\nu(i\omega_n)]^{-1}, \quad \xi_p = \frac{p^2}{2m} - \mu, \quad \lambda_\nu = \frac{1}{2m} \left(\frac{\nu\pi}{d} \right)^2, \quad (2.15)$$

is the zeroth order Matsubara–Green function, ω_n is the Matsubara frequency, the superscript ‘–’ denotes the time inversion and s is the surface area of the film;

$$(\text{p-pairing}) \quad V_{\nu\nu'}(\mathbf{p}, \mathbf{p}')_{\alpha\beta\gamma\delta} = \frac{3}{2} g_1 P_1(\hat{\mathbf{p}}\hat{\mathbf{p}}' \sin\theta_\nu \sin\theta_{\nu'} + \cos\theta_\nu \cos\theta_{\nu'}) (\delta_{\alpha\gamma}\delta_{\beta\delta} + \delta_{\alpha\delta}\delta_{\beta\gamma}), \quad (2.16)$$

and

$$(\text{s-pairing}) \quad V_{\nu\nu'}(\mathbf{p}, \mathbf{p}')_{\alpha\beta\gamma\delta} = \frac{1}{2} g_0 (\delta_{\alpha\gamma}\delta_{\beta\delta} - \delta_{\alpha\delta}\delta_{\beta\gamma}). \quad (2.17)$$

Let us underline that equations (2.11)–(2.15) are formally equivalent to the corresponding ones for dirty superconductors in the nonergodic regime [22].

After performing the summation over $\mathbf{p}$ in (2.13) and (2.14) one obtains for the p-paired and s-paired states, respectively:

$$\tilde{\omega}_n(\nu) = \omega_n + \Gamma \cos^2 \theta_\nu \sum_{\cos \theta_{\nu'}} \cos^2 \theta_{\nu'} \frac{\tilde{\omega}_n(\nu')}{[\tilde{\omega}_n^2(\nu') + \Delta^2 \sin^2 \theta_{\nu'}]^{1/2}}, \quad (2.18)$$

$$1 = T^{\frac{3}{2}} \frac{\pi^2 \lambda_1}{p_F d} \sum_{n, \nu} \frac{\sin^2 \theta_\nu}{[\tilde{\omega}_n^2(\nu) + \Delta^2 \sin^2 \theta_\nu]^{1/2}}, \quad (2.19)$$

$$\tilde{\omega}_n(\nu) = \omega_n + \Gamma \cos^2 \theta_\nu \sum_{\cos \theta_{\nu'}} \cos^2 \theta_{\nu'} \frac{\tilde{\omega}_n(\nu')}{[\tilde{\omega}_n^2(\nu') + \Delta^2]^{1/2}}, \quad (2.20)$$

$$1 = T \frac{\pi^2 \lambda_0}{p_F d} \sum_{n, \nu} \frac{1}{[\tilde{\omega}_n^2(\nu) + \Delta^2]^{1/2}}, \quad (2.21)$$

where $\tilde{\omega}_n(\nu) = \omega_n + i\Sigma_\nu(i\omega_n)$, $\lambda_1 = N_0 g_1$, $\lambda_0 = N_0 g_0$, g_0 and g_1 are bulk coupling parameters, and $\Gamma = (\pi^4/2md^2)(w^2/d^4)\nu_0^5$.

Equations (2.18)–(2.21) contain the required information about the superfluid film on a rough substrate.

3. Gapless superfluidity

In order to discuss the gapless superfluidity we introduce the density of states

$$N(\omega) = -\frac{1}{\pi} \text{Im} \sum_{p, \nu} G_{\nu}(i\omega_n, p) \Big|_{i\omega_n = \omega + i\delta}$$

$$= \begin{cases} N_0 \frac{\pi}{p_F d} \text{Im} \sum_{\nu} \frac{\omega_{\nu}}{[\Delta^2 - \omega_{\nu}^2]^{1/2}}, & \text{for the s-pairing,} \\ N_0 \frac{\pi}{p_F d} \text{Im} \sum_{\nu} \frac{\omega_{\nu}}{[\Delta^2 \sin^2 \theta_{\nu} - \omega_{\nu}^2]^{1/2}}, & \text{for the p-pairing.} \end{cases} \quad (3.1)$$

In the above ω_{ν} is the analytical continuation of $i\tilde{\omega}_n(\nu)$:

$$\omega_{\nu} = i\tilde{\omega}_n(\nu) \Big|_{i\omega_n = \omega + i\delta}$$

$$= \begin{cases} \omega + \Gamma \cos^2 \theta_{\nu} \sum_{\cos \theta_{\nu'}} \cos^2 \theta_{\nu'} \frac{\omega_{\nu'}}{[\Delta^2 - \omega_{\nu'}^2]^{1/2}}, & \text{s-pairing,} \\ \omega + \Gamma \cos^2 \theta_{\nu} \sum_{\cos \theta_{\nu'}} \cos^2 \theta_{\nu'} \frac{\omega_{\nu'}}{[\Delta^2 \sin^2 \theta_{\nu'} - \omega_{\nu'}^2]^{1/2}}, & \text{p-pairing.} \end{cases} \quad (3.2)$$

Here $\cos^2 \theta_{\nu} = \left(\frac{\nu}{\nu_0}\right)^2$, $\sum_{\cos \theta_{\nu}} \equiv \frac{1}{\nu_0} \sum_{\nu}$,

and N_0 is the bulk density of states at the Fermi surface.

To evaluate the above equations it is useful to introduce the function $U(\omega/\Delta)$ [14]:

$$\frac{\omega_{\nu}}{\Delta} = \frac{\omega}{\Delta} + \zeta \cos^2 \theta_{\nu} U\left(\frac{\omega}{\Delta}\right),$$

where $\zeta = \Gamma/\Delta$. Then, eqs. (3.1) and (3.2) are

$$N(\omega) = N_0 \frac{\pi}{p_F d} \text{Im} \sum_{\nu} \frac{\frac{\omega}{\Delta} + \zeta \cos^2 \theta_{\nu} U\left(\frac{\omega}{\Delta}\right)}{\left(1 - \left(\frac{\omega}{\Delta} + \zeta \cos^2 \theta_{\nu} U\left(\frac{\omega}{\Delta}\right)\right)^2\right)^{1/2}}, \quad (3.3)$$

$$U\left(\frac{\omega}{\Delta}\right) = \sum_{\cos \theta_{\nu}} \frac{\frac{\omega}{\Delta} \cos^2 \theta_{\nu} + \zeta \cos^4 \theta_{\nu} U\left(\frac{\omega}{\Delta}\right)}{\left(1 - \left(\frac{\omega}{\Delta} + \zeta \cos^2 \theta_{\nu} U\left(\frac{\omega}{\Delta}\right)\right)^2\right)^{1/2}}, \quad (3.4)$$

for the s-paired state, and

$$N(\omega) = N_0 \frac{\pi}{p_F d} \text{Im} \sum_{\nu} \frac{\frac{\omega}{\Delta} + \zeta \cos^2 \theta_{\nu} U\left(\frac{\omega}{\Delta}\right)}{\left(\sin^2 \theta_{\nu} - \left(\frac{\omega}{\Delta} + \zeta \cos^2 \theta_{\nu} U\left(\frac{\omega}{\Delta}\right)\right)^2\right)^{1/2}}, \quad (3.5)$$

$$U\left(\frac{\omega}{\Delta}\right) = \sum_{\cos \theta_\nu} \frac{\frac{\omega}{\Delta} \cos^2 \theta_\nu + \zeta \cos^4 \theta_\nu U\left(\frac{\omega}{\Delta}\right)}{\left(\sin^2 \theta_\nu - \left[\frac{\omega}{\Delta} + \zeta \cos^2 \theta_\nu U\left(\frac{\omega}{\Delta}\right)\right]^2\right)^{1/2}}, \quad (3.4)$$

for the p-paired system.

The numerical solutions of (3.3) and (3.4) for several values of ζ are plotted in fig. 2. There exists critical roughness ζ_c above which a finite density of states appears at the Fermi level. One observes the characteristic gapless behavior, similar to that in superconductors with paramagnetic impurities [26, 15]. The same effect is observed in p-paired superfluids and the shape of the function $N(\omega)$ is demonstrated in [14].

Our main objective is to find the critical value of ζ which leads to the transition from the superfluid to the gapless superfluid phase.

At the Fermi level ($\omega = 0$), eqs. (3.4) and (3.6) are

$$U_1 = \sum_{\cos \theta_\nu} \zeta \frac{\cos^4 \theta_\nu}{[2z_\nu]^{1/2}} [U_1(1 + \cos \phi_\nu)^{1/2} + a_\nu U_2(1 - \cos \phi_\nu)^{1/2}], \quad (3.5)$$

$$U_2 = \sum_{\cos \theta_\nu} \zeta \frac{\cos^4 \theta_\nu}{[2z_\nu]^{1/2}} [U_2(1 + \cos \phi_\nu)^{1/2} - a_\nu U_1(1 - \cos \phi_\nu)^{1/2}], \quad (3.6)$$

where $U = U_1 + iU_2$,

$$z_\nu = \begin{cases} [(1 + \zeta^2(U_2^2 - U_1^2) \cos^4 \theta_\nu)^2 + 4\zeta^4 U_1^2 U_2^2 \cos^8 \theta_\nu]^{1/2}, & \text{s-pairing,} \\ [(\sin^2 \theta_\nu + \zeta^2(U_2^2 - U_1^2) \cos^4 \theta_\nu)^2 + 4\zeta^4 U_1^2 U_2^2 \cos^8 \theta_\nu]^{1/2}, & \text{p-pairing,} \end{cases} \quad (3.7)$$

$$\cos \phi_\nu = \begin{cases} \frac{1 + \zeta^2(U_2^2 - U_1^2) \cos^4 \theta_\nu}{z_\nu}, & \text{s-pairing,} \\ \frac{\sin^2 \theta_\nu + \zeta^2(U_2^2 - U_1^2) \cos^4 \theta_\nu}{z_\nu}, & \text{p-pairing,} \end{cases} \quad (3.8)$$

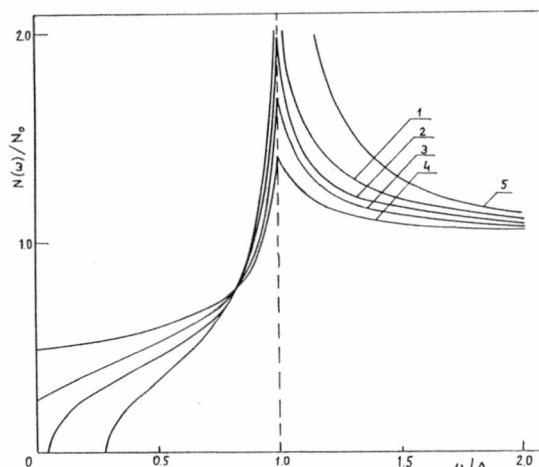


Fig. 2. The density of states of quasiparticle excitations for the s-paired system; 1, $\zeta = 2$; 2, $\zeta = 4$; 3, $\zeta = 6$; 4, $\zeta = 10$; 5, $\zeta = 63$. here $p_F d = 63$.

$$a_\nu = \begin{cases} -\text{sign}(U_1 U_2), & \text{for } U_1 U_2 \neq 0, \\ -1, & \text{for } U_1 U_2 = 0. \end{cases} \quad (3.11)$$

It is easy to find out that the system of eqs. (3.7) and (3.8) has the following solutions only:

$$U = (U_1, 0), \quad (3.12)$$

$$U = (0, U_2). \quad (3.13)$$

After a straightforward calculation one can see that the solution (3.12) does not lead to a finite density of states on the Fermi surface. Only when $U = iU_2$ and $U_2 \neq 0$ we have the gapless superfluidity. As consequence, combining (3.13) and (3.8), we get the following condition for the studied transition in the s- and p-paired states, respectively:

$$1 = \zeta \sum_{\cos \theta_\nu} \frac{\cos^4 \theta_\nu}{[1 + \zeta^2 U_2^2 \cos^4 \theta_\nu]^{1/2}} \quad \text{s-pairing} \quad (3.14)$$

$$1 = \zeta \sum_{\cos \theta_\nu} \frac{\cos^4 \theta_\nu}{[\sin^2 \theta_\nu + \zeta^2 U_2^2 \cos^4 \theta_\nu]^{1/2}} \quad \text{p-pairing} \quad (3.15)$$

These equations are to be solved numerically. The smallest value of ζ which leads to the nonzerot solutions for U_2 in (3.14) and (3.15) is the searched for critical parameter.

In the next section we present the numerical solutions of (3.14) and (3.15). Having solved the equations (2.19) and (2.21) we evaluate the dimensionless parameter $p_F^4 w^2$ being the direct measure of this critical substrate roughness in units of interatomic distance

4. Numerical results

The evolution of the critical value of the parameter ζ with the film thickness is presented in fig. 1. For the two considered types of potential (s-wave, p-wave) one observes the characteristic oscillator behavior of ζ (fig. 1). This effect is strictly connected with the discrete spectrum of single-quasiparticle excitations and resembles the de Haas–van Alphen effect. The difference is that here we control the number of the energy levels inside the Fermi sphere by the thickness of the film.

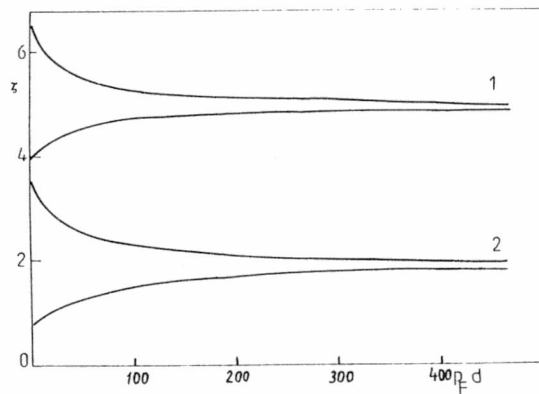


Fig. 3. The envelopes of the oscillations presented in fig. 1, long range behavior; 1, s- and 2, p-pairing.

On the long range scale the oscillatory amplitude of ζ is suppressed (fig. 3). The dashed line in fig. 1 represents the ζ oscillations in a system without the cutoff ansatz (eq. (2.9)). It seems impossible that for some thicknesses there would exist the gapless superfluidity in the limit of perfectly smooth boundaries ($\zeta = 0$), i.e., a pair breaking mechanism in a system without impurities. That is why in this simple mathematical model the cutoff operation is performed.

In order to draw the phase diagram we have to solve the gap eqs. (2.19) and (2.21) with the critical value of ζ .

Following standard methods we transform these equations into:

$$\ln \frac{T}{T_c} + \left(\frac{p_F d}{\pi \nu_c} - 1 \right) \ln \left(1.13 \frac{\omega_D}{T_c} \right) = 3.56 \frac{T/T_c}{\Delta/\Delta_0} \frac{1}{\nu_c} \sum_{n \geq 0, \nu} \frac{1}{\left(\left(\frac{(2n+1)T/T_c}{\Delta/\Delta_0} 1.78 + \zeta U \cos^2 \theta_\nu \right)^2 + 1 \right)^{1/2}} - \sum_{n \geq 0} \frac{1}{n + 1/2}, \quad (4.1)$$

for the s-paired system, and

$$\ln \frac{T}{T_c} + \left(\frac{2/3}{\pi/(p_F d) \sum_\nu \sin^2 \theta_\nu} - 1 \right) \ln \left(1.13 \frac{\omega_D}{T_c} \right) = \frac{1}{\sum_\nu \sin^2 \theta_\nu} 3.56 \frac{T/T_c}{\Delta/\Delta_0} \sum_{n \geq 0, \nu} \frac{\sin^2 \theta_\nu}{\left(\left(\frac{(2n+1)T/T_c}{\Delta/\Delta_0} 1.78 + \zeta U \cos^2 \theta_\nu \right)^2 + \sin^2 \theta_\nu \right)^{1/2}} - \sum_{n \geq 0} \frac{1}{n + 1/2}, \quad (4.2)$$

for the p-pairing.

Simultaneously with the gap equations, we solve the equations for U , where

$$\frac{\tilde{\omega}_n(\nu)}{\Delta} = \frac{(2n+1)\pi T}{\Delta} + \zeta U \left(\frac{\omega}{\Delta} \right) \cos^2 \theta_\nu, \quad (4.3)$$

and the renormalizing eqs. (2.18) and (2.20) have the form

$$U \left(\frac{\omega}{\Delta} \right) = \sum_{\cos \theta_\nu} \cos^2 \theta_\nu \frac{\frac{(2n+1)T/T_c}{\Delta/\Delta_0} 1.78 + \zeta U \left(\frac{\omega}{\Delta} \right) \cos^2 \theta_\nu}{\left(\left(\frac{(2n+1)T/T_c}{\Delta/\Delta_0} 1.78 + \zeta U \left(\frac{\omega}{\Delta} \right) \cos^2 \theta_\nu \right)^2 + 1 \right)^{1/2}}, \quad (4.4)$$

$$U \left(\frac{\omega}{\Delta} \right) = \sum_{\cos \theta_\nu} \cos^2 \theta_\nu \frac{\frac{(2n+1)T/T_c}{\Delta/\Delta_0} 1.78 + \zeta U \left(\frac{\omega}{\Delta} \right) \cos^2 \theta_\nu}{\left(\left(\frac{(2n+1)T/T_c}{\Delta/\Delta_0} 1.78 + \zeta U \left(\frac{\omega}{\Delta} \right) \cos^2 \theta_\nu \right)^2 + \sin^2 \theta_\nu \right)^{1/2}}, \quad (4.5)$$

for s-pairing and p-pairing, respectively. Here T_c is the bulk critical temperature, Δ_0 the bulk zero-temperature gap parameter, ω_D is the BCS cutoff parameter. In the numerical calculations we have taken $\omega_D = 100 T_c$. Figures 4 and 5 present the solution of eq. (4.1). The gap parameter for the

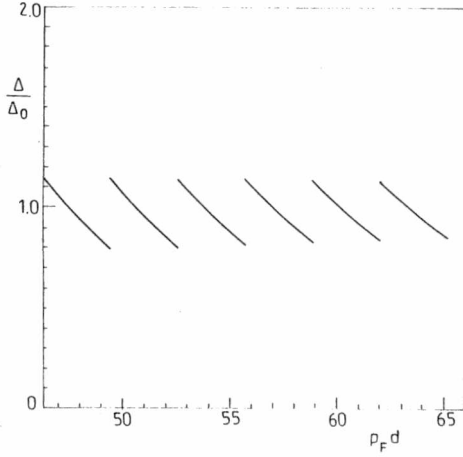


Fig. 4. The energy gap as a function of the film thickness for the s-paired state; here $T = 0.01 T_c$.

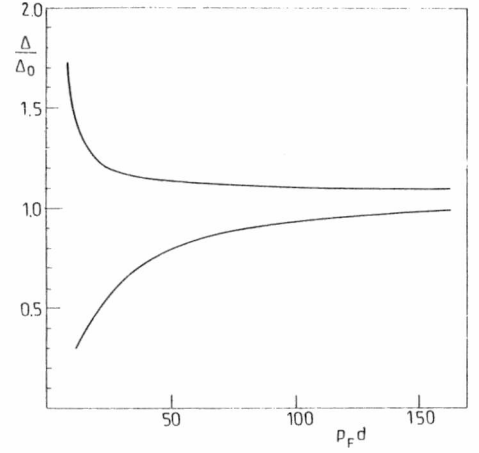


Fig. 5. The envelope of the gap vs. film thickness dependence in fig. 4; here also $T = 0.01 T_c$.

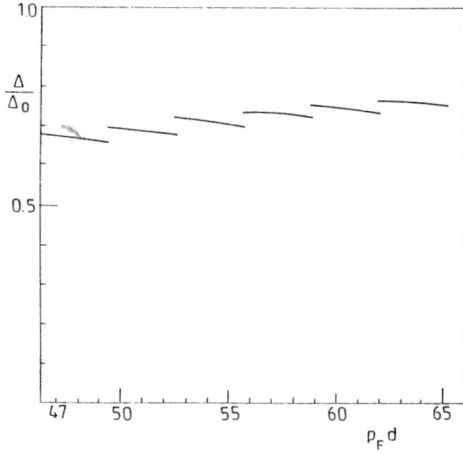


Fig. 6. The energy gap as a function of $p_F d$ for the p-paired state; $T = 0.01 T_c$.

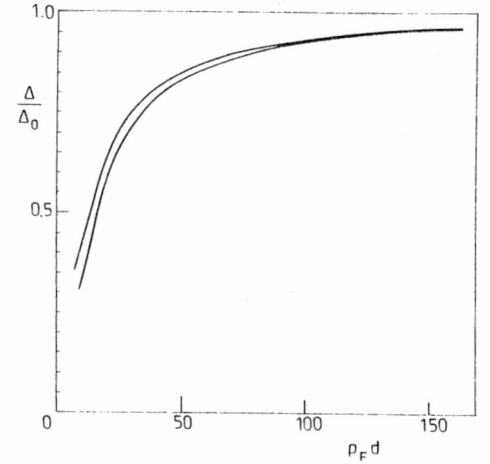


Fig. 7. The envelope of the graph Δ/Δ_0 vs. $p_F d$ in fig. 6; $T = 0.01 T_c$.

p-paired state is plotted in figs. 6 and 7. The graphs of obtained solutions are similar to those in the limit of perfectly smooth boundaries [17–19].

As a final result we plotted the phase diagram for the transition superfluid–gapless superfluid phase figs. 8–11, where we used the formula

$$\frac{p_F^4 w^2}{\Delta_0 / \mu} = \frac{(p_F d)^6}{\pi^4 \nu_0^5} \frac{\Delta}{\Delta_0} \zeta, \quad (4.6)$$

p_F , μ , Δ_0 , T_c are bulk values.

5. Conclusions

As can be seen from figs. 8–11, there are characteristic oscillations of the critical roughness:

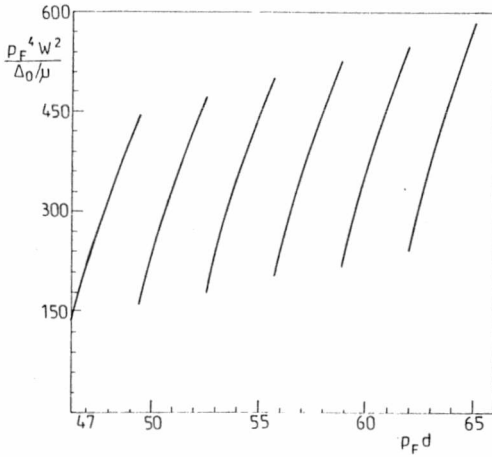


Fig. 8. The oscillations of the critical surface roughness parameter (3.21), p-pairing; $T = 0.01 T_c$.

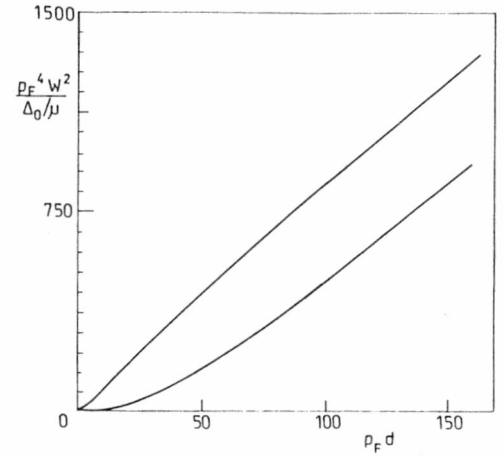


Fig. 9. The envelope for the function in fig. 8; $T = 0.01 T_c$.

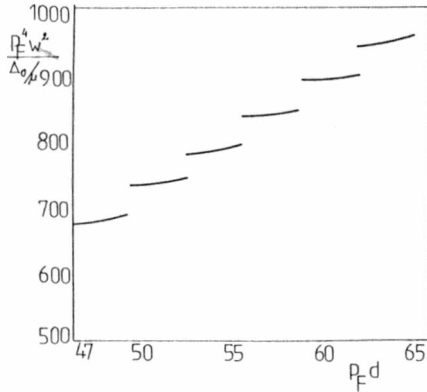


Fig. 10. The critical surface roughness vs. film thickness, s-pairing; $T = 0.01 T_c$.

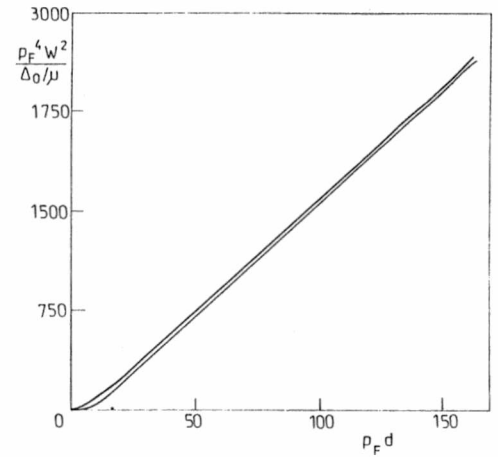


Fig. 11. The envelope to the fig. 10; $T = 0.01 T_c$.

parameter. The amplitude of these oscillations increases with the layer thickness for the triplet state (figs. 8 and 9) and remains almost unchanged for the singlet state (figs. 10 and 11). Note also that the system with s-pairing is less sensitive to surface roughness than the p-paired one (compare figs. 9 and 11).

The position of the lower envelope in p-wave, fig. 9, depends on our choice of cutoff operation and here is some room for discussion and a proper choice of the cutoff threshold in $\sin \theta_v$ (eq. (2.9)).

If we put no restrictions on $\sin \theta_v$ we would have the situation where for certain values of film thickness $\zeta_c \rightarrow 0$.

Figures 8 and 10 resemble the typical characteristics of the de Haas–van Alphen effect.

The oscillations of ζ , fig. 3, are smaller for the s-pairing and their amplitude is decreasing faster with layer thickness than in the case of the p-pairing.

The main result of this work is that we can control the superfluid phase transition by changing the film thickness and choosing a substrate.

For very thin films, i.e., when $\nu_c \leq 10$, the theory presented here should not be taken seriously, since in this range the model breaks down [18–20, 14].

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OSCILLATIONS OF CRITICAL PARAMETERS IN THIN FILMS OF ^3He

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In this paper we consider some aspects of thin films ($p_F^{-1} \ll d \leq \xi_0$) of ^3He . d is the film's thickness and ξ_0 - the correlation length of the Cooper pair. We investigate the influence of surface roughness on the superfluid transition. It was suggested that in thin films of ^3He on a rough substrate there may exist a gapless superfluid phase¹ due to a diffusive scattering at the boundary. The peculiar feature of the gapless state is that we may have both an energy gap and a nonzero density of states at the Fermi surface.

We consider the ABM phase which was found to be stable in thin film in this thickness range². The boundary conditions imposed on a single-particle wave function are

$$\Psi(x, y, d) + u(x, y) \frac{\partial}{\partial z} \Psi(x, y, z) \Big|_{z=d} = 0, \quad \Psi(x, y, 0) = 0. \quad (1)$$

Here $u(x, y)$ represents height of the irregularities on the surface liquid-solid and is a gaussian random function with a zero mean and a δ -type correlation. The second of eqs. (1) means an infinite potential wall at the other boundary.

Following standard methods of the Abrikosov-Gorkov theory we obtain the gap equation as a function of film thickness and degree of surface roughness. Details of this derivation can be found elsewhere³. Solving the gap equation for the parameter of the critical surface roughness ζ giving the transition to a nonzero density of states at the Fermi surface, we arrive at

$$1 = \zeta \sum_{\cos \theta_v} \frac{\cos^4 \theta_v}{[\sin^2 \theta_v + \zeta^2 U^2 \cos^4 \theta_v]^{\frac{1}{2}}}, \quad (2)$$

where $\cos \theta_v = \frac{v}{v_0}$, v being the number of one of the parallel Fermi

circles, $v_0 = \left[\frac{2md^2}{\pi^2} \mu \right]^{\frac{1}{2}}$ and μ is the chemical potential. We look

for the smallest value of ζ leading to a nonzero solution for U . Figs. 1-4 present results of numerical calculations.

The final result is the phase diagram for the transition: superfluid-gapless superfluid phase, Figs. 3 and 4, where we use the formula

$$\frac{p_F^4 w^2}{\Delta_0 / \mu} = \frac{(p_F d)^6}{\pi^4 v_0^5} \frac{\Delta}{\Delta_0} \zeta. \quad (3)$$

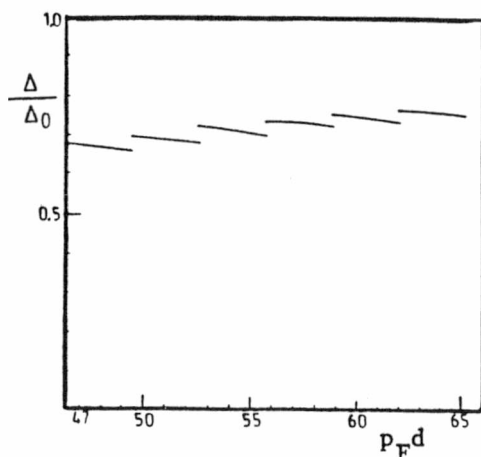


Fig. 1. The energy gap as a function of film thickness.

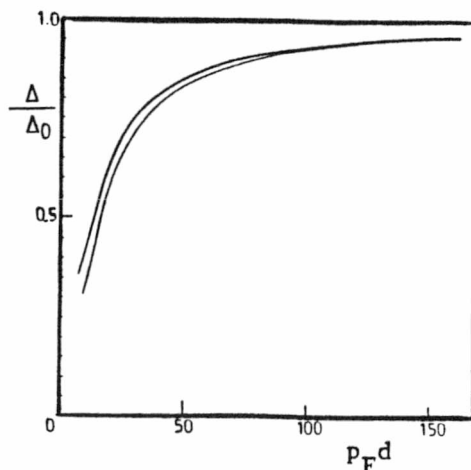


Fig. 2. The envelope of oscillations in Fig. 1.

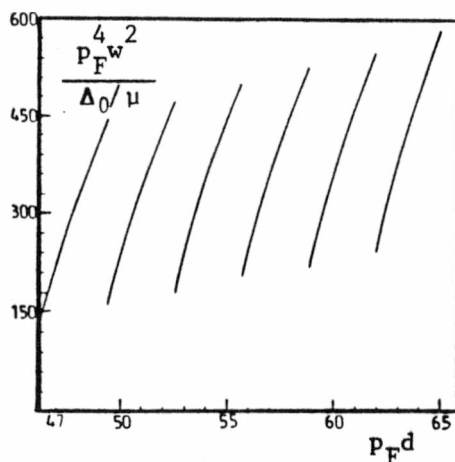


Fig. 3. The critical surface roughness as a function of film thickness.

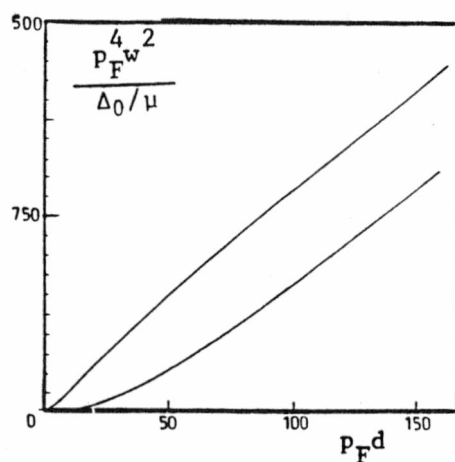


Fig. 4. The envelope of oscillations in Fig. 3.

Here p_F , μ , Δ_0 and T_c are bulk values. $p_F^4 w^2$ is a direct measure of the critical surface roughness in units of interatomic distance, w denotes the average height of 'bumps'. Our calculations are performed for $T = 0.01 T_c$. Let us note that Fig. 3 resembles typical characteristics of the de Haas - van Alphen effect. It is worth mentioning in this context that this and other interesting effects also appear in thin films of ^3He in magnetic fields, and we plan to report on it in near future.

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THIN FILM OF A SUPERFLUID FERMI SYSTEM IN A MAGNETIC FIELD

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We study the problem of a thin film of neutral fermions in the presence of a magnetic field and scattering on random, uncorrelated irregularities at the boundaries of the film. An attractive interaction of s-wave type is assumed. We solve the equation for the energy gap and demonstrate its dependence on both the field and the scattering strength. The phase diagram is found. There exists a critical point (T_c^*, H_c^*) . For temperatures below T_c^* the phase transition is of the first order and of the second order above it. As expected, the surface scattering significantly reduces the values of the critical field H_c and temperature T_c . The critical point changes its position with increasing scattering rate but is not removed from the phase diagram until superfluidity is destroyed.

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1. Introduction

In this paper we describe the behavior of a thin film of a superfluid Fermi liquid with s-wave pairing in magnetic field. We consider the film thickness range $p_F^{-1} \ll d \leq \xi_0$, where $p_F = (3\pi^2 n)^{1/3}$ is the bulk Fermi momentum, n is the bulk density and $\xi_0 \approx 15$ to 40 nm is the zero temperature coherence length, i.e. de Broglie length of the Cooper pair. For this thickness range we deal with the three-dimensional dynamics of unpaired fermions affected by the boundary conditions and the two-dimensional behavior of the superfluid component. The field is

assumed here to be strong, i.e. $\mu_B H \sim k_B T_c \ll \mu$, where T_c is the critical temperature of the normal–superfluid transition at zero field, μ denotes the chemical potential, μ_B is the Bohr magneton and k_B is the Boltzmann’s constant. We discuss in detail the phase transitions due to the magnetic field including effects of restricted geometry. These effects in superconductors were originally studied by Maki and Tsuneto [1, 2] who found the change of the order of the superconducting transition resulting from Pauli paramagnetism and its dependence on the concentration of impurities. Also, there appeared a detailed theory of Fermi liquids in a magnetic field by Jacak and Krzyżanowski [3]. They showed the dependence of the critical parameters (critical magnetic field H_c , critical temperature T_c) and the order of the superfluid phase transition on Fermi liquid interactions between quasiparticles.

The motivation for this work comes from a number of recent experimental studies of various aspects of superfluidity in confined geometries [4–7]. The effect of the substrate and its irregularities on properties of superfluid thin films is important from both experimental [4–7] and theoretical [8–13] points of view. Our aim was to investigate the influence of substrate’s roughness on the phase transition in the s-paired superfluid. This system may be adequate to describe the expected superfluidity of ^3He in mixtures of ^3He and ^4He [14] (for some concentrations of ^3He). The approach presented here may also serve as a guide in doing similar calculations for anisotropic superfluids which are already investigated experimentally [4–7] in the thickness range of about 100 nm, i.e. a little above the thickness limit studied here. As far as we know, at present there are no experimental results for a critical temperature of these ^3He films in magnetic field.

In our analysis we neglect the Fermi liquid interactions between quasiparticles and treat the system as a Fermi gas, expecting the interactions with the substrate to be more significant in the thin film geometry.

2. Formulation of the problem

We use the model of a thin film proposed by Tešanović and Valls [13] which takes into account the substrate’s roughness via a random Gaussian height function $u(x, y)$:

$$\langle u(x, y) \rangle = 0, \quad (1)$$

$$\langle u(x, y)u(x', y') \rangle = w^2 \delta(x - x') \delta(y - y'). \quad (2)$$

The parameter w describes the absolute magnitude of the irregularities. This description [13] is equivalent to the well-known Abrikosov and Gorkov [15] method of studying superconductors with impurities. There is however one difference here. The interaction with the substrate’s irregularities is introduced by suitable boundary conditions [18] for the single-particle fermion wave function $\Psi(x, y, d+u(x, y)) = 0$, and $\Psi(x, y, 0) = 0$, which affect only the dynamics of the three-dimensional unpaired fermions. As a consequence of the inequality $w \ll d \leq \xi_0$ we consider the superfluid component as purely two-dimensional and do not impose any additional

constraints on it. This means that the energy of one-particle excitations is renormalized as in Eq. (5), whereas the anomalous self-energy Δ is not directly changed by the scattering at the boundaries. In other words we treat the particle-particle interaction differently from the particle-hole interaction, as in the case of the time reversal symmetry breaking perturbation or a spatial variation of the order parameter [19]. This situation causes the gapless behavior of the system [12, 19].

The boundary conditions written above lead to the quantization of the energy levels in the film. The Fermi sphere degenerates into a set of Fermi circles. Further below we make use of ν_c , the total number of Fermi circles below the chemical potential and $\nu_0 = [2md^2\mu/\pi^2]^{1/2}$. ν_c is the nearest integer less than ν_0 . Refs. [8, 12, 13] contain a more detailed description of the problem.

In the equations below $\nu = 1, \dots, \nu_c$ denotes one of the ν_c Fermi circles. We begin by writing the Abrikosov-Gorkov equations [15] including the Pauli terms:

$$G_\nu(i\omega_n, p) = G_{0\nu}(i\omega_n, p) + G_{0\nu}(i\omega_n, p)\Delta(p)F_\nu^\dagger(i\omega_n, p), \quad (3)$$

$$F_\nu^\dagger(i\omega_n, p) = -G_{0\nu}^{\psi-}(i\omega_n, p)\Delta^\dagger(p)G_\nu(i\omega_n, p), \quad (4)$$

(the spin indices are suppressed here — they will be used elsewhere if confusion might arise), completed by two self-consistent equations for the normal $\Sigma_\nu(i\omega_n)$ and anomalous Δ parts of the self-energy [12]:

$$\Sigma_\nu(i\omega_n) = \frac{\omega^2}{2m^2d^2} \left(\frac{\nu\pi}{d}\right)^2 \frac{1}{S} \sum_{p, \nu'} \left(\frac{\nu'\pi}{d}\right)^2 G_{\nu'}(i\omega_n, p), \quad (5)$$

$$\Delta_{\alpha\beta}^\dagger = -T \sum_{p, \nu, n} \frac{1}{2} g_0 (\delta_{\alpha\gamma} \delta_{\beta\delta} - \delta_{\alpha\delta} \delta_{\beta\gamma}) F_{\nu\gamma\delta}^\dagger(i\omega_n, p). \quad (6)$$

$G_{0\nu}(i\omega_n, p)$ is the normal Matsubara-Green function [3]:

$$G_{0\nu\alpha\beta}^{-1}(i\omega_n, p) = [i\omega_n - \xi_p - \lambda_\nu - \Sigma_\nu(i\omega_n)]\delta_{\alpha\beta} - h^i \sigma_{\alpha\beta}^i, \quad (7)$$

where $\xi_p = (p^2/2m) - \mu$, $\lambda_\nu = (1/2m)(\nu\pi/d)^2$, ω_n is the Matsubara frequency, the superscript ' $\psi-$ ' in Eq.(4) denotes the time inversion and S is the surface area of the film. $G_\nu(i\omega_n, p)$ and $F_\nu(i\omega_n, p)$ are the normal and anomalous superfluid Matsubara-Green functions respectively, $h = \mu_B H$, g_0 — the bulk s-wave coupling parameter. For each Fermi circle ν we have a coupled set of equations (3), (4), (5) and (6). Solving Eqs. (3) and (4) we find

$$G_\nu^{-1}(i\omega_n, p) = \left[i\omega_n - \Sigma_\nu(i\omega_n) - \xi_\nu(p) - \frac{i\omega_n - \Sigma_\nu(i\omega_n) + \xi_\nu(p)}{R^2} \Delta^2 \right] \hat{I} - \left(1 + \frac{\Delta^2}{R^2} \right) h^i \sigma^i, \quad (8)$$

$$F_\nu^{\dagger-1}(i\omega_n, p) =$$

$$\frac{1}{\Delta} \left\{ [i\omega_n - \Sigma_\nu(i\omega_n)]^2 - \xi_\nu^2(p) - \Delta^2 + h^2 - 2[i\omega_n - \Sigma_\nu(i\omega_n)] h^i \sigma^i \right\} (i\sigma_y), \quad (9)$$

where $R^2 = (i\omega_n - \Sigma_\nu(i\omega_n) + \xi_\nu(p))^2 - h^2$, $\xi_\nu(p) = \xi_p + \lambda_\nu$. After performing summation over p in Eqs. (5) and (6) we obtain

$$1 = g_0 N_0 \frac{\pi^2}{p_F d} T \sum_{n, \nu} |z_\nu|^{-\frac{1}{2}} \sin \frac{\varphi(n, \nu)}{2}, \quad (10)$$

$$U(i\omega_n) = \sum_{\cos \theta_\nu} \cos^2 \theta_\nu |z_\nu|^{-\frac{1}{2}} \left\{ [\omega_n + \Gamma U(i\omega_n) \cos^2 \theta_\nu] \sin \frac{\varphi(n, \nu)}{2} + \text{sign}(\omega_n + \Gamma U(i\omega_n) \cos^2 \theta_\nu) h \cos \frac{\varphi(n, \nu)}{2} \right\}, \quad (11)$$

where

$$\Sigma_\nu(i\omega_n) = -i\Gamma U(i\omega_n) \cos^2 \theta_\nu, \quad \Gamma = \frac{\pi^4}{2md^2} \frac{w^2}{d^4} \nu_0^5, \quad (12)$$

$$\cos^2 \theta_\nu = \left(\frac{\nu}{\nu_0} \right)^2, \quad \sum_{\cos \theta_\nu} \equiv \frac{1}{\nu_0} \sum_\nu,$$

$$|z_\nu| = \left\{ ((\omega_n + \Gamma U(i\omega_n) \cos^2 \theta_\nu)^2 + \Delta^2 - h^2)^2 + 4h^2 [\omega_n + \Gamma U(i\omega_n) \cos^2 \theta_\nu]^2 \right\}^{\frac{1}{2}}, \quad (13)$$

$$\cos \varphi(n, \nu) = |z_\nu|^{-1} \left\{ -[\omega_n + \Gamma U(i\omega_n) \cos^2 \theta_\nu]^2 - \Delta^2 + h^2 \right\}, \quad (14)$$

$$\cos \frac{\varphi(n, \nu)}{2} = \sqrt{\frac{1 + \cos \varphi(n, \nu)}{2}}, \quad \sin \frac{\varphi(n, \nu)}{2} = \sqrt{\frac{1 - \cos \varphi(n, \nu)}{2}}. \quad (15)$$

$N_0 = (m/\pi)p_F S d$ is the bulk density of states at the Fermi surface. Equations (10) and (11) contain all the required information about the superfluid film on a rough substrate in magnetic field. The gap equation in zero field was found in Ref. [12].

3. The gap equation near the continuous phase transition

Assuming continuous change of the order parameter Δ we expand the gap equation (10), or rather its form more convenient for this purpose:

$$1 = \frac{1}{2} T g_0 N_0 \frac{\pi^2}{p_F d} i \sum_{n, \nu} \left(\frac{1}{\sqrt{(i\tilde{\omega}_n(\nu) + h)^2 - \Delta^2}} + \frac{1}{\sqrt{(i\tilde{\omega}_n(\nu) - h)^2 - \Delta^2}} \right) \quad (16)$$

in powers of Δ around $\Delta = 0$; $\tilde{\omega}_n(\nu)$ is defined here as: $\tilde{\omega}_n(\nu) = \omega_n + \Gamma U(i\omega_n) \cos^2 \theta_\nu$. We also assume particle-hole symmetry and that surface imperfections do not spoil this symmetry (see Appendix A). Thus, for small values of Δ we obtain

$$\ln \frac{T}{T_{c0}} = -f_0 \left(\frac{\Gamma}{T}, \frac{h}{T} \right) - \frac{1}{2} f_1 \left(\frac{\Gamma}{T}, \frac{h}{T} \right) \left(\frac{\Delta}{2\pi T} \right)^2 + \frac{3}{8} f_2 \left(\frac{\Gamma}{T}, \frac{h}{T} \right) \left(\frac{\Delta}{2\pi T} \right)^4, \quad (17)$$

where

$$f_0 \left(\frac{\Gamma}{T}, \frac{h}{T} \right) = \left\langle \text{Re} \psi \left(\frac{1}{2} + \rho_\nu \right) - \psi \left(\frac{1}{2} \right) \right\rangle, \quad (18)$$

$$f_1 \left(\frac{\Gamma}{T}, \frac{h}{T} \right) =$$

$$\sum_{n \geq 0} \left(\left\langle \text{Re} \left(n + \frac{1}{2} + \rho_\nu \right)^{-3} \right\rangle - \frac{\Gamma}{2\pi T} \frac{\nu_c}{\nu_0} \left\langle \text{Re} \cos^2 \theta_\nu \left(n + \frac{1}{2} + \rho_\nu \right)^{-2} \right\rangle^2 \right), \quad (19)$$

$$\begin{aligned} f_2 \left(\frac{\Gamma}{T}, \frac{h}{T} \right) = & \sum_{n \geq 0} \left\{ \left\langle \text{Re} \left(n + \frac{1}{2} + \rho_\nu \right)^{-5} \right\rangle - 3 \left\langle \text{Re} \cos^2 \theta_\nu \left(n + \frac{1}{2} + \rho_\nu \right)^{-2} \right\rangle \right. \\ & \times \left\langle \text{Re} \cos^2 \theta_\nu \left(n + \frac{1}{2} + \rho_\nu \right)^{-4} \right\rangle \frac{\Gamma}{2\pi T} \frac{\nu_c}{\nu_0} + \frac{4}{3} \left\langle \text{Re} \cos^4 \theta_\nu \left(n + \frac{1}{2} + \rho_\nu \right)^{-3} \right\rangle \\ & \times \left\langle \text{Re} \cos^2 \theta_\nu \left(n + \frac{1}{2} + \rho_\nu \right)^{-2} \right\rangle^2 \left(\frac{\Gamma}{2\pi T} \right)^2 \left(\frac{\nu_c}{\nu_0} \right)^2 \Big\}, \end{aligned} \quad (20)$$

$$\rho_\nu = \frac{\Gamma' \cos^2 \theta_\nu}{2\pi T} + \frac{ih}{2\pi T}, \quad \langle \dots \rangle \equiv \frac{1}{\nu_c} \sum_{\nu=1}^{\nu_c} (\dots), \quad \Gamma' = \frac{\nu_c(\nu_c + \frac{1}{2})(\nu_c + 1)}{3\nu_0^3} \Gamma, \quad (21)$$

$\psi(x)$ is the digamma function. T_{c0} is the critical temperature at zero field and in the limit of ideally smooth boundaries, i.e. $w = 0$. It is given by

$$T_{c0} = \frac{2\omega_C e^\gamma}{\pi} \exp \left(-\frac{p_F d}{\pi \nu_c N_0 g_0} \right) = 1.13 \omega_C \exp \left(-\frac{p_F d}{\pi \nu_c N_0 g_0} \right). \quad (22)$$

ω_C is the energy cut-off parameter. It is easy to show the relation between T_{c0} and T_c , the critical temperature of the bulk liquid:

$$\ln \frac{T_{c0}}{T_c} = \left(1 - \frac{p_F d}{\pi \nu_c} \right) \ln \left(1.13 \frac{\omega_C}{T_c} \right). \quad (23)$$

Equation (17) resembles the gap equation in the vicinity of the second order phase transition for superconductors with impurities [2]. Let us note that when the film thickness d increases to infinity or when the measure of the irregularities on the surface w is vanishingly small, then

$$\Gamma = \frac{\pi^4}{2md^2} \frac{w^2}{d^4} \nu_0^5 \rightarrow 0, \quad (24)$$

$$\rho_\nu \rightarrow \rho = \frac{ih}{2\pi T}, \quad (25)$$

and the functions f_0 , f_1 and f_2 reach the same limits as for clean superconductors in Ref. [1].

The critical field at the phase transition of second order is obtained from Eq. (17) by setting $\Delta = 0$. Then (17) becomes simplified:

$$\ln \frac{T}{T_0} = - \left\langle \operatorname{Re} \psi \left(\frac{1}{2} + \rho_\nu \right) - \psi \left(\frac{1}{2} \right) \right\rangle. \quad (26)$$

4. The order of the phase transition

We find the free energy from the formula [15]:

$$F_s - F_n = \int_0^\Delta d \left(\frac{1}{g} \right) \Delta^2, \quad (27)$$

where $d \left(\frac{1}{g} \right)$ is determined by Eqs. (16) and (17),

$$d \left(\frac{1}{g} \right) = N_0 \frac{\pi \nu_c}{p_F d} \left(-f_1 \left(\frac{\Gamma}{T}, \frac{h}{T} \right) \frac{\Delta}{(2\pi T)^2} + \frac{3}{2} f_2 \left(\frac{\Gamma}{T}, \frac{h}{T} \right) \frac{\Delta^3}{(2\pi T)^4} \right) d\Delta. \quad (28)$$

Performing integration in (27) we get

$$F_s - F_n = -N_0 \frac{\pi \nu_c}{p_F d} \left[\frac{1}{4} f_1 \left(\frac{\Gamma}{T}, \frac{h}{T} \right) \frac{\Delta^4}{(2\pi T)^2} - \frac{1}{4} f_2 \left(\frac{\Gamma}{T}, \frac{h}{T} \right) \frac{\Delta^6}{(2\pi T)^4} \right]. \quad (29)$$

In the Landau and Lifshitz theory the change of the order of the phase transition follows from the change of sign of the function f_1 . For $f_1 > 0$ the transition is of second order, whereas for $f_1 < 0$ it is of first order. In the region where the phase transition is of first order the gap equation has two solutions. Figure 1(a)–(d) shows the phase diagrams for different values of Γ' (see Appendix B). For $\Gamma' = 0$ the coordinates of the critical point are $T_c^* = 0.561 T_{c0}$ and $h_c^* = 0.610 \Delta_0$, where Δ_0 is the magnitude of the order parameter at zero field ($h = 0$) and in the limit of smooth boundaries ($\Gamma' = 0$). We investigate the influence of the substrate's irregularities on the critical point (T_c^*, h_c^*) , i.e. we find solution to the equation $f_1 = 0$ for different values of Γ' (see Eq. (19) and Appendix B). The diffuse scattering significantly reduces both the critical field and temperature as can be seen in Figs. 2 and 3. There is a critical value of Γ' , $\Gamma'_c \approx 3.4 \Delta_0$ above which only the normal fluid can exist. Our numerical solution shows the existence of the first order phase transition for all values $\Gamma' \leq 3.4 \Delta_0$. The critical point is present on the phase diagram until the superfluidity is destroyed by the surface scattering (solid lines on Figs. 2 and 3).

Due to the quantization of energy levels, the coordinates T_c^* and h_c^* of the critical point are discontinuous functions of the film's thickness. We illustrate this in Fig. 4. Here we use the parameter of the absolute surface roughness

$p_F^4 w^2 / (\Delta_0 / \mu)$ which is a measure of surface irregularities in units of interatomic distance:

$$\frac{\Gamma'}{\Delta_0} = \frac{p_F^4 w^2}{\Delta_0 / \mu} \frac{\pi^4 \nu_0^2 \nu_c (\nu_c + 1) (\nu_c + \frac{1}{2})}{3(p_F d)^6}. \quad (30)$$

Figure 5 presents the trend of oscillations of the coordinates of the critical point. Various aspects of size effects without magnetic field were studied previously [8, 12, 13]. Finally, we present the solution of the gap equation. We find the order parameter as a function of magnetic field, Fig. 6, and temperature, Fig 7.

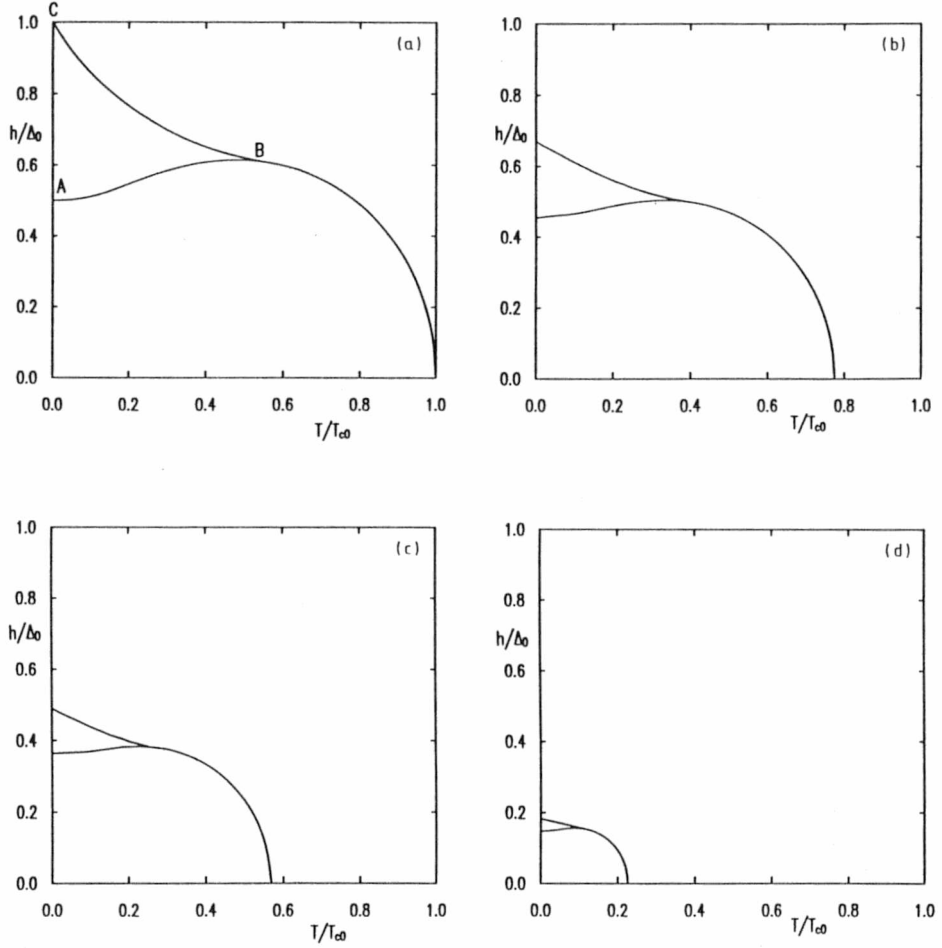


Fig. 1. The phase diagram for: (a) $\Gamma'/\Delta_0 = 0.0$, (b) $\Gamma'/\Delta_0 = 0.5$, (c) $\Gamma'/\Delta_0 = 1.0$, (d) $\Gamma'/\Delta_0 = 2.0$. The lines AB and BC represent the supercooling field and overheating field respectively. In the region ABC the gap equation has two solutions.

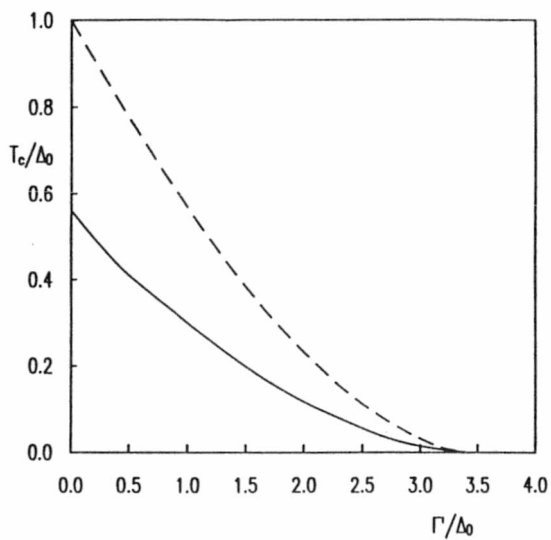


Fig. 2. Critical temperature at zero field (broken line) and the temperature coordinate of the critical point T_c^*/T_{c0} (solid line) as a function of the scattering rate Γ'/Δ_0 .

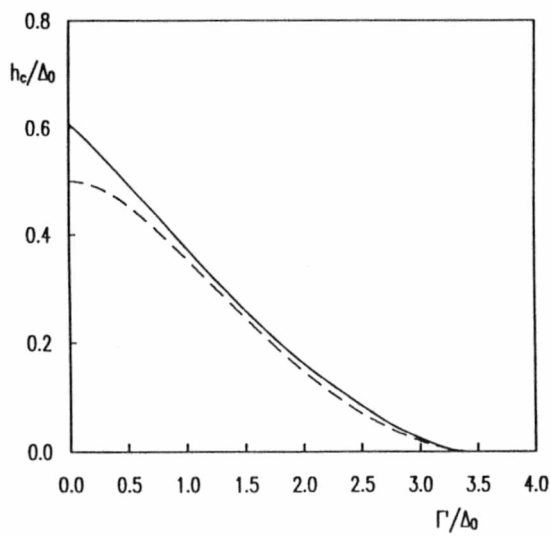


Fig. 3. The dependence of the critical field for the second order transition at $T = 0$ (broken line) and the magnetic field coordinate of the critical point h_c^*/Δ_0 (solid line) on the scattering rate Γ'/Δ_0 .

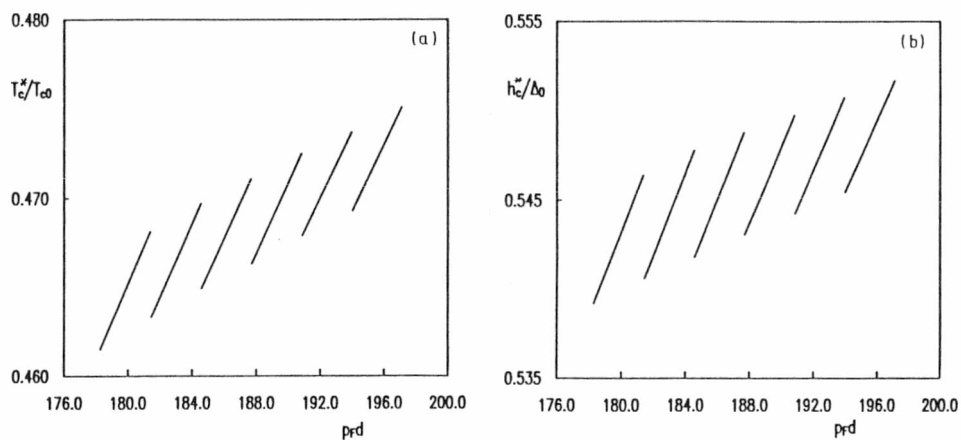


Fig. 4. Oscillations of (a) T_c^*/T_{c0} and (b) h_c^*/Δ_0 with changing thickness. The value of the parameter of the absolute surface roughness [13] $p_F^4 w^2/(\Delta_0/\mu)$ is set to 500.

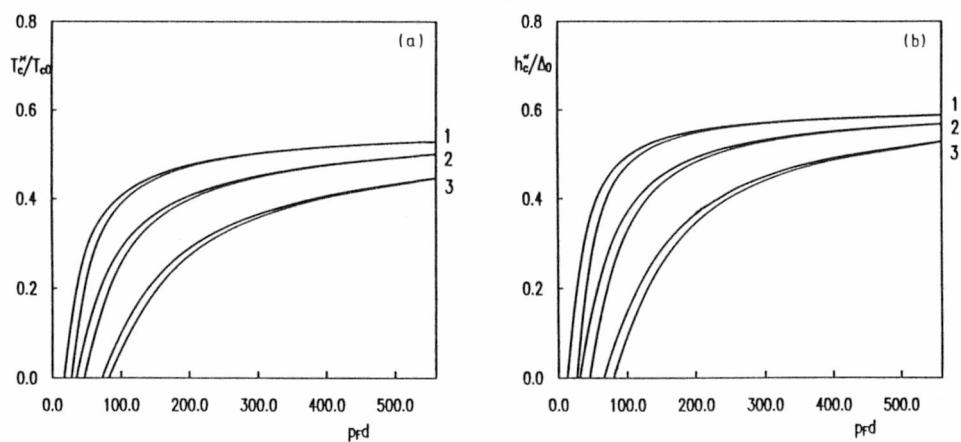


Fig. 5. The envelope of oscillations of (a) T_c^*/T_{c0} and (b) h_c^*/Δ_0 for $p_F^4 w^2/(\Delta_0/\mu) = 500$ (1), 1000 (2), 2000 (3).

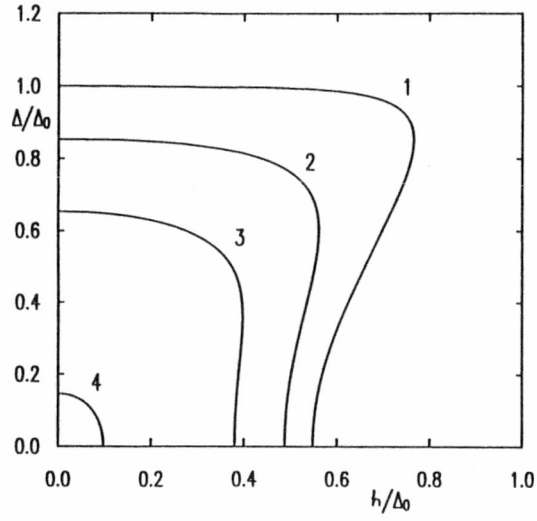


Fig. 6. The energy gap as a function of magnetic field at $T = 0.2 T_{c0}$. The lines on the graph represent (1) $\Gamma'/\Delta_0 = 0.0$, (2) $\Gamma'/\Delta_0 = 0.5$, (3) $\Gamma'/\Delta_0 = 1.0$, (4) $\Gamma'/\Delta_0 = 2.0$.

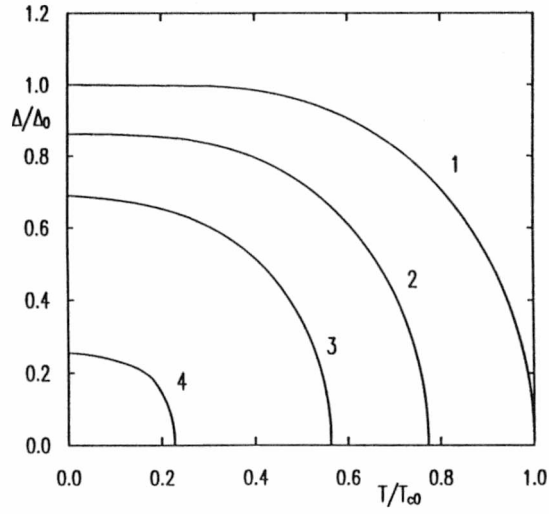


Fig. 7. The energy gap as a function of reduced temperature at $h = 0$; (1) $\Gamma'/\Delta_0 = 0.0$, (2) $\Gamma'/\Delta_0 = 0.5$, (3) $\Gamma'/\Delta_0 = 1.0$, (4) $\Gamma'/\Delta_0 = 2.0$.

5. Conclusions

We have studied the phase transition from the s-paired superfluid to the normal state in a thin film under the influence of a magnetic field and a surface scattering. The interaction of the fluid with the substrate modifies the spectrum of one-particle excitations (Eq. (5)), while the particle-particle correlations are not directly changed. Although the influence of different perturbations on the phase transition in superconductors was extensively studied in the past, we will summarize some of those results and compare them with ours.

Maki and Tsuneto [1] showed that due to the Pauli paramagnetism the order of the phase transition in magnetic field may change from second to first when temperature decreases. However, in superconductors this behavior is possible only in a very thin film [2] ($d \approx 10 \text{ \AA}$) due to a strong diamagnetism. When the system is doped with magnetic impurities the critical point (T_c^*, h_c^*) changes its position on the phase diagram [1]. There exists a critical concentration of impurities that makes the phase transition of only the second order. The scattering on magnetic impurities breaks the time reversal symmetry. As it was stressed before, we consider the model in which the geometrical restrictions impose in Eq. (5), (6) asymmetry analogous to the broken time reversal symmetry [15, 19]. We found a strong dependence of the order of the phase transition on the scattering rate I'/Δ_0 , (Fig. 1). Nevertheless our numerical analysis shows the permanent existence of the first order phase transition, (Figs. 2 and 3). In other words the point (T_c^*, h_c^*) is not removed from the phase diagram. This behavior of the critical point is similar to its dependence on the strength of the Fermi liquid interaction [3], when this latter is taken into account. It was pointed out by Jacak and Krzyżanowski [3] that there is still a change of the phase transition order when the value of the dimensionless Landau amplitude F_0^a is in the interval $-1 < F_0^a < 1$. This resemblance is not surprising since the inclusion of scattering on substrate's irregularities (Eq. (5)) corresponds to the definition of the Fermi interactions between quasi-particles [3, 15]. Our model is suitable for describing neutral s-paired systems. This kind of superfluidity (if it is realized) in a thin film ($d \leq 40 \text{ nm}$) could provide an opportunity to study the influence of spin paramagnetism and interaction with a substrate on the phase transition.

In the case of a thin film of fermions with p-pairing interactions, e.g. superfluid ^3He , these effects could be observable in a low magnetic field [20], $H \leq 20 \text{ G}$, and temperature much below the critical temperature T_c [21, 22] due to the strong orientational influence of the external magnetic field on the order parameter [22, 23].

Acknowledgments

We would like to thank Dr. L. Jacak for suggesting the problem studied here and for helpful discussions with one of us (GH).

Appendix A. Particle-hole symmetry

We use the assumption that the reflections of particles from the surface do not affect the particle-hole symmetry, i.e. the scattering does not convert the particle excitations into hole excitations and vice versa. It means that we look for solutions of the equation:

$$\tilde{\omega}_n(\nu) = \omega_n + \Gamma \cos^2 \theta_\nu \sum_{\cos \theta_{\nu'}} \cos^2 \theta_{\nu'} \frac{\tilde{\omega}_n(\nu')}{[\tilde{\omega}_n^2(\nu') + \Delta^2]^{\frac{1}{2}}}, \quad (31)$$

which in the limit $\Delta \rightarrow 0$ becomes

$$\tilde{\omega}_n(\nu)|_{\Delta=0} = \omega_n + \Gamma' \cos^2 \theta_\nu \text{sign}(\tilde{\omega}_n(\nu)) = \omega_n + \Gamma' \cos^2 \theta_\nu \text{sign} \omega_n. \quad (32)$$

The second equality is possible due to our assumption of the particle-hole symmetry. The Andreev reflection [16, 17] which reverses character of excitations might modify somewhat the results presented here.

Appendix B

To calculate the critical magnetic field for the continuous transition we rewrite Eq. (26)

$$\ln \frac{T}{T_{c0}} = - \left\langle \text{Re} \psi \left(\frac{1}{2} + \frac{\Gamma' \cos^2 \theta_\nu}{2\pi T} + \frac{ih}{2\pi T} \right) - \psi \left(\frac{1}{2} \right) \right\rangle \quad (33)$$

in a form convenient for numerical analysis:

$$\ln \frac{T}{T_{c0}} = - \frac{1}{\nu_c} \sum_{\nu, n \geq 0} \frac{(n + \frac{1}{2} + \rho_1 \cos^2 \theta_\nu) \rho_1 \cos^2 \theta_\nu + \rho_2^2}{(n + \frac{1}{2}) [(n + \frac{1}{2} + \rho_1 \cos^2 \theta_\nu) + \rho_2^2]}, \quad (34)$$

where

$$\rho_1 = \frac{\Gamma'/\Delta_0}{T/T_{c0}} 0.281, \quad \rho_2 = \frac{h/\Delta_0}{T/T_{c0}} 0.281. \quad (35)$$

First we assign a value to the parameter of relative surface roughness Γ'/Δ_0 then at a temperature T/T_{c0} Eq. (34) is solved for a critical field h/Δ_0 . This gives the lower curve in Fig. 1. To obtain the overheating curve in Fig. 1 we look for the highest h , at a given temperature, for which there exists a nonzero solution of the gap equation (16). The following form of (16) is suitable for numerical treatment:

$$\ln \frac{T}{T_{c0}} = 2\eta \frac{1}{\nu_c} \sum_{\nu, n \geq 0} |z_\nu|^{-\frac{1}{2}} \sin \frac{\varphi(n, \nu)}{2} - \sum_{n \geq 0} \frac{1}{n + \frac{1}{2}}, \quad (36)$$

where

$$|z_\nu| = \left\{ \left[\left((2n+1)\eta + \frac{\Gamma'}{\Delta} \cos^2 \theta_\nu U(i\omega_n) \right)^2 - \left(\frac{h}{\Delta} \right)^2 + 1 \right]^2 + 4 \left(\frac{h}{\Delta} \right)^2 \left[(2n+1)\eta + \frac{\Gamma'}{\Delta} \cos^2 \theta_\nu U(i\omega_n) \right]^2 \right\}^{1/2}, \quad (37)$$

$$\cos \varphi(n, \nu) = |z_\nu|^{-1} \left\{ - \left[(2n+1)\eta + \frac{\Gamma'}{\Delta} \cos^2 \theta_\nu U(i\omega_n) \right]^2 + \left(\frac{h}{\Delta} \right)^2 - 1 \right\}, \quad (38)$$

$$\eta = \frac{T/T_{c0}}{\Delta/\Delta_0} 1.78. \quad (39)$$

Equation (36) is solved together with a version of (11):

$$U(i\omega_n) = \sum_{\cos \theta_\nu} \frac{\cos^2 \theta_\nu}{|z_\nu|^{1/2}} \left\{ \left[(2n+1)\eta + \frac{\Gamma'}{\Delta} \cos^2 \theta_\nu U(i\omega_n) \right] \sin \frac{\varphi(n, \nu)}{2} + \text{sign} \left((2n+1)\eta + \frac{\Gamma'}{\Delta} \cos^2 \theta_\nu U(i\omega_n) \right) \frac{h}{\Delta} \cos \frac{\varphi(n, \nu)}{2} \right\}. \quad (40)$$

The critical point is found from $f_1 = 0$. For this purpose we rewrite (19) in the following way:

$$f_1(\rho_1, \rho_2) = \sum_{n \geq 0} \left\{ \left\langle \frac{(n + \frac{1}{2} + \rho_1 \cos^2 \theta_\nu)^3 - 3\rho_2^2(n + \frac{1}{2} + \rho_1 \cos^2 \theta_\nu)}{[(n + \frac{1}{2} + \rho_1 \cos^2 \theta_\nu)^2 + \rho_2^2]^3} \right\rangle - \frac{3\nu_0^2}{(\nu_c + 1)(\nu_c + \frac{1}{2})} \rho_1 \left\langle \frac{\cos^2 \theta_\nu [(n + \frac{1}{2} + \rho_1 \cos^2 \theta_\nu)^2 - \rho_2^2]}{[(n + \frac{1}{2} + \rho_1 \cos^2 \theta_\nu)^2]^2} \right\rangle \right\}, \quad (41)$$

ρ_1 and ρ_2 are defined in (35). Equation (41) is solved together with Eq. (34).

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THE INFLUENCE OF VAN DER WAALS FORCES ON T_c IN THIN FILM OF He^3

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Recently it has been reported that the transition temperature to superfluid state of He^3 in thin film is highly affected by various types of substratum. In this note we try to elucidate this effect by the inclusion of the Van der Waals forces into consideration. Though the model used is very simplified the proper behaviour is found.

Recently Harrison et al (1) have reported that the transition temperature to superfluid state of He^3 in thin film is highly reduced from its bulk value. This suppression decreases, however, when the substrate is coated with a monolayer of He^4 . Some authors suggested (2) that the absorbed He^4 changes the nature of scattering on substrate from diffuse to specular. In the present short note we intend to check up the influence on transition temperature in thin He^3 film of the Van der Waals forces in the range of substratum. This forces could be also used as the convenient model of various types of substrata (with inclusion of He^4 atoms coating too). Many authors describe this forces via the Lenhard-Jones potential (instead of the infinite potential wall which models the surface of the thin film). In order to check the tendency only we simplify highly this model and consider the potential well in the z direction (perpendicular to the infinite film) according to the formula:

$$V(z) = \begin{cases} V > 0 & \text{for } 0 < z \leq a; \\ 0 & \text{for } a < z < d \\ \infty & \text{for } z \leq 0 \text{ or } z \geq d \end{cases}$$

The one-particle energy attains then the form: $e_{p,v} = f_v + p^2/2m$, with $p = (p_x, p_y)$. The spectrum f_v was found numerically by the use of the quasiclassical methods. It is clear that instead of the homogeneous set of Fermi circles (as for simple well, cf. (3)) we deal now with similar circle set but inhomogeneous one. Note also that, roughly speaking, in the vicinity of substratum we deal with some number (which is dependent of well parameters) of "bound states". This states will influence the chemical

potential of He^3 thin film system. There is easy also to calculate the transition temperature to superfluid phase in such described thin film. The gap equation has the form:

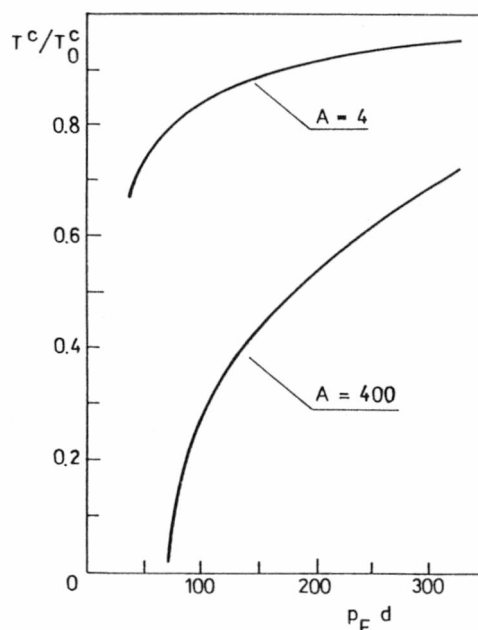


Figure 1
 T_c versus the thickness of the film for two types of Van der Waals forces ($A = 2mVa^2/\pi^2$)

$$1 = 3gmS/2\pi \sum_V \int_0^c de \frac{\sin^2 \theta_V}{E_V} \tanh \frac{E_V}{2T}$$

with

$$E_V = (e^2 + \Delta^2 \sin^2 \theta_V)^{1/2};$$

$$\sin \theta_V = (1 - f_V/\mu)^{1/2},$$

where μ is the chemical potential in thin film for the same density of He^3 as in bulk; S is the area of the film; g is the pairing constant; c is energetical cut-off parameter.

For the transition temperature the following formula holds:

$$T_c/T_{c0} = \exp((1 - 1/(3\pi/(2p_F d) \sum \sin^2 \theta_V)) * \ln(2ce^{\gamma}/\pi T_{c0})),$$

where T_{c0} is the transition temperature to superfluid state in the bulk and p_F is the Fermi radius in the bulk.

The results for the transition temperature T_c in the thin film of He^3 are presented in the Figs 1 and 2, for several values of parameters for the model potential. In calculations we have assumed $c/T_{c0} = 100$, taking in mind that the cut-off parameter in the thin film can differ from its value in the bulk. In the Fig.1 we present the decrease of T_c with the decrease of the film thickness. Note that the curve for $A=4$ (where $A=2mVa^2/\pi^2$) is similar to the T_c dependence for the substratum coated with He^4 , while $A=400$ coincides better with clean substratum. In the Fig.2 T_c is presented as the function of parameter a which imitates the "radius" of Van der Waals forces.

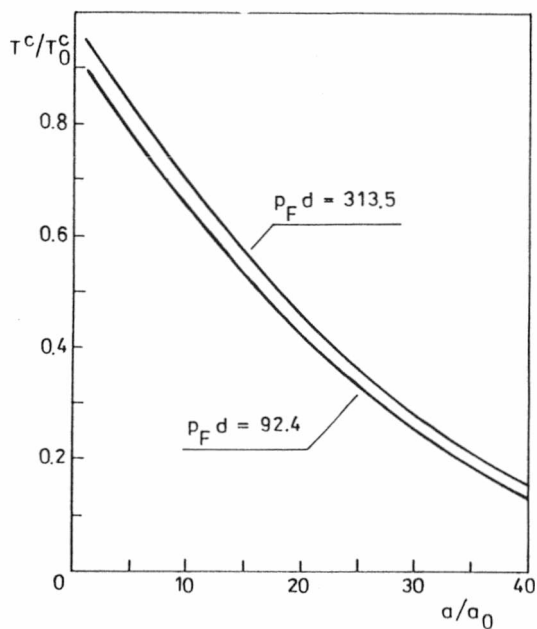


Figure 2
 T_c versus parameters of the Van der Waals forces ($2mVa_0^2 = 4\pi^2$).

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INFLUENCE OF VAN DER WAALS FORCES ON $^3\text{He-A}$ SUPERFLUID FILM FLOW AT $T=0$.

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Abstract

The superfluid A phase of ^3He in thin films is studied at temperature $T=0$. A simple model of van der Waals interaction between the superfluid and the substrate is proposed. The influence of this interaction on the critical superflow for film thicknesses $d \leq \xi_0$ is analyzed.

Pacs no. 67.50

1. Introduction

A remarkable progress in low temperature experimental techniques has made possible studies of superfluid ^3He confined to lengths of the order of the coherence length $\xi(T)$ [1-5]. Almost all measurements focused on the maximal critical current are carried out in a wide range of temperature, not only near T_c . Despite that, previous theoretical studies were limited to the Ginzburg-Landau region only [9]. Especially the most recent experiments of Xu and Crooker [5] are beyond the range of validity of theories presented so far.

In this communication we consider the superflow and the effect of the substrate potential on superflow in the opposite temperature limit $T=0$. This is the first of the forthcoming microscopical studies of superfluidity in thin films [15].

In their recent experiments, Steel et al. [1,2] found strong dependence of the transition temperature and the critical superfluid current on the properties of the interface between the superfluid and the solid. The presence of ^4He adsorbed at the substrate's surface increased substantially T_c and the superflow which remained reduced however from their bulk values. Apparently the adsorbed ^4He reduces pairbreaking at the interface and changes the character of scattering from

strongly diffuse to specular. In addition the van der Waals interaction of ^3He quasiparticles with the substrate is changed [14]. The superfluid system is isolated by ^4He atoms from the adsorbent.

The latter effect is studied in this letter. Following our former report [6] we use a potential step to simulate the Lennard-Jones potential in a film of thickness d :

$$V(z) = \begin{cases} V > 0 & \text{for } 0 < z \leq a \\ 0 & \text{for } a < z < d \\ \infty & \text{for } z \leq 0 \text{ or } z \geq d \end{cases}, \quad (1)$$

a and V are substrate-dependent parameters .

This perturbation allows for a drastic change of the single particle energy spectrum compared with the case of the infinite potential well usually used for thin films [8,10]. Instead of the ordinary homogenous set of Fermi circles we have unequally-spaced energy levels characterized by the single particle energy spectrum [6]:

$$e_{p,i} = f_i + \frac{p^2}{2m}, \quad (2)$$

where m is the ^3He atomic mass, p is a two-dimensional

momentum and f_i ($i=1,2,\dots,\nu_c$) is a discrete energy spectrum of the potential well $V(z)$. The number of Fermi circles is denoted by $\nu_c = \text{integer}(\nu_0)$, where ν_0 is related to the Fermi energy μ : $f_{\nu_0} = \mu$. For further details see [8,10].

2. Basic relations

Symmetry considerations yield the form of the order parameter Δ for the A phase in two dimensions [9]:

$$\Delta = (3/2)^{1/2} (\hat{\Delta}_1 \hat{p}_x + i \hat{\Delta}_2 \hat{p}_y) \sigma_3 i \sigma_2, \quad (3)$$

where σ_i ($i=1,2,3$) - are Pauli matrices.

Taking the energy spectrum $\{e_{p,i}\}$ as a set of one particle Matsubara-Green function's poles and after following standard methods [5,6], the gap equation in the presence of the superfluid flow $\mathbf{v}$ is obtained:

$$\Delta_1 = 3gN_0 \frac{\pi}{\omega p_0} \Delta_1 \sum_{i=1}^{\nu_c} \sin^2 \vartheta_i \int_0^{\omega} d\varepsilon \int_0^{\pi/2} \frac{d\phi}{\pi} \frac{\cos^2 \phi}{E_i} \left[\operatorname{tgh}(E_i^+/2T) + \operatorname{tgh}(E_i^-/2T) \right], \quad (4)$$

$$\Delta_2 = 3gN_0 \frac{\pi}{\omega p_0} \Delta_2 \sum_{i=1}^{\nu_c} \sin^2 \vartheta_i \int_0^{\omega} d\varepsilon \int_0^{\pi/2} \frac{d\phi}{\pi} \frac{\sin^2 \phi}{E_i} \left[\operatorname{tgh}(E_i^+/2T) + \operatorname{tgh}(E_i^-/2T) \right], \quad (5)$$

where

$$E_i = \left(\varepsilon^2 + \frac{3}{2} (\Delta_1^2 \cos^2 \phi + \Delta_2^2 \sin^2 \phi) \sin^2 \vartheta_i \right)^{1/2},$$

$$E_i^{+(-)} = E_i + (-) v p_f \sin \vartheta_i \cos \phi, \quad \sin^2 \vartheta_i = 1 - \frac{f_i}{\mu},$$

T is a temperature, N_0 represents the bulk density of states at the Fermi surface, p_0 denotes the Fermi momentum and g, ω are the bulk coupling constant and the energetical cut-off parameter respectively. p_f is the two-dimensional Fermi momentum [6] determined by the three-dimensional Fermi momentum p_0 :

$$p_f = \left(2p_0^3 d / (3\pi \sum_{i=1}^{\nu_c} \sin^2 \vartheta_i) \right)^{1/2}. \quad (6)$$

Standard methods [7] of calculating the superfluid current j_s lead to :

$$j_s = (3\pi^2)^{-1/3} n^{2/3} \left(v p_0 - \frac{(1.5\pi)^{1/2}}{d p_0 s_d} \sum_{i=1}^c \sin \vartheta_i \right) \quad (7)$$

$$\times \int_0^{\omega} d\varepsilon \int_{-\pi/2}^{\pi/2} \frac{d\phi}{\pi} \cos \phi \left[\operatorname{tgh} (E_i^+ / 2T) - \operatorname{tgh} (E_i^- / 2T) \right] ,$$

$$\text{where } s_d = \left(\frac{1}{p_0 d} \sum_{i=1}^c \sin^2 \vartheta_i \right)^{1/2} .$$

Free energy of the superfluid state relative to the normal state is found by appropriate integration [7] of equations (4) and (5). Unfortunately, cumbersome numerical evaluations are required in order to solve all these equations. The problem is much simpler in the limit $T=0$.

At temperature $T=0$ the gap equation (4),(5) reduces to a convenient form for further numerical calculations :

$$\begin{pmatrix} \Delta_1 \\ \Delta_2 \end{pmatrix} = 3gN_o \frac{\pi}{dp_o} \sum_{i=1}^{\nu_c} \sin^2 \vartheta_i \int_0^{\omega} \frac{d\phi}{\pi} \int_0^{\omega} d\varepsilon \begin{pmatrix} \Delta_1 \cos^2 \phi \\ \Delta_2 \sin^2 \phi \end{pmatrix} \\ \times \frac{1}{E_i} \left[1 + \Theta(E_i - v_{p_f} \sin \vartheta_i \cos \phi) - \Theta(v_{p_f} \sin \vartheta_i \cos \phi - E_i) \right], \quad (8)$$

The superflow is now :

$$j_s = (3/8\pi)^{1/6} s_d n^{2/3} \left[v_{p_f} - E(v_{p_f} - (1.5)^{1/2} \Delta_1) \frac{(2/3)^{1/2} (v_{p_f})^2 - (3/2)^{1/2} \Delta_1^2}{(2(v_{p_f})^2/3 + \Delta_2^2 - \Delta_1^2)^{1/2}} \right] \quad (9)$$

and the free energy ΔF at $T=0$ is given by:

$$\Delta F = \left[v_{p_f} (v_{p_f} - E(v_{p_f} - (1.5)^{1/2} \Delta_1) \frac{(v_{p_f})^2 - 1.5 \Delta_1^2}{((v_{p_f})^2 + 1.5 \Delta_2^2 - 1.5 \Delta_1^2)^{1/2}}) \right. \\ \left. - 0.75(\Delta_1^2 + \Delta_2^2) \right] \times 0.5gN_o \pi s_d^2, \quad (10)$$

where $\Theta(x) = \begin{cases} 1 & \text{for } x \geq 0 \\ 0 & \text{for } x < 0 \end{cases}$.

3. Results

The solution of the gap equation (8) at $T=0$ is presented in Fig.1. It should be emphasized that this diagram is universal for every film thickness and potential $V(z)$. All information about the real film parameters (i.e. thickness and potential) enters this relation through $\Delta_0 = \Delta(v=0, T=0)$ and p_f (6). We find two superfluid phases :

the A-like phase ($\Delta_1 > 0, \Delta_2 > 0$) and the Polar-like phase ($\Delta_1 = 0$). The first order transition between these two phases takes place at $vp_f \simeq 1.07\Delta_0$ (Fig.2). It can be seen from the Fig.1 that the relation between the order parameter (Δ_1, Δ_2) and the superfluid velocity v in a thin film is very similar to that of the bulk B phase [7].

The superflow as a function of superfluid velocity v is obtained from (9) and shown in Fig.3. Here we introduced a quantity $\Delta_s = s_d \Delta_0$, whereas $n = p_0^3 / 3\pi^2$ is a total ^3He bulk density. This diagram is also invariant to the changes of both the thickness of the film and the potential $V(z)$.

Taking into account the phase transition (Fig.2) and superflow drawn in Fig.3 we can determine the critical (i.e. maximal) superflow j_{sc} at $T=0$ as $j_{sc} = j_s(v p_f = 1.07\Delta_0)$.

The resulting superfluid current which is given by the formula:

$$j_{sc} = (3/8\pi)^{1/6} \Delta_s n_s^{2/3} \quad (11)$$

is presented for a wide range of film thicknesses in Fig.4. It is easy to notice that the potential $V(z)$ simulating the van der Waals field of a substrate substantially changes values of superfluid currents. Not only the field but mainly finite dimensions depress superfluidity. There is a critical thickness of the film at which the superflow vanishes. This critical value depends on the potential $V(z)$. We can conclude that this study proves that the superflow is strongly reduced by a van der Waals field.

To give our results some experimental background we present in Fig.5 the results of measurements of Steel et al. These experiments compare the ^3He superfluid flow rates in a film covering a substrate with a layer of ^4He adsorbed at the interface with those on a bare metallic surface. There is a qualitative agreement of our calculations with the experimental results. However, due to not compatible definitions of flow rates and slightly different temperature range a direct comparison is not possible yet. Several measurements

of supeflows on various surfaces carried out by Harrison et al. [1,2] resulted in a statement that small critical currents support other evidence for an alternate dissipation mechanism that comes in before pair-breaking [1]. We suggest that the interaction considered in this communication may be a candidate for such a mechanism.

We have discussed a very simple model of a thin film. The assumption of a constant film thickness led us to the one dimensional quantum well approximation for this system. The van der Waals-like field was also chosen as a uniform function of the distance from the adsorbent. It is obvious that in real systems these assumptions do not hold and as a result the density of ^3He atoms changes when approaching the substrate's surface. This implies that the phase transition from the normal to the superfluid state does not occur immediately in a whole sample but appears progressively in different regions of a film. This effect should be taken into account by considering additional interfaces between phases. However our studies are mainly devoted to films of thickness equal or smaller than the zero-temperature coherence length ξ_0 and in this limit the complications mentioned above are not relevant. In so chosen

geometrical regime the two-dimensional superfluid phases should occur. It was shown by Brusov and Popov [12] that there are two degenerate phases : A- and B-like which minimize the free energy, but the A-like phase is stabilized by a superflow [11] and there is no need to consider the B-like phase as far as the determination of a critical superflow is concerned.

Although our model is quite simple we have obtained the results consistent with available experimental data.

Finally we would like to add ,that due to the orientational effects [13], films with thickness of a several ξ_0 can also be analyzed within this model.

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5. Figure captions

Fig.1. The order parameter at $T=0$ as a function of superfluid velocity v for the A-like phase (the solid line):

Δ_2 - the longitudinal, Δ_1 - the transversal amplitudes and for the polar-like phase (the dashed line).

Fig.2. The free energy at $T=0$ of the A-like phase (the solid line) and the polar-like phase (the dashed line) versus superfluid velocity v .

Fig.3. The superflow j_s at $T=0$ as a function of superfluid velocity v for the A-like phase (the solid line) and the polar-like phase (the dashed line).

Fig.4. The critical superflow j_{sc} at $T=0$ as a function of film thickness for five different potentials :

(1) $V_a=0.1$, $d/a=100$

(2) $V_a=0.5$, $d/a=100$

(3) $V_a=10$, $d/a=100$

(4) $V_a=50$, $d/a=50$

(5) $V_a=50$, $d/a=10$

where

$V_a = 2mVa^2/\pi^2$, and d is the maximum thickness.

Δ_o^b is the bulk gap at $T=0$.

Fig.5. Flow rate versus film thickness for pure ^3He (the black symbols) and with ^4He monolayer (the white symbol measured by Steel et al [2]).

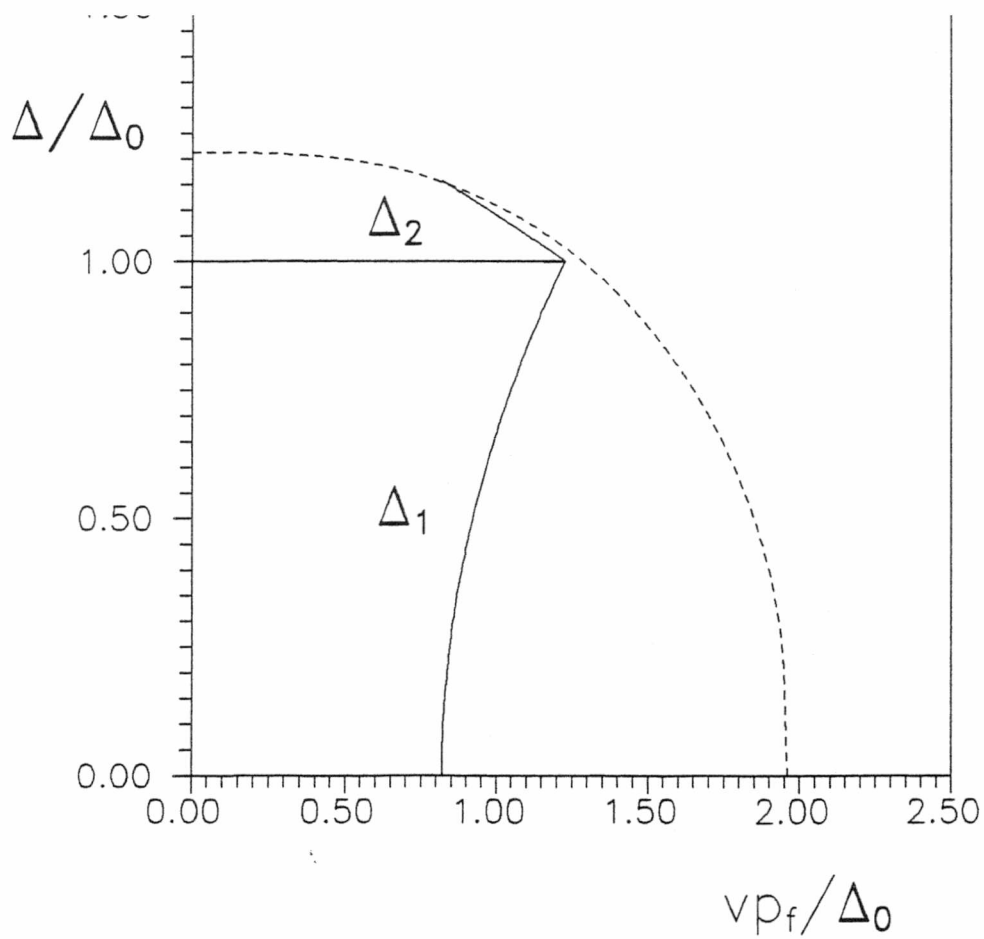


Fig.1

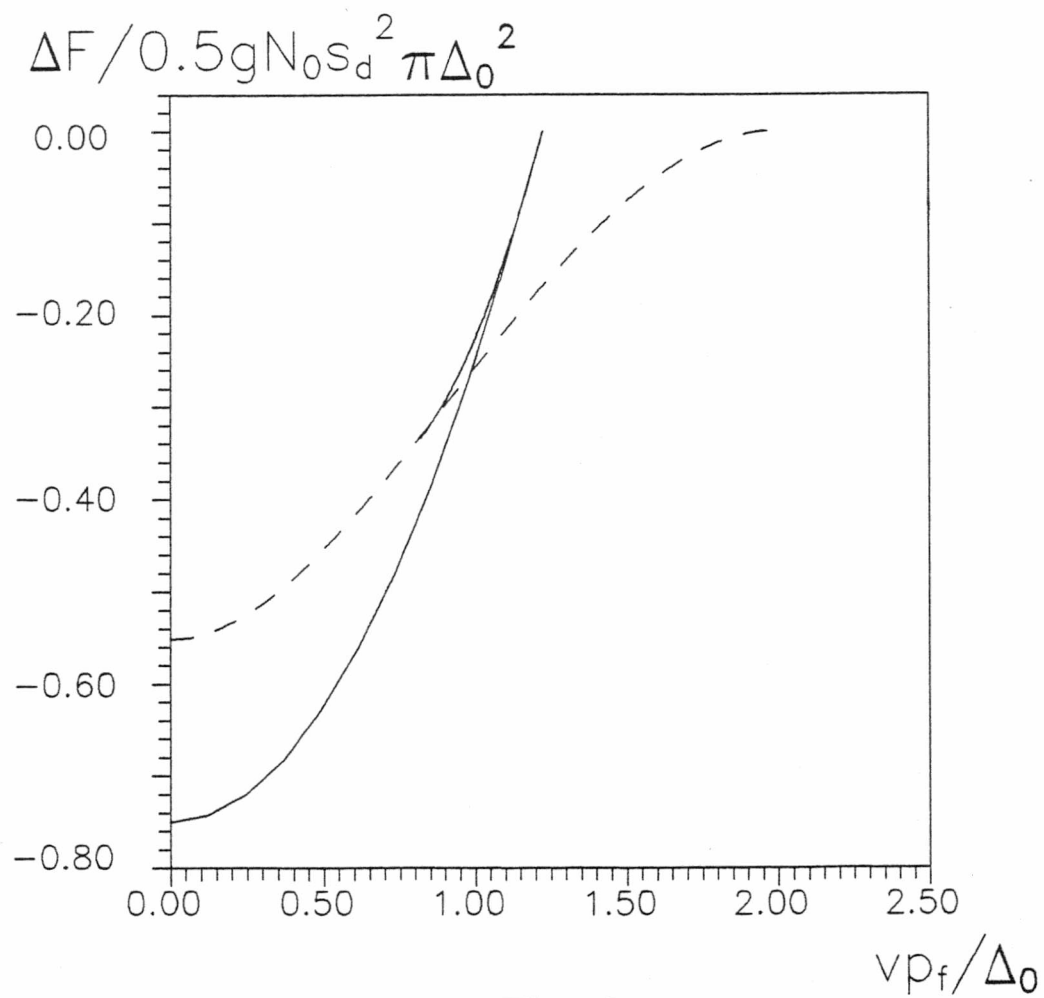


Fig.2

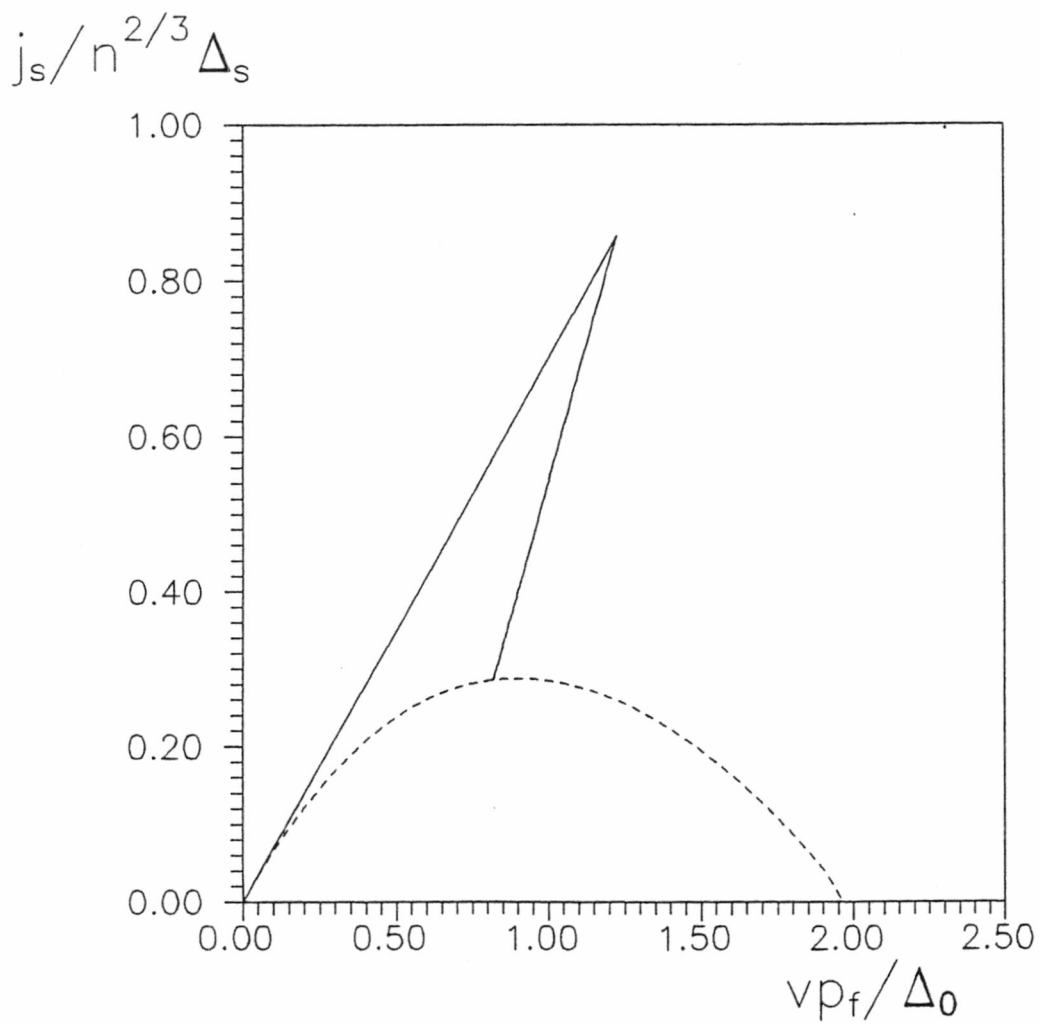


Fig.3

$$j_{sc}/n^{2/3}\Delta_0^b$$

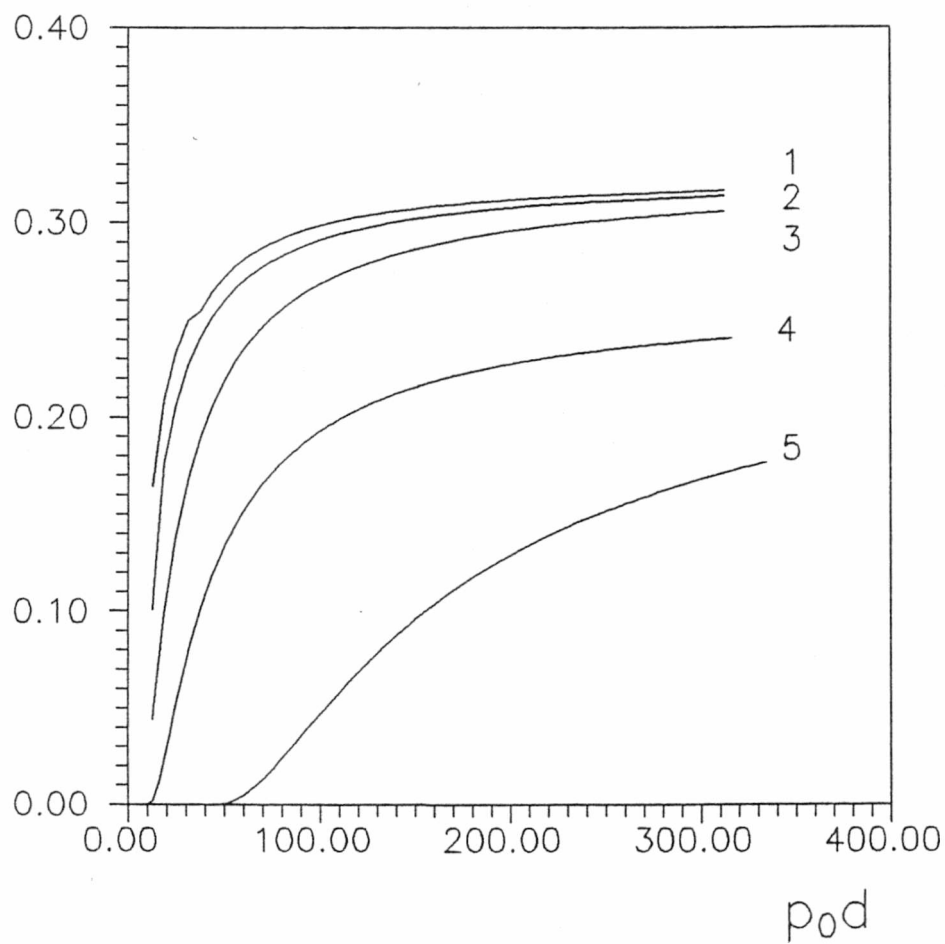


Fig.4

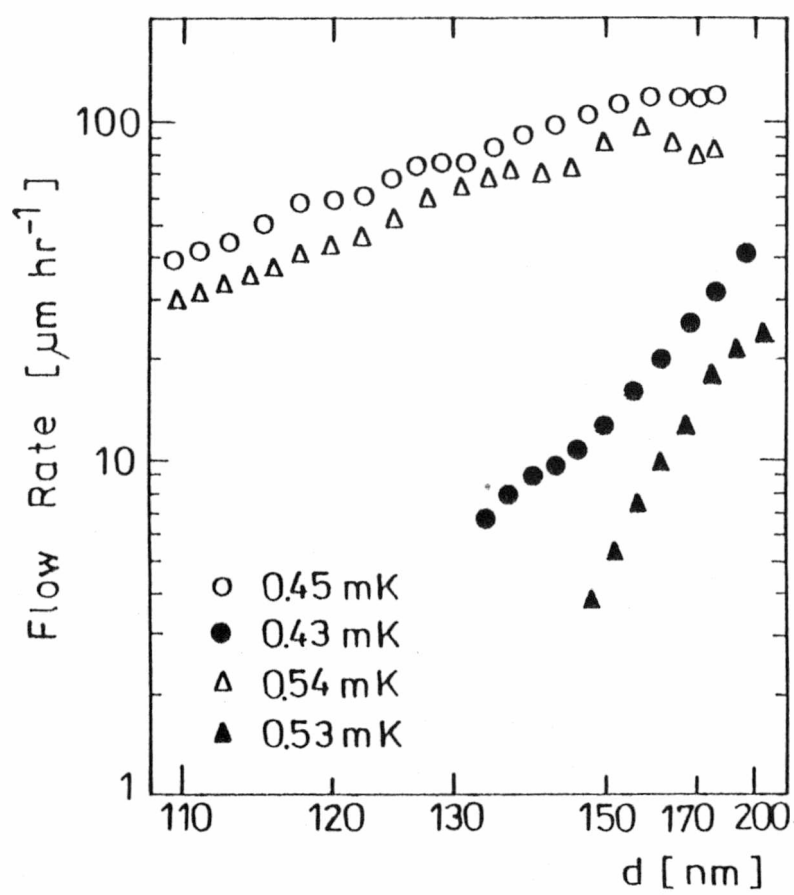


Fig. 5

THEORY OF SUPERFLOW IN TWO-DIMENSIONAL STATES OF ^3He
IN AN ARBITRARY POTENTIAL WELL .

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ABSTRACT

Two-dimensional superfluid states of ^3He are studied within a microscopic model, including effects of a substrate potential and the superflow. The energetical stability of these states is analyzed. The phase transition temperature and the critical superflow are found and compared to experiment. The effect of substrate coating with ^4He is discussed.

1. INTRODUCTION

Recent progress in experimental low temperature techniques during the last few years allowed several groups [1-7] to obtain superfluid ^3He in confined geometries. They reported existence of superfluid currents in thin films with thickness d approaching the zero-temperature coherence length $\xi_0 \cong 65\text{nm}$ (at $P=0$ bar) [1-7]. Although experimental efforts focused on determination of the superfluid density and the critical temperature T_c^F of the superfluid transition, the theoretical description of these phenomena is still not sufficient. There are studies by Fetter and Ullah [8] and Jacobsen and Smith [9] in which the superflow is calculated, but in the Ginzburg-Landau regime only. Some experiments are carried out in a different temperature limit [2] and it is not possible to compare them with these theoretical predictions. In this paper the superfluidity is studied within the microscopical theory and results obtained are valid for all temperatures. As it was mentioned in our first paper [10] for the $T=0$ limit, our aim is to investigate the influence of the substrate potential on the superfluid state. One of the experimental groups [1,4,5] reported that both the superflow and the critical temperature strongly depend on the kind of substrate. These two quantities are considerably enhanced when the adsorbent is coated with ^4He . Freeman et al.[6] suggested that ^4He

atoms smooth out the substrate irregularities and change the scattering of ^3He quasiparticles off the wall from diffuse to specular. In our work we want to describe another effect caused by covering the substrate with ^4He . We suggest that not only the kind of substrate scattering but also the interaction of ^3He quasiparticles with the substrate through the van der Waals forces is weakened by isolating layers of ^4He . Using a simple model with a substrate potential we find the measurable quantities like the maximal superflow j_{sc} and the critical temperature T_c^F . We study the two-dimensional states. Therefore our results are correct for films not thicker than ξ_0 , a little thinner than those experimentally accessible now. However as it is explained further in the text, our analysis may also be relevant for films with thickness d of the order of the temperature dependent coherence length $\xi(T)$, especially at low temperatures which is the limit not covered in a literature so far. The other thickness limit for this model is $d \gg p_0^{-1}$, where $p_0 = (3\pi^2 n)^{1/3}$ is the bulk Fermi momentum and n is the bulk density of ^3He . This condition ensures that the interactions between quasiparticles are almost the same as for the bulk system.

In this paper we use the atomic units $\hbar = k_B = 1$.

2. MODEL

We discuss a thin film of a constant thickness d . The z -axis is chosen orthogonal to the film surface. This system is approximated by the infinite potential well [11] :

$$V_1(z) = \begin{cases} 0 & \text{for } 0 < z < d \\ \infty & \text{for } z \leq 0 \text{ or } z \geq d \end{cases} . \quad (2.1)$$

In order to study the influence of the substrate forces on the adsorbed fluid we add the van der Waals-like field [12] :

$$V_2(z) = \begin{cases} V & \text{for } 0 \leq z \leq a \\ 0 & \text{for } z < 0 \text{ or } z > a \end{cases} , \quad (2.2)$$

where V and a are the amplitude and the range of the potential respectively.

The potential modelling the thin film is a superposition of V_1 and V_2 :

$$V(z) = V_1(z) + V_2(z) . \quad (2.3)$$

This simple potential takes the repulsive part of the substrate field into account, whereas the almost flat long range attractive part is approximated by the bottom of the well. This approximation is consistent with potentials calculated by means of variational methods [13,14] and based on the well known Lennard-Jones or Aziz formulas. As it was shown by Krotscheck et al. [13]

^4He layers isolate ^3He from the substrate and displace it to the region of a weaker potential. The point in our approximation is to simulate by a step potential the effect of coating the substrate with ^4He . Different values of the potential $V_2(z)$ correspond to different ^4He coverages of the surface. It should be added that potentials calculated in [13,14] have rather intricate oscillatory shape and lead to existence of some surface states. Fortunately we deal with films of thickness much larger than the interatomic distance ($d \gg p_0^{-1}$) and the number of states substantially exceeds this very limited number of bound states. That is we can neglect the surface states in a macroscopic effect like superfluidity. The potential $V(z)$ may be treated as the simplified effective mean field acting on the superfluid.

Simple considerations show that the only parameters, which characterize the quasiparticle energy spectrum are d/a and V/μ , where μ is the chemical potential of the system. In Figs.1 and 2 we show the single particle energy spectrum ϵ_ν for such a potential and its dependence on the above parameters for fixed μ . From now on this field will be given by two dimensionless parameters d/a and Va^2 .

The energy spectrum of that system is :

$$\epsilon_\nu(p) = \frac{p^2}{2m} + \epsilon_\nu, \quad \nu = 1, \dots, \nu_c, \quad (2.4)$$

where $\nu_c = \max\{ \nu : \epsilon_\nu \leq \mu \}$, $\mathbf{p} = (p_x, p_y)$ is the two-dimensional momentum and m is the mass of ^3He atom. The Fermi momentum of the quantum state number ν is $p_F \sin \vartheta_\nu$, where $\sin \vartheta_\nu = (1 - \epsilon_\nu / \mu)^{1/2}$. The inclusion of the potential $V_2(z)$ makes the energy levels unequally spaced.

For a given density n , the two-dimensional Fermi momentum p_F and the bulk Fermi momentum $p_0 = (3\pi^2 n)^{1/3}$ are connected by :

$$p_F = p_0 \left[2 p_0 d \left(3\pi \sum_{\nu=1}^{\nu_c} \sin^2 \vartheta_\nu \right)^{-1} \right]^{1/2}. \quad (2.5)$$

3. FORMALISM

We introduce the Matsubara-Green function for noninteracting system of fermions in a thin film [11, 15]:

$$\mathcal{G}_{0\nu}(i\omega_n, \mathbf{p}) = \left[i\omega_n - \epsilon_\nu(\mathbf{p}) + \mu \right]^{-1}, \quad (3.1)$$

where $\omega_n = (2n+1)\pi T$ and $\epsilon_\nu(\mathbf{p})$ is defined by Eq. (2.4).

The Green functions $\mathcal{G}_\nu(i\omega_n, \mathbf{p})$ and $\mathcal{F}_\nu(i\omega_n, \mathbf{p})$ for a superfluid state are determined by Abrikosov-Gorkov equations [16]:

$$\mathcal{G}_{\nu}(i\omega_n, \mathbf{p}) = \mathcal{G}_{0\nu}(i\omega_n, \mathbf{p}) + \mathcal{G}_{0\nu}(i\omega_n, \mathbf{p}) \Delta_{\nu}(\mathbf{p}) \mathcal{F}_{\nu}^{+}(i\omega_n, \mathbf{p}), \quad (3.2)$$

$$\mathcal{F}_{\nu}^{+}(i\omega_n, \mathbf{p}) = - \mathcal{G}_{0\nu}^{-}(i\omega_n, \mathbf{p}) \Delta_{\nu}^{+}(\mathbf{p}) \mathcal{G}_{\nu}(i\omega_n, \mathbf{p}) \quad . \quad (3.3)$$

The order parameter $\Delta_{\nu}(\mathbf{p})$ is found from the following self-consistent gap equation :

$$\Delta_{\nu}^{+}(\mathbf{p}) = -T \sum_{\mathbf{p}_1, \nu_1, n} V_{\nu\nu_1} \left[\hat{\mathbf{p}}, \hat{\mathbf{p}}_1 \right] \mathcal{F}_{\nu_1}^{+}(i\omega_n, \mathbf{p}_1) \quad , \quad (3.4)$$

where

$$V_{\nu\nu_1} \left[\hat{\mathbf{p}}, \hat{\mathbf{p}}_1 \right]_{\alpha\beta\gamma\delta} = \frac{3}{2}g \left[\hat{p}_p \hat{p}_1 \sin\vartheta_{\nu} \sin\vartheta_{\nu_1} + \cos\vartheta_{\nu} \cos\vartheta_{\nu_1} \right] \times \\ \left[\delta_{\alpha\gamma} \delta_{\beta\delta} + \delta_{\alpha\delta} \delta_{\beta\gamma} \right] \quad (3.5)$$

is the p-pairing interaction for thin films [11,15], g is the bulk coupling constant, the superscript '-' in Eq.(3.3) denotes the time inversion.

The presence of a superflow in a system is introduced by a Galilean transformation [17,18] :

$$\mathbf{p} \longrightarrow \mathbf{p} + m\mathbf{v} \quad , \quad (3.6)$$

where $\mathbf{v}$ is a superfluid velocity and is assumed to be independent of spatial coordinates. With accuracy to linear terms in $\mathbf{v}$, which is consistent with the assumption that $v \ll p_F/m$, this transformation implies a shift of Matsubara frequencies :

$$i\omega_n \longrightarrow i\omega_n - \mathbf{v}\mathbf{p} . \quad (3.7)$$

Since we have restricted our considerations to films of thickness $d \leq \xi_0$, the order parameter $\Delta_\nu(\mathbf{p})$ is not a function of z and superfluidity is two-dimensional. The order parameter will be discussed more thoroughly further in the paper.

4. TRANSITION TEMPERATURE

The superfluid transition temperature T_c^F [12] is obtained from Eqs. (3.2)-(3.4) :

$$\frac{T_c^F}{T_c^B} = \left[\frac{2\omega_c e^\gamma}{\pi T_c^B} \right] \left[1 - 2p_0 d \left[3\pi \sum_{\nu=1}^{\nu_c} \sin^2 \vartheta_\nu \right]^{-1} \right] \quad (4.1)$$

where T_c^B is the bulk critical temperature and ω_c is the energy cut-off parameter. In calculations we have assumed $\omega_c/T_c^B = 100$ according to appropriate BCS relations. ω_c should be interpreted as the limit upper value of definite quasiparticle excitations in a system.

The critical temperature T_c^F as a function of film thickness for some values of potential $V(z)$ is shown in Fig.3. There is a qualitative agreement between presented relations and the experimental results [2,3]. As it is seen from Fig.3, the van der Waals field substantially reduces T_c^F . This is consistent with the

experimental results of Harrison et al. [1,4], when different ^4He coverages of the surface correspond to different values of the potential $V(z)$. However, it is not possible to fit the measured transition temperature as a function of thickness with a simple $V(z)$ field. The small jumps in theoretically calculated T_C^F (see Fig.3) are due to quantum size effects. The similar behavior of measured T_C^F [2,3] is of the same origin, however in some cases it can be associated to experimental errors.

5. SUPERFLUID PHASES

Now we turn our attention to the order parameter $\Delta_{\nu}(\mathbf{p})$. Note that both the angular momentum L and the spin S of a pair of quasiparticles are unit vectors. The order parameter $\Delta_{\nu}(\mathbf{p})$ is represented by the spherical tensors $|J, m\rangle$ [19] with J denoting the total angular momentum of a Cooper pair and m its projection on the z -axis. It is obvious that for the two-dimensional momentum $\mathbf{p}$ the angular momentum projection m_L attains only two values $m_L = \pm 1$ and simple considerations lead to the representation of $\Delta_{\nu}(\mathbf{p})$ by six spherical tensors $|J, m\rangle$:

$$\Delta_\nu(\mathbf{p}) = \Delta \sin\vartheta_\nu \left[a_{0,0}|0,0\rangle + a_{1,0}|1,0\rangle + a_{1,1}|1,1\rangle + \right. \\ \left. a_{1,-1}|1,-1\rangle + a_{2,2}|2,2\rangle + a_{2,-2}|2,-2\rangle \right] . \quad (5.1)$$

The order parameter for a p-paired system is conventionally defined as [20] :

$$\Delta_\nu(\mathbf{p}) = \Delta \left[\mathbf{d}(\hat{\mathbf{p}}, \nu) \hat{\sigma} \right] i\sigma_y , \quad (5.2)$$

where $\hat{\sigma} = [\sigma_x, \sigma_y, \sigma_z]$ are Pauli matrices.

The vector $\mathbf{d}(\hat{\mathbf{p}}, \nu)$ is represented by a tensor d_{ij} [20] :

$$d_i(\hat{\mathbf{p}}, \nu) = d_{ij} \hat{p}_j \sin\vartheta_\nu . \quad (5.3)$$

Using the explicit form of the tensor $|J, m\rangle$ we can write

$$d_{ij} = \frac{1}{\sqrt{3}} a_{0,0} \delta_{ij} + \frac{1}{\sqrt{2}} a_{1,0} \left[\delta_{iy} \delta_{jx} - \delta_{ix} \delta_{jy} \right] + \\ \frac{1}{2} i a_{1,1} \delta_{iz} \left[\delta_{jx} + i \delta_{jy} \right] + \frac{1}{2} i a_{1,-1} \delta_{iz} \left[\delta_{jx} - i \delta_{jy} \right] \\ + \frac{1}{2} a_{2,2} \left[\delta_{ix} + i \delta_{iy} \right] \left[\delta_{jx} + i \delta_{jy} \right] + \\ \frac{1}{2} a_{2,-2} \left[\delta_{ix} - i \delta_{iy} \right] \left[\delta_{jx} - i \delta_{jy} \right] . \quad (5.4)$$

This general form of the tensor d_{ij} can also be obtained by setting $\hat{p}_z = 0$ in $\mathbf{d}(\hat{\mathbf{p}})$ for a three-dimensional superfluid [21].

The gap equation (3.4) can be written in a convenient form [19] :

$$d_{ij} \hat{p}_j \sigma_i i\sigma_y = T_{jl}^{ik} d_{kl} \hat{p}_l \sigma_k i\sigma_y , \quad (5.5)$$

where T is the operator acting on the right side of Eq.(3.4) , the upper and lower indices refer to spin and orbital degrees of freedom respectively. In absence of a magnetic field the tensor T_{jl}^{ik} is independent of spin indices :

$$T_{jl}^{ik} = T_{jl} \delta_{ik} . \quad (5.6)$$

Then the gap equation (5.5) reduces to :

$$d_{ij} \hat{p}_j = T_{jl} d_{il} \hat{p}_l . \quad (5.7)$$

We obtain two main unitary phases which are invariants of the Eq.(5.7) -

the two-dimensional A phase (2-A) :

$$d_{ij} = \frac{1}{2} i \left[(a_{1,1} + a_{1,-1}) \delta_{jx} + i (a_{1,1} - a_{1,-1}) \delta_{jy} \right] \delta_{iz} , \quad (5.8)$$

and the two-dimensional B phase (2-B) :

$$d_{ij} = \left[\frac{1}{\sqrt{3}} a_{0,0} \delta_{jx} - \frac{1}{\sqrt{2}} a_{1,0} \delta_{jy} \right] \delta_{ix} + \left[\frac{1}{\sqrt{3}} a_{0,0} \delta_{jy} + \frac{1}{\sqrt{2}} a_{1,0} \delta_{jx} \right] \delta_{iy} . \quad (5.9)$$

It is convenient to transform these tensors and write the order parameter as :

$$d = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \Delta_1 & i\Delta_2 \end{bmatrix} , \quad (5.10)$$

$$\Delta_\nu(\mathbf{p}) = \sqrt{1.5} \sin\vartheta_\nu \left[\Delta_1 \hat{p}_x + i\Delta_2 \hat{p}_y \right] \sigma_z i\sigma_y \quad (5.11)$$

for the 2-A state and

$$d = \begin{bmatrix} \Delta_1 & -\Delta_2 \\ \Delta_2 & \Delta_1 \\ 0 & 0 \end{bmatrix} , \quad (5.12)$$

$$\Delta_\nu(\mathbf{p}) = \sqrt{1.5} \sin\vartheta_\nu \left[\left(\Delta_1 \hat{p}_x - \Delta_2 \hat{p}_y \right) \sigma_x + \left(\Delta_2 \hat{p}_x + \Delta_1 \hat{p}_y \right) \sigma_y \right] i\sigma_y . \quad (5.13)$$

for the 2-B state.

The amplitudes Δ_1, Δ_2 are determined by the gap equation (3.4) and usually are different for each phase. The above phases (5.10)-(5.13) are generalization to the case with broken rotational symmetry of

two-dimensional phases obtained by Brusov and Popov [22]. The superflow in the plane of the film breaks the symmetry of the system with respect to rotations about the z -axis. However, each phase corresponds to a different quantum number m (the z -component of the total angular momentum), namely the 2-A phase corresponds to $m=1$ whereas the 2-B one is associated with $m=0$, as in the case of an unbroken rotational symmetry. The first of these phases may be also interpreted as the bulk phase in a proximity of the substrate surface with the angular momentum vector $\mathbf{l}$ orthogonal to the surface plane [20,23,24]. That is the superfluid state in films with thickness $d \cong \xi(T)$ is approximated at low temperature by the two-dimensional A phase (5.10)-(5.11). Also the bulk B phase, due to the orientational wall effects, can be thought of as the two-dimensional B phase (5.12)-(5.13) in a proximity of the surface [25].

It was proved [22] that the 2-A and the 2-B phases are degenerate and stable when there is no flow in the system. The problem of a superfluid current in the Ginzburg-Landau regime for thicker films ($d > \xi(T)$), where the bulk phases occur, was studied by Fetter and Ullah [8].

Next we present results of the microscopical theory for two-dimensional superfluids.

6. THE TWO-DIMENSIONAL A PHASE

The gap equation (3.4) for the 2-A phase is :

$$\Delta_1 = 3gN_0 \frac{\pi}{p_0 d} \Delta_1 \sum_{\nu=1}^{\nu_c} \sin^2 \vartheta_{\nu} \int_0^{\omega_c} d\varepsilon \int_0^{\pi/2} \frac{d\phi}{\pi} \frac{\cos^2 \phi}{E_{\nu}} \left[\operatorname{tgh} \frac{E_{\nu}^+}{2T} + \operatorname{tgh} \frac{E_{\nu}^-}{2T} \right], \quad (6.1)$$

$$\Delta_2 = 3gN_0 \frac{\pi}{p_0 d} \Delta_2 \sum_{\nu=1}^{\nu_c} \sin^2 \vartheta_{\nu} \int_0^{\omega_c} d\varepsilon \int_0^{\pi/2} \frac{d\phi}{\pi} \frac{\sin^2 \phi}{E_{\nu}} \left[\operatorname{tgh} \frac{E_{\nu}^+}{2T} + \operatorname{tgh} \frac{E_{\nu}^-}{2T} \right], \quad (6.2)$$

$$\text{where } E_{\nu} = \left[\varepsilon^2 + \frac{3}{2} \left(\Delta_1^2 \cos^2 \phi + \Delta_2^2 \sin^2 \phi \right) \sin^2 \vartheta_{\nu} \right]^{1/2}, \quad (6.3)$$

$$E_{\nu}^{\pm} = E_{\nu} \pm v p_F \sin \vartheta_{\nu} \cos \phi. \quad (6.4)$$

Solution to Eqs.(6.1) and (6.2) was found numerically and is presented for some temperatures in Fig.4a-c. The superfluid gap function for finite flow has two nondegenerate components : the component Δ_2 perpendicular to the flow is enhanced , the component Δ_1 parallel to the flow is suppressed. There is a phase transition from the 2-A phase ($\Delta_1 \neq 0$ and $\Delta_2 \neq 0$) to the two-dimensional polar phase ($\Delta_1 = 0$), which is only a special case of the 2-A state (5.10)-(5.11). For sufficiently low temperatures this transition is of the first order and can be determined by calculating the

free energy $F_S^A(\Delta_1, \Delta_2)$. Let us remark that the gap function plays the role of the order parameter in the system, and the gap equations (6.1)-(6.2) are understood as stationary type conditions of the free energy $F_S^A(\Delta_1, \Delta_2)$. That is $\frac{\partial}{\partial \Delta_1} F_S^A(\Delta_1, \Delta_2) = 0$ and $\frac{\partial}{\partial \Delta_2} F_S^A(\Delta_1, \Delta_2) = 0$ coincide with the Eqs. (6.1) and (6.2) respectively. Hence, the free energy $F_S^A(\Delta_1, \Delta_2)$ of the system can be obtained [17] by integrating the gap equations. If we write the Eq. (6.1) as $f_1(\Delta_1, \Delta_2) = 0$ and the Eq. (6.2) as $f_2(\Delta_1, \Delta_2) = 0$ then the free energy $F_S^A(\Delta_1, \Delta_2)$ is :

$$F_S^A(\Delta_1, \Delta_2) - F_N = \int_0^{\Delta_1} f_1(\Delta'_1, \Delta_2) d\Delta'_1 + \int_0^{\Delta_2} f_2(0, \Delta'_2) d\Delta'_2, \quad (6.5)$$

where F_N is the free energy of the normal phase. From Eq. (6.5) we obtain :

$$\begin{aligned} F_S^A(\Delta_1, \Delta_2) - F_N = & 3gN_0 \frac{\pi}{p_0} \sum_{\nu=1}^{\nu_c} \int_0^{\omega_c} d\epsilon \int_0^{\pi/2} \frac{d\phi}{\pi} \left[\left(\Delta_1^2 \cos^2 \phi + \Delta_2^2 \sin^2 \phi \right) \right. \\ & \times \frac{1}{E_\nu} \sin^2 \vartheta_\nu \left[\operatorname{tgh} \frac{E_\nu^+}{2T} + \operatorname{tgh} \frac{E_\nu^-}{2T} \right] - \\ & \left. \frac{4}{3} T \left[\ln \left(\cosh \frac{E_\nu^+}{2T} \right) + \ln \left(\cosh \frac{E_\nu^-}{2T} \right) - \right. \right. \\ & \left. \left. \ln \left(\cosh \frac{(\epsilon + \nu p_F \sin \vartheta_\nu \cos \phi)}{2T} \right) - \ln \left(\cosh \frac{(\epsilon - \nu p_F \sin \vartheta_\nu \cos \phi)}{2T} \right) \right] \right]. \end{aligned} \quad (6.6)$$

For the two-dimensional polar phase (2-P) we have :

$$F_S^P(\Delta_2) = F_S^A(\Delta_1=0, \Delta_2) . \quad (6.7)$$

Phase diagrams calculated from Eqs.(6.6) and (6.7) are shown in Fig.5a-c. The transition from the two-dimensional A phase to the normal state is always through the two-dimensional polar phase.

Finally we determine the superflow j_s [17,18]:

$$j_s = \frac{1}{\omega} T \sum_{\mathbf{p}, \nu, n} \frac{1}{m} (\mathbf{p} + m\mathbf{v}) \mathcal{G}_\nu(i\omega_n, \mathbf{p}) , \quad (6.8)$$

where ω is the volume of the sample.

After straightforward algebraic evaluations we find :

$$j_s = \frac{1}{(3\pi^2)^{1/3}} n^{2/3} \left[v p_0 - \left(p_0 d \sum_{\nu=1}^{\nu_c} \sin^2 \vartheta_\nu / 6\pi \right)^{-1/2} \times \right. \\ \left. \sum_{\nu=1}^{\nu_c} \sin^2 \vartheta_\nu \int_0^{\omega_c} d\epsilon \int_0^{\pi/2} \frac{d\phi}{\pi} \cos \phi \left[\operatorname{tgh} \frac{E_\nu^+}{2T} - \operatorname{tgh} \frac{E_\nu^-}{2T} \right] \right] . \quad (6.9)$$

This equation allows to draw the superflow j_s as a function of superfluid velocity $\mathbf{v}$ (see Fig.6a-c) . Comparing superflows with phase diagrams (Fig.5) we conclude that the maximal superflow j_{sc} is attained in the 2-A phase . Now it is easy to find the relation between the critical superflow j_{sc} and the temperature.

This function is plotted for different values of potential $V(z)$ in Fig.7a and for a constant potential and varying film thickness in Fig.7b. As could be expected the superflow j_{sc} changes substantially with variations of the substrate field.

Before the eventual comparison of the theoretical results with experimental data, we will discuss the properties of the 2-B phase.

7. THE TWO-DIMENSIONAL B PHASE

The gap equation (3.4) for the 2-B phase is :

$$\begin{pmatrix} \Delta_1 \\ \Delta_2 \end{pmatrix} = 3gN_0 \frac{\pi}{p_0 d} \begin{pmatrix} \Delta_1 \\ \Delta_2 \end{pmatrix} \times \sum_{\nu=1}^{\nu_c} \sin^2 \vartheta_{\nu} \int_0^{\omega_c} d\epsilon \int_0^{\pi/2} \frac{d\phi}{\pi} \frac{\cos^2 \phi}{E_{\nu}} \left[\operatorname{tgh} \frac{E_{\nu}^+}{2T} + \operatorname{tgh} \frac{E_{\nu}^-}{2T} \right], \quad (7.1)$$

$$\begin{pmatrix} \Delta_1 \\ \Delta_2 \end{pmatrix} = 3gN_0 \frac{\pi}{p_0 d} \begin{pmatrix} \Delta_1 \\ \Delta_2 \end{pmatrix} \times \sum_{\nu=1}^{\nu_c} \sin^2 \vartheta_{\nu} \int_0^{\omega_c} d\epsilon \int_0^{\pi/2} \frac{d\phi}{\pi} \frac{\sin^2 \phi}{E_{\nu}} \left[\operatorname{tgh} \frac{E_{\nu}^+}{2T} + \operatorname{tgh} \frac{E_{\nu}^-}{2T} \right], \quad (7.2)$$

$$\text{where } E_{\nu} = \left[\epsilon^2 + \frac{3}{2} (\Delta_1^2 + \Delta_2^2) \sin^2 \vartheta_{\nu} \right]^{1/2}, \quad (7.3)$$

and E_{ν}^+ is defined by Eq. (6.4).

Because this set of equations is invariant to the transformation $\Delta_1 \leftrightarrow \Delta_2$, the only quantity we can determine is $\Delta^2 \equiv \Delta_1^2 + \Delta_2^2$ - it is presented for five different temperatures in Fig. 8; Δ has the interpretation of the magnitude of the order parameter for the 2-B phase.

The methods described in section 5 lead to the formula for free energy :

$$\begin{aligned} F_S^B(\Delta) - F_N = & 3gN_{op} \frac{\pi}{d} \sum_{\nu=1}^{\nu_c} \int_0^{\omega_c} d\epsilon \int_0^{\pi/2} \frac{d\phi}{\pi} \left[\frac{\Delta^2 \sin^2 \vartheta_{\nu}}{2E_{\nu}} \times \right. \\ & \left. \left[\operatorname{tgh} \frac{E_{\nu}^+}{2T} + \operatorname{tgh} \frac{E_{\nu}^-}{2T} \right] - \right. \\ & \left. \frac{4}{3} T \left[\ln \left(\cosh \frac{E_{\nu}^+}{2T} \right) + \ln \left(\cosh \frac{E_{\nu}^-}{2T} \right) - \right. \right. \\ & \left. \left. \ln \left(\cosh \frac{(\epsilon + \nu p_F \sin \vartheta_{\nu} \cos \phi)}{2T} \right) - \ln \left(\cosh \frac{(\epsilon - \nu p_F \sin \vartheta_{\nu} \cos \phi)}{2T} \right) \right] \right]. \end{aligned} \quad (7.4)$$

The superflow j_s (6.8) is given by the Eq. (6.9) with E_{ν} from Eq. (7.3). From Eqs. (6.9), (7.3) and (7.4) we see that the superflow j_s and the free energy $F_S^B(\Delta)$ depend on the gap magnitude Δ only. Comparing Eq. (6.6) with Eq. (7.4) one gets $F_S^A(v=0) = F_S^B(v=0)$ since

at $v=0$ in both phases $\Delta_1 = \Delta_2$ for symmetry reasons. The degeneracy of these phases is consistent with results of Brusov and Popov [22]. In the presence of a superflow the numerical calculations show the stability of the 2-A phase against the 2-B phase. We present this result for the temperature $T=0.7T_c^F$ in Fig.9 . To complete this analysis the superflow j_s for the two-dimensional B phase is drawn in Fig.10 .

8. COMPARISON TO EXPERIMENT

In this chapter we compare our results with the experimental data [1-7]. We calculate the critical superflows for films with thicknesses and transition temperatures measured by Daunt et al. [5]. We fit the potential $V(z)$ (2.3) in order to reproduce the measured transition temperature T_c^F for a given film. This procedure is single-valued. Next we find the critical superflow j_{sc} as a function of temperature. The obtained results are drawn in Fig.11. The values of j_{sc} in our model are almost an order of magnitude higher than the measured ones [5]. Nevertheless we have got a characteristic temperature dependence of the superflow :

$$j_{sc} \propto \left[1 - \frac{T}{T_c^F} \right]^{3/2}, \quad (8.1)$$

which is consistent with the experimental results [3,5].

This relation holds for temperatures from $T \approx 0.2T_C^F$ to $T = T_C^F$, that is almost in the entire range of temperature (see Fig.11).

Next we calculate the critical superflows for films investigated by Xu and Crooker [2]. In Fig.12 we draw the superfluid densities as a function of reduced temperature T/T_C^F . They are described by the universal curve (see Fig.12). The superfluid density ρ_{SC} is an order of magnitude higher than those measured by Xu and Crooker [2]. The conclusion is that there is a different mechanism responsible for a strong suppression of the superflow in a thin film. However, the influence of the isolating ^4He layers on T_C^F [1,4] and j_{SC} [1,4,6,7] can be partially explained by a change of a substrate field (see Figs.3 and 7a). Since the presented microscopical model is quite general, other kinds of potential $V(z)$ can not change the functions drawn in Figs.11 and 12 significantly. A potential influences the results through the energy spectrum ε_ν (2.4). To reproduce a transition temperature T_C^F appropriate to the experiment, each kind of potential has to give nearly the same set of energies ε_ν (4.1) and can not change the results. For this reason we do not discuss the applicability of used potentials, even more realistic ones must give almost the same results.

9. CONCLUSION

We have considered three superfluid two-dimensional states of ^3He in a presence of a superflow. The two-dimensional A state is stabilized against the two-dimensional B state by the superflow (Fig.9). There is a phase transition from the 2-A state to the 2-P state when the superfluid velocity increases (Fig.5). The maximal superflow j_{sc} is achieved in the 2-A state. For a wide range of temperature j_{sc} is described by the power law (8.1), which is consistent with the theoretical results of Jacobsen and Smith [9] and Fetter and Ullah [8] in the Ginzburg-Landau limit. Our theoretical results are about an order of magnitude higher than the experimental ones. We have tried to explain the lowered superfluidity by the interaction of ^3He quasiparticles with the van der Waals-like field of the adsorbent. Whereas the effect of coating a substrate with ^4He [1,4,6,7] can be partially explained by this model, the measured superflow values can not be obtained. In experiments [1-7] the onset of dissipation occurs at relatively low superflows. This effect may be due to a very complex structure of a substrate, for instance the large scale irregularities may cause the thinning effect of a film and lower the current. There are always uncertainties surrounding geometric characterization in these experiments. Some measurements [1,4,5] are performed in the u-tube geometry of the

film-flow apparatus. Xu and Crooker [2] estimate that about the equivalent of 100Å thick film is contained in the scratches. This is also supported by the suggestion in [4], that the main influence of the substrate, rough or smooth, seems to be its influence on the film thickness. An additional mechanism limiting the critical current may be due to a vortex creation and motion [4,7]. This effect is expected to be strongly dependent on the substrate structure [26].

We have used several approximations in our method. The superfluid velocity $\mathbf{v}$ and the order parameter are assumed uniform in space [18]. Moreover, the gap function is two-dimensional so the model is applicable rather to films with thickness $d \leq \xi_0$. We do not include the Fermi liquid forces, because we do not expect them to play a significant role compared to the influence of a substrate. As far as we know, there is no well-defined Fermi liquid interaction in a proximity of a solid surface. It should be added that the density of an adsorbed ^3He film is a function of a distance from the substrate and in general is not constant, as it was assumed in our paper. These corrections to the model certainly will change the results. However, we do not believe them to influence the calculated quantities substantially.

The theory presented here can be applied to thin films with more realistic and more complex potential of

the substrate. This can be done by a suitable choice of the energy spectrum ε_p (2.4).

In the following paper we analyze the effect of surface roughness on superflow and stability of the superfluid states studied in this paper.

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FIGURE CAPTIONS

Fig.1 The single particle energy spectrum for $d/a = 2$

- and 1. $2mVa^2 = 90000\pi^2$, 2. $2mVa^2 = 10000\pi^2$,
3. $2mVa^2 = 2500\pi^2$, 4. $2mVa^2 = 900\pi^2$;

the dashed line represents the energy spectrum of the infinite potential well.

Fig.2 The single particle energy spectrum for

- $2mVa^2 = 10000\pi^2$ and 1. $d/a = 2$, 2. $d/a = 1.5$,
3. $d/a = 1.1$, 4. $d/a = 50$;

the dashed line : the energy spectrum of the potential well.

Fig.3 The phase transition temperature T_c^F as a function of film thickness d for several values of substrate field :

1. $d/a = 50$, $2mVa^2 = 2500\pi^2$
2. $d/a = 40$, $2mVa^2 = 10000\pi^2$
3. $d/a = 30$, $2mVa^2 = 90000\pi^2$
4. $d/a = 50$, $2mVa^2 = 100\pi^2$
5. $d/a = 20$, $2mVa^2 = 100\pi^2$
6. $d/a = 30$, $2mVa^2 = 100\pi^2$.

The results of Xu and Crooker are represented by the black squares and those of Harrison et al. by the open squares (pure ^3He) and the crosses (one ^4He layer).

Fig.4 The order parameter as a function of superfluid velocity v for the 2-A phase (the solid line) and the 2-P phase (the dashed line) at
a) $T = 0.1T_C^F$, b) $T = 0.4T_C^F$, c) $T = 0.9T_C^F$,
for the film thickness $p_0 d = 100$ and potential constants $d/a = 100$, $2mVa^2 = \pi^2$.
 $\Delta_0 = \Delta(v = 0, T = 0)$.

Fig.5 The energy of the 2-A phase (the solid line) and the 2-P phase (the dashed line) vs. the superfluid velocity v at
a) $T = 0.1T_C^F$, b) $T = 0.4T_C^F$, c) $T = 0.9T_C^F$,
for the film thickness $p_0 d = 100$ and potential constants $d/a = 100$, $2mVa^2 = \pi^2$.

Fig.6 The superflow as a function of superfluid velocity v for the 2-A phase (the solid line) and the 2-P phase (the dashed line) at
a) $T = 0.1T_C^F$, b) $T = 0.4T_C^F$, c) $T = 0.9T_C^F$,
for the film thickness $p_0 d = 100$ and potential constants $d/a = 100$, $2mVa^2 = \pi^2$.

Fig.7 The critical superflow as a function of temperature :

- a) at a constant film thickness $p_0 d = 100$ and
1. $d/a = 10$, $2mVa^2 = 100\pi^2$
 2. $d/a = 5$, $2mVa^2 = 400\pi^2$
 3. $d/a = 2.9$, $2mVa^2 = 1225\pi^2$
 4. $d/a = 2$, $2mVa^2 = 2500\pi^2$,

b) at a constant potential rate $2mVa^2 = \pi^2$ and

decreasing film thickness :

1. $d/a = 100$, $p_0 d = 100$

2. $d/a = 25$, $p_0 d = 25$

3. $d/a = 10$, $p_0 d = 10$.

Δ_0^B is the bulk value of the order parameter at

$v = 0$ and $T = 0$.

Fig.8 The order parameter for the 2-B phase vs. the

superfluid velocity v at

1. $T = 0.1T_C^F$, 2. $T = 0.3T_C^F$, 3. $T = 0.5T_C^F$,

4. $T = 0.7T_C^F$, 5. $T = 0.9T_C^F$,

for the film thickness $p_0 d = 100$ and potential constants $d/a = 100$, $2mVa^2 = \pi^2$.

Fig.9 The energy of the 2-B phase (the short-dashed

line), the 2-A phase (the solid line) and the 2-P phase (the dashed line) vs. the superfluid velocity v at $T = 0.7T_C^F$, for the film thickness $p_0 d = 100$ and potential constants $d/a = 100$, $2mVa^2 = \pi^2$.

Fig.10 The superflow of the 2-B phase vs. the superfluid

velocity v at

1. $T = 0.1T_C^F$, 2. $T = 0.3T_C^F$, 3. $T = 0.5T_C^F$,

4. $T = 0.7T_C^F$, 5. $T = 0.9T_C^F$,

for the film thickness $p_0 d = 100$ and potential constants $d/a = 100$, $2mVa^2 = \pi^2$.

Fig.11 The critical superflow as a function of temperature for films with thickness d and critical temperature T_C^F :

1. $d = 1200 \text{ \AA}$, $T_C^F/T_C^B = 0.94$
2. $d = 1100 \text{ \AA}$, $T_C^F/T_C^B = 0.84$
3. $d = 1000 \text{ \AA}$, $T_C^F/T_C^B = 0.78$.

Fig.12 The superfluid density ρ_{SC} vs. the temperature T/T_C^F .

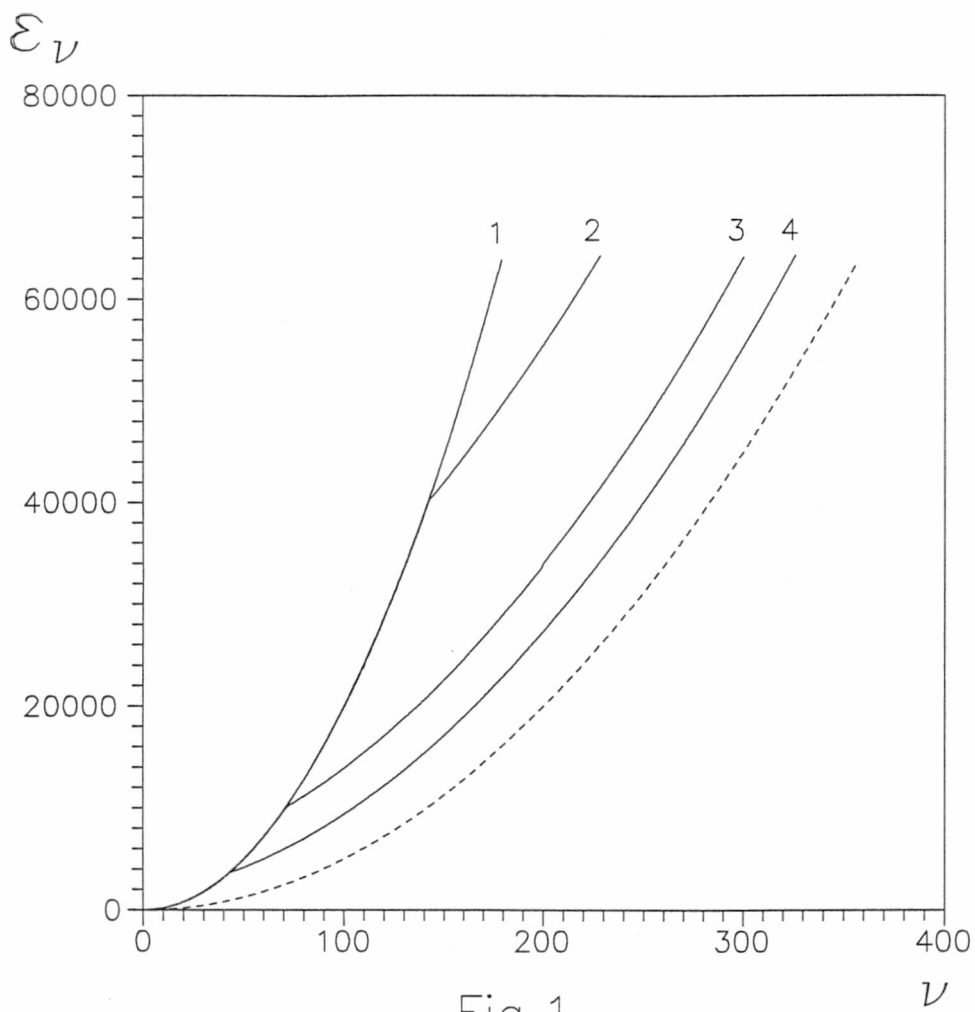


Fig.1

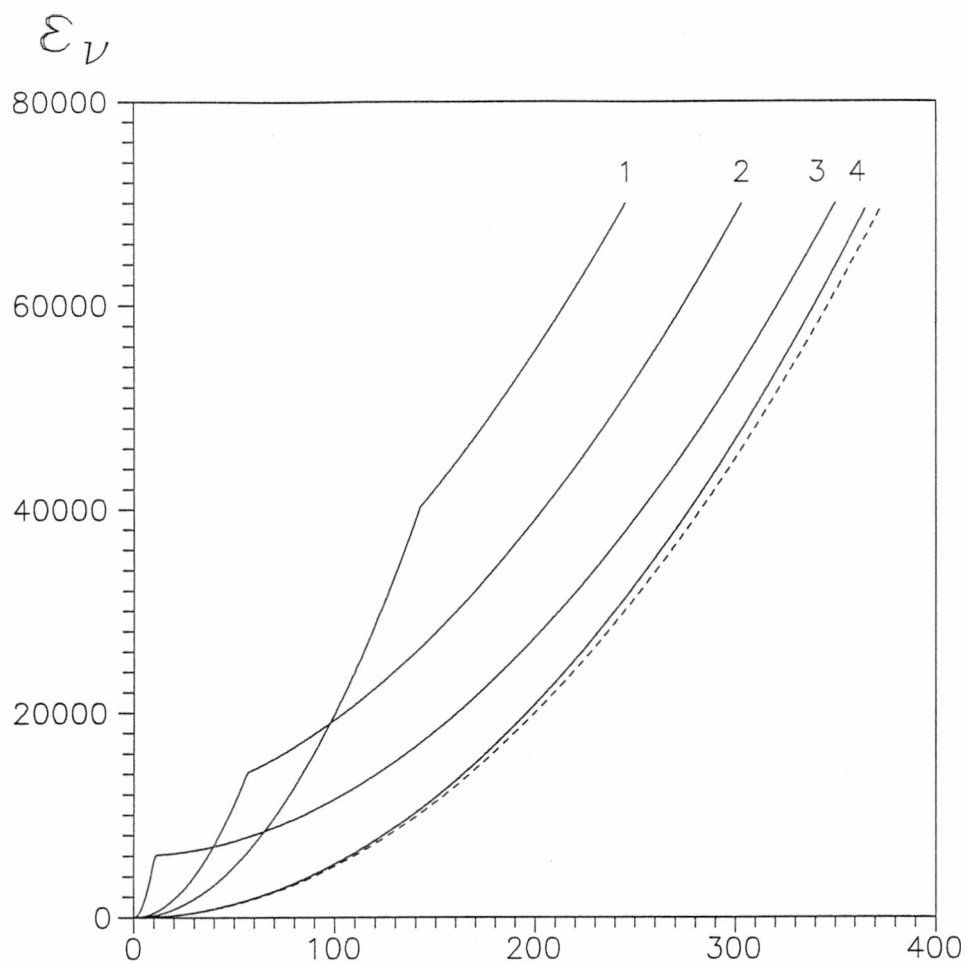
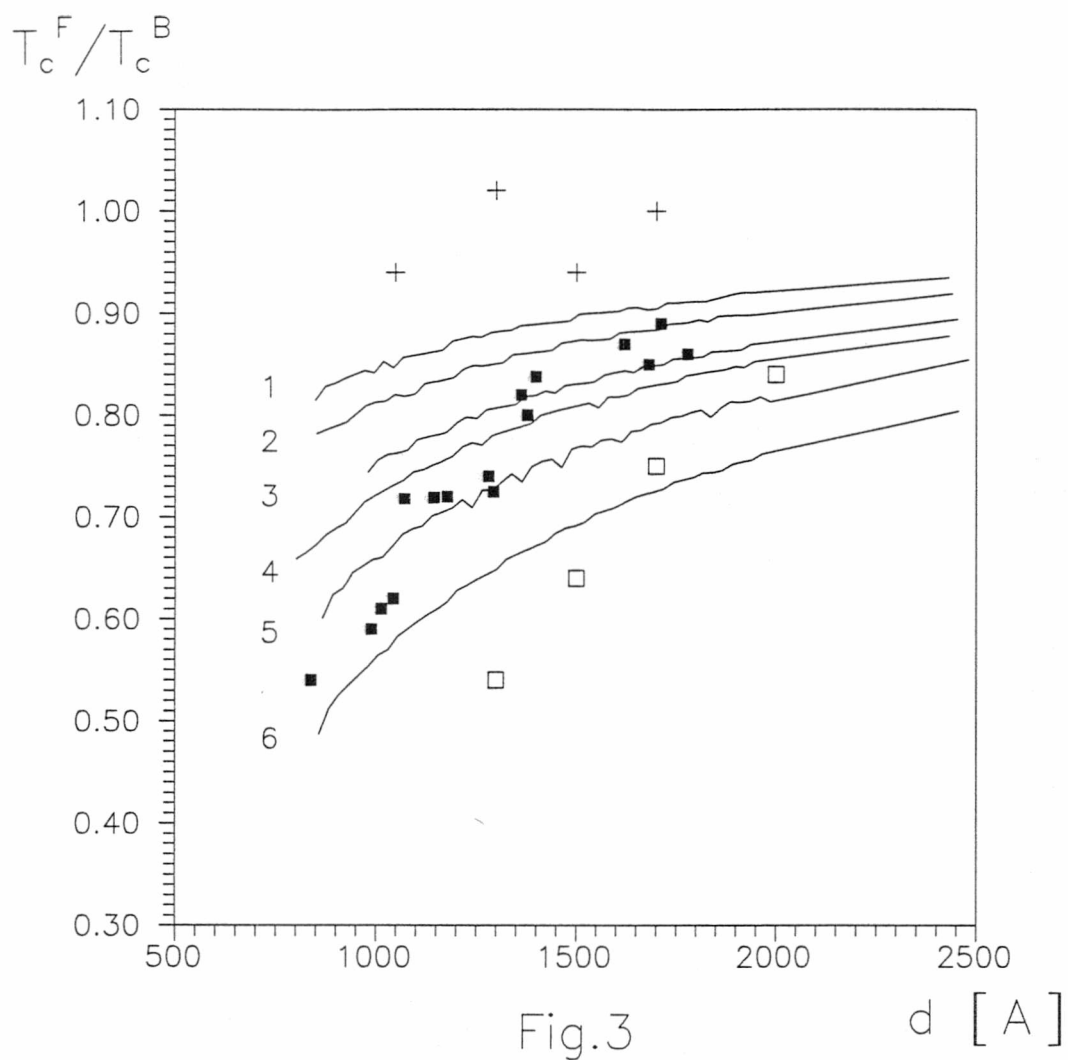


Fig.2

ν



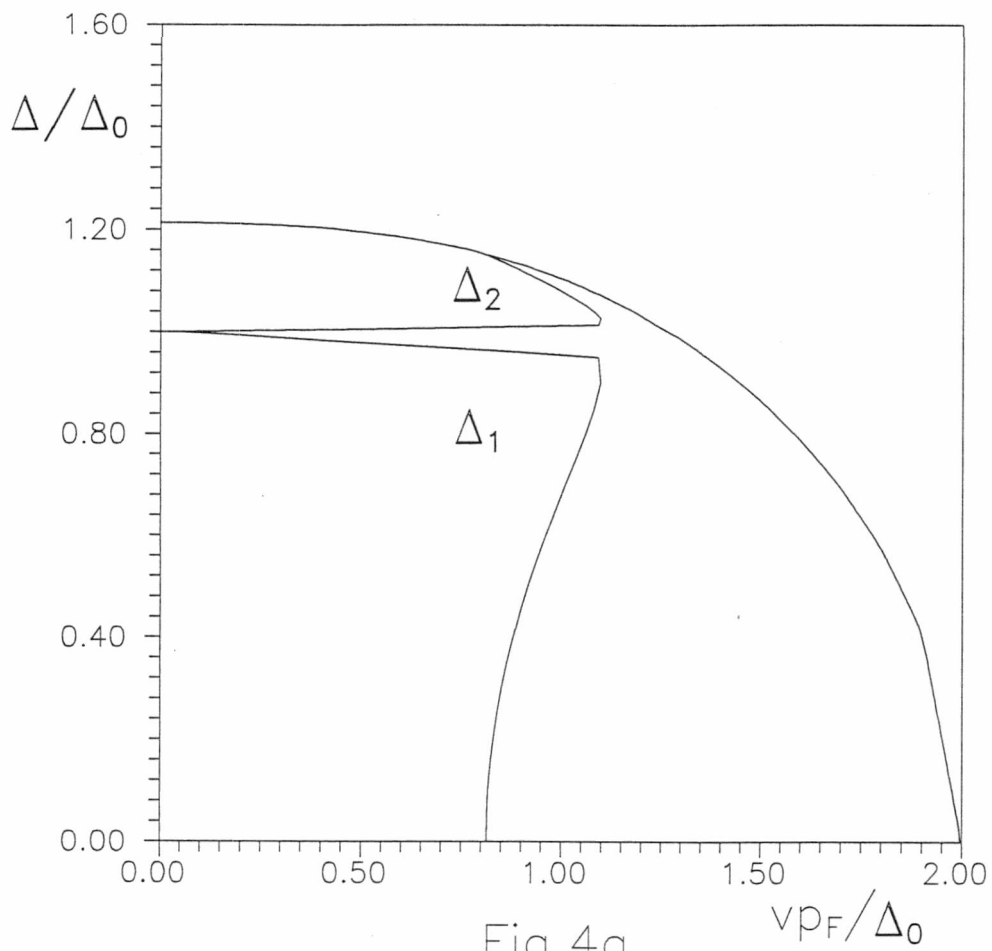
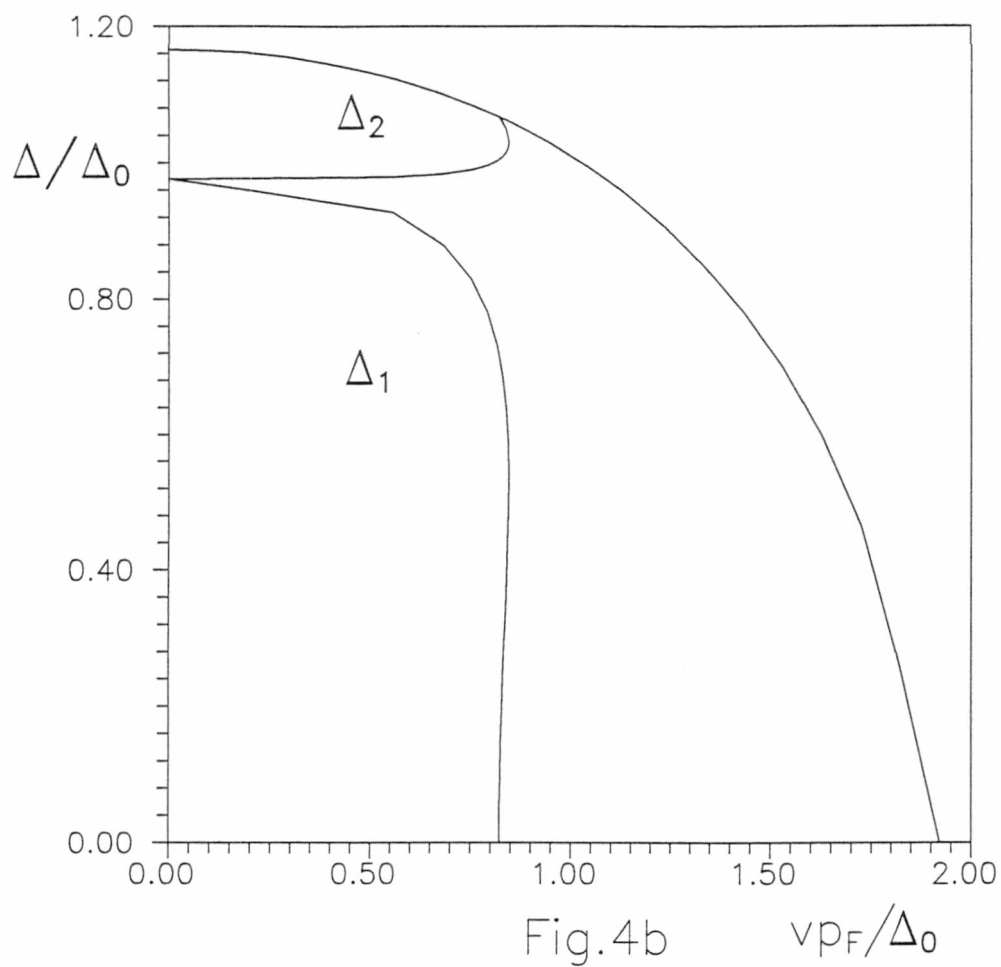
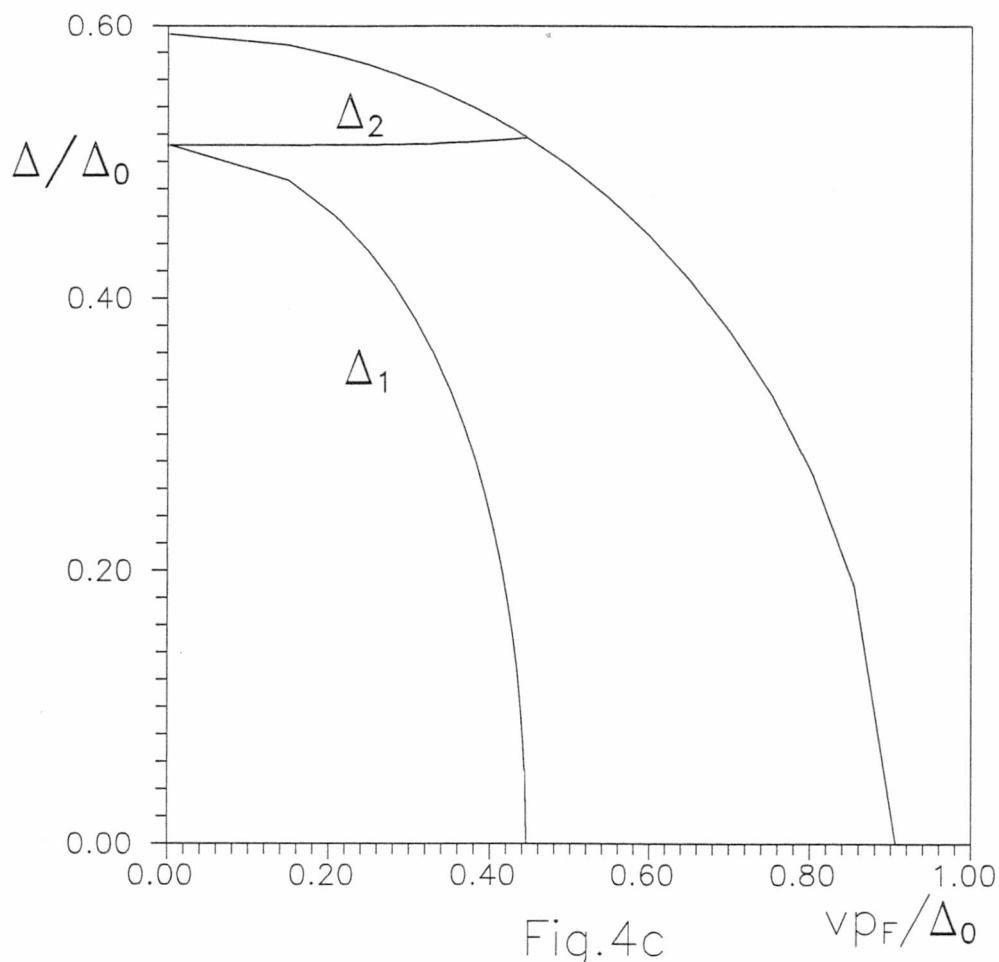
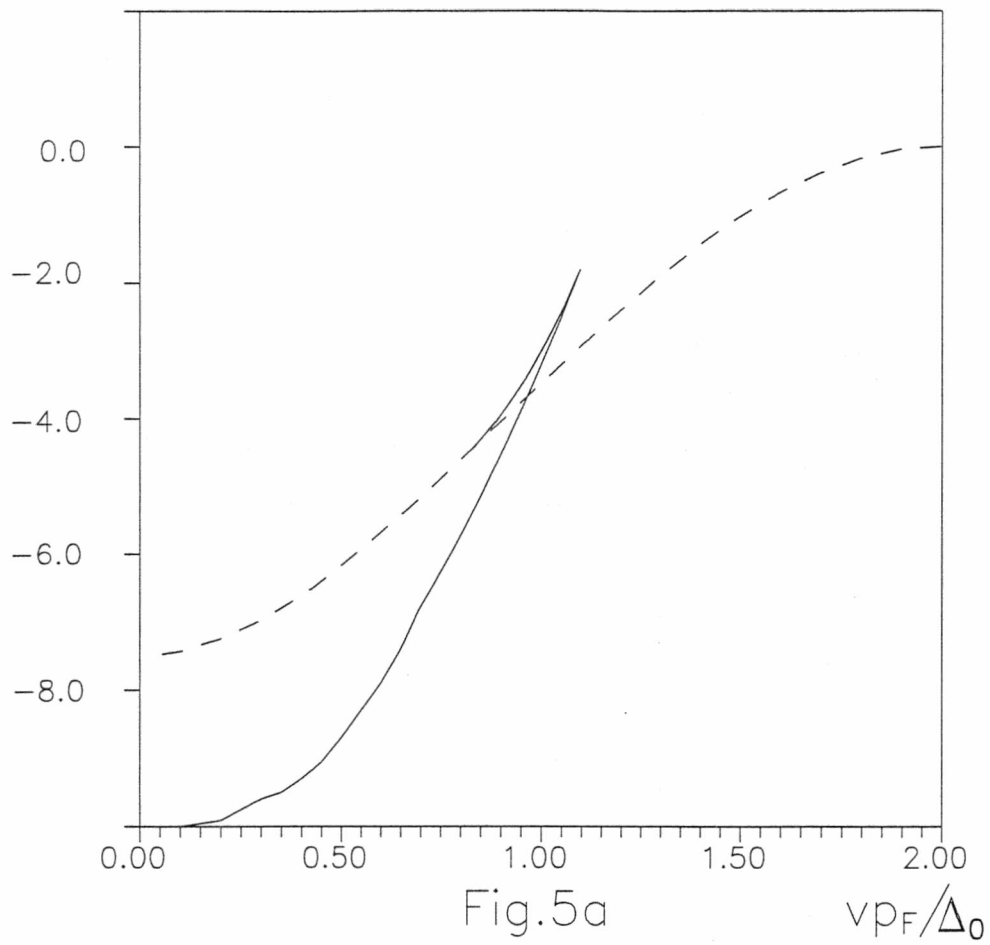


Fig.4a





$$\Delta F / 1.5 g N_0 (p_{0d})^{-1} \Delta_0^2$$



$$\Delta F / 1.5gN_0(p_0d)^{-1}\Delta_0^2$$

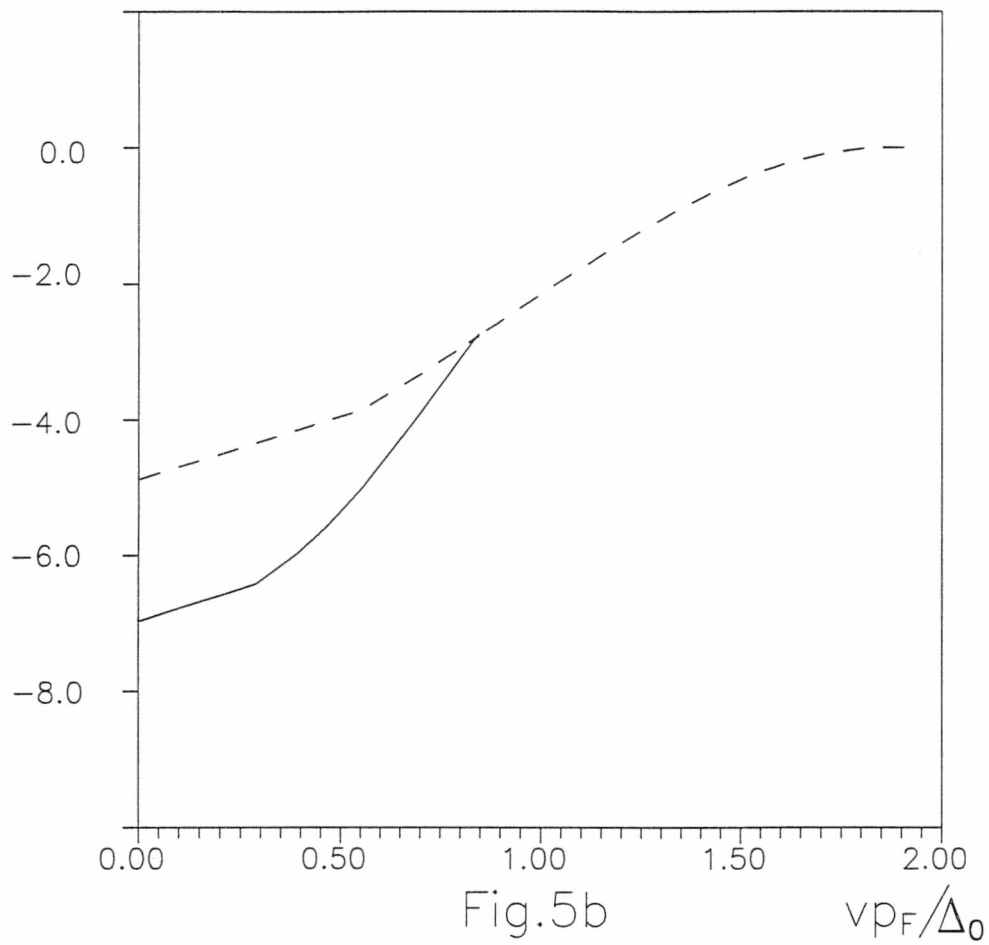
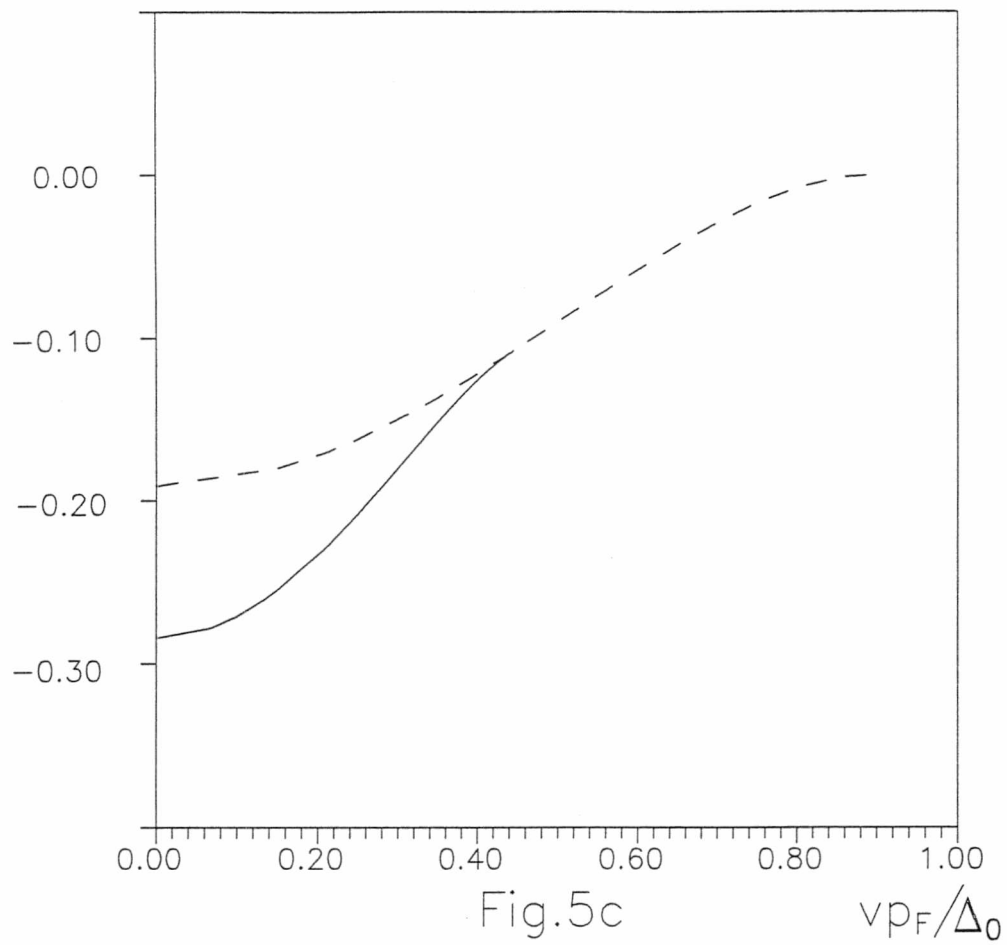
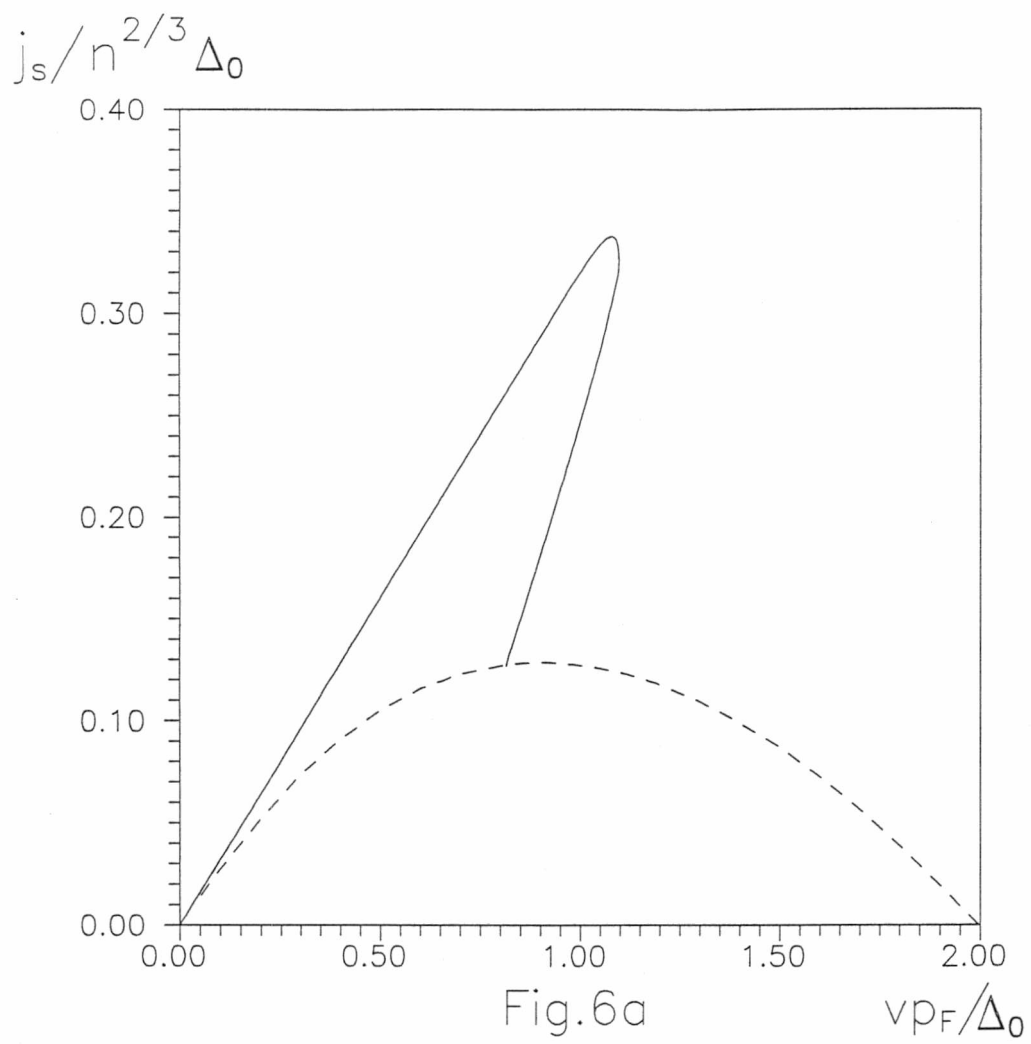


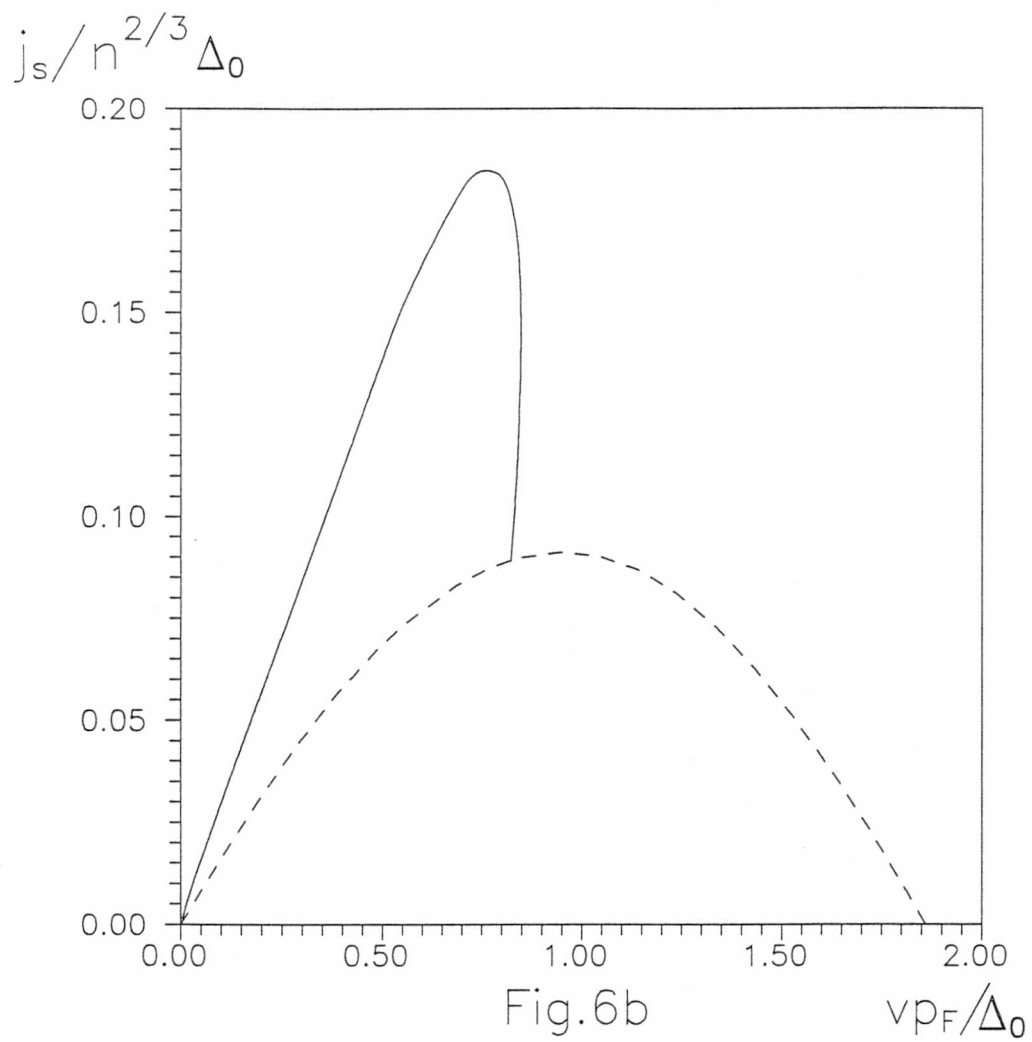
Fig.5b

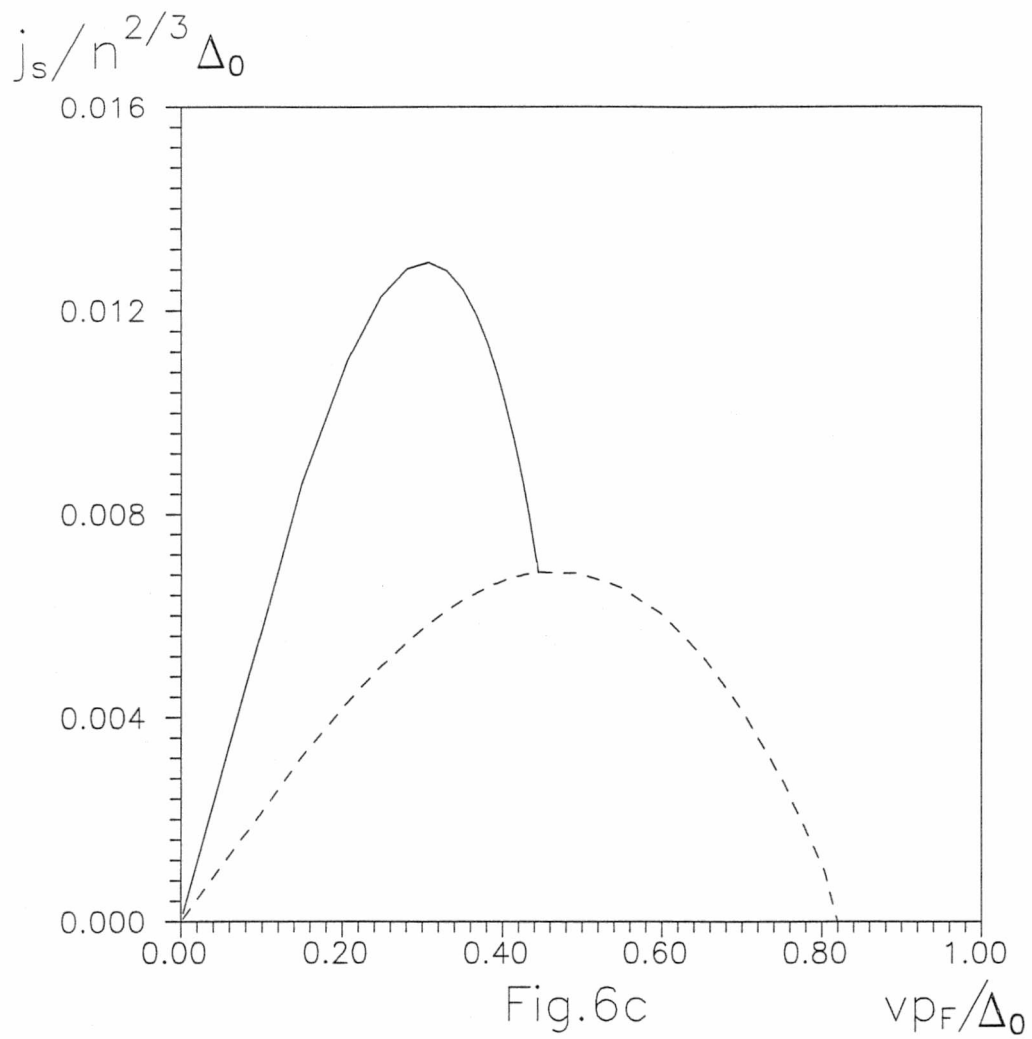
vp_F / Δ_0

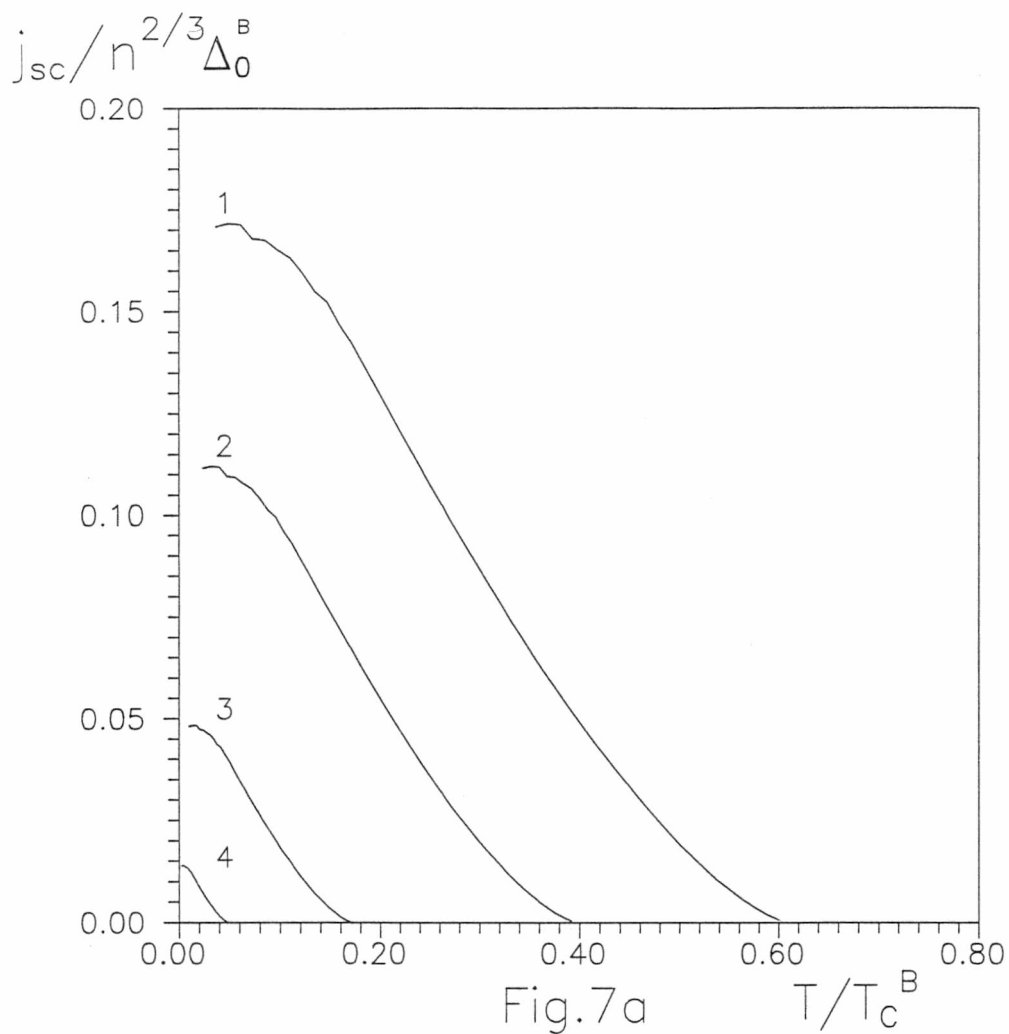
$$\Delta F / 1.5 g N_0 (p_0 d)^{-1} \Delta_0^2$$



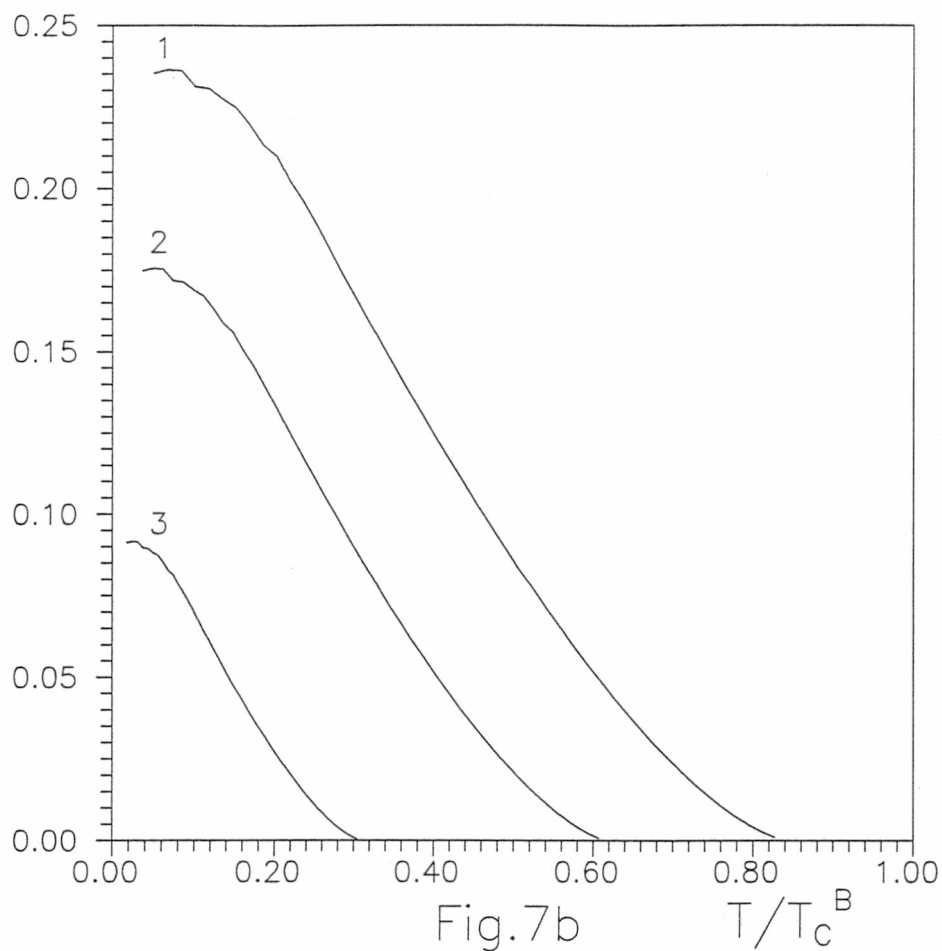


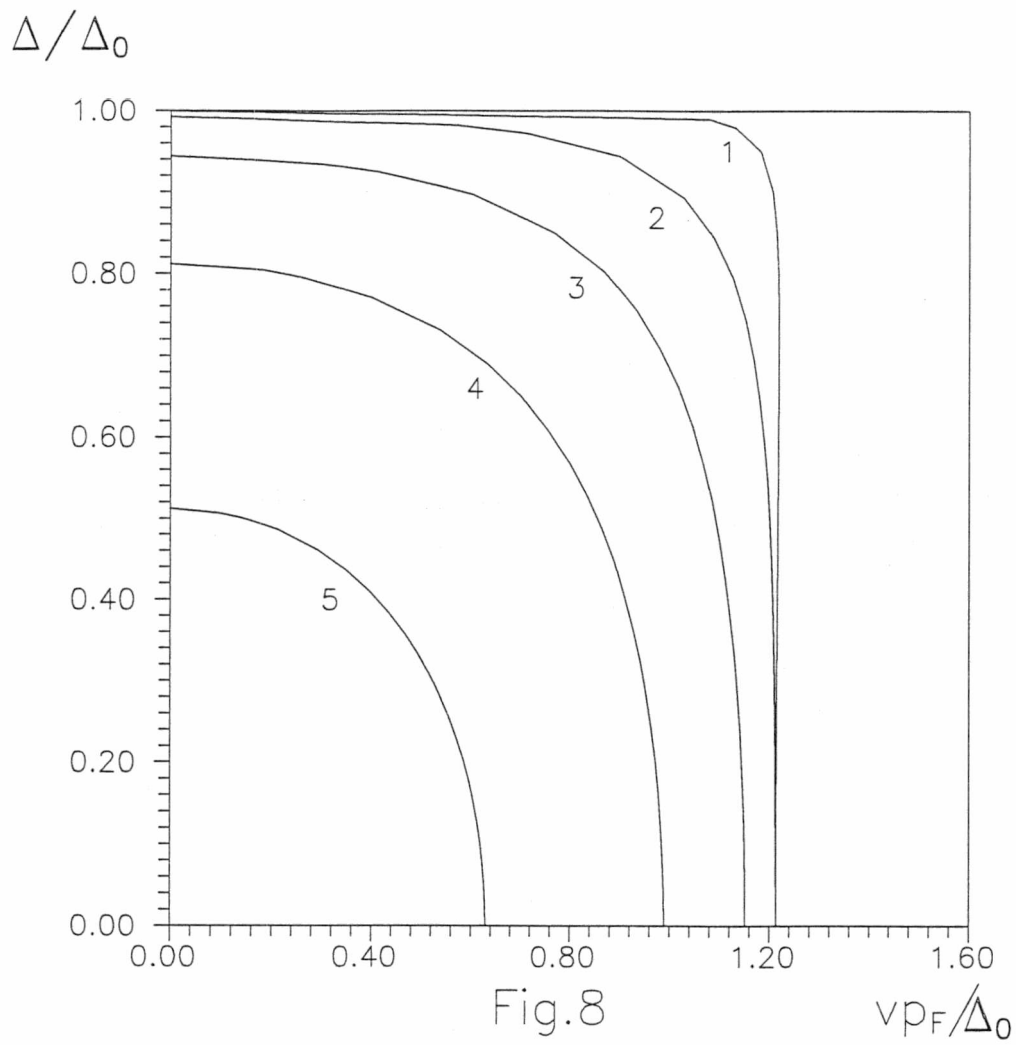






$$j_{sc}/n^{2/3}\Delta_0^B$$





$$\Delta F / 1.5gN_0(p_0d)^{-1}\Delta_0^2$$

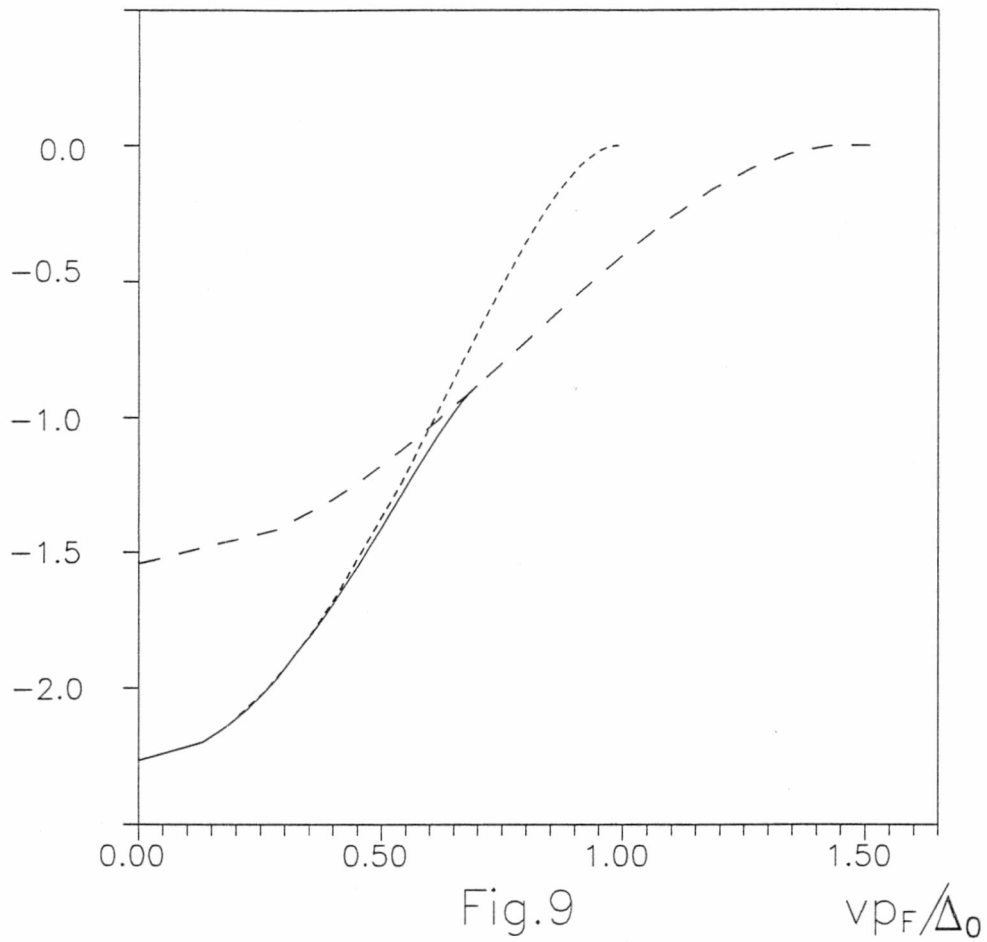
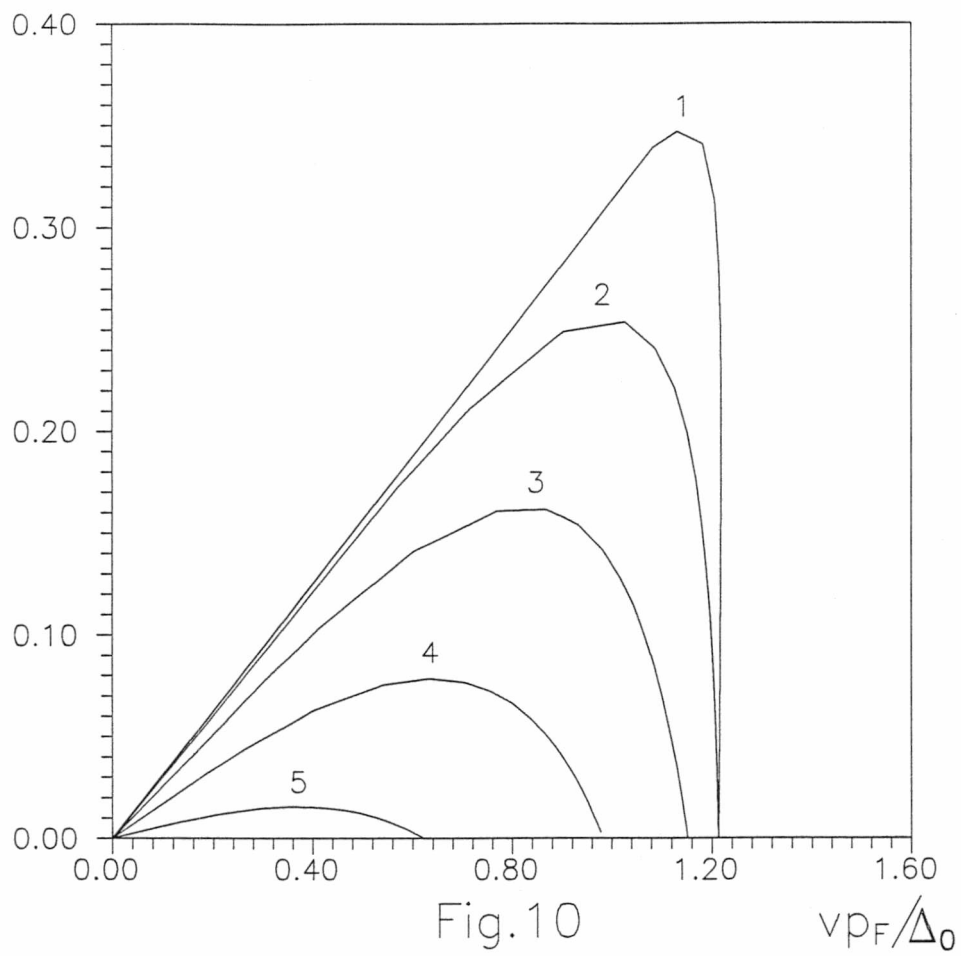


Fig.9

$$j_s / n^{2/3} \Delta_0$$



$(j_{sc}/n)^{2/3} \text{ (mm/sec)}^{2/3}$

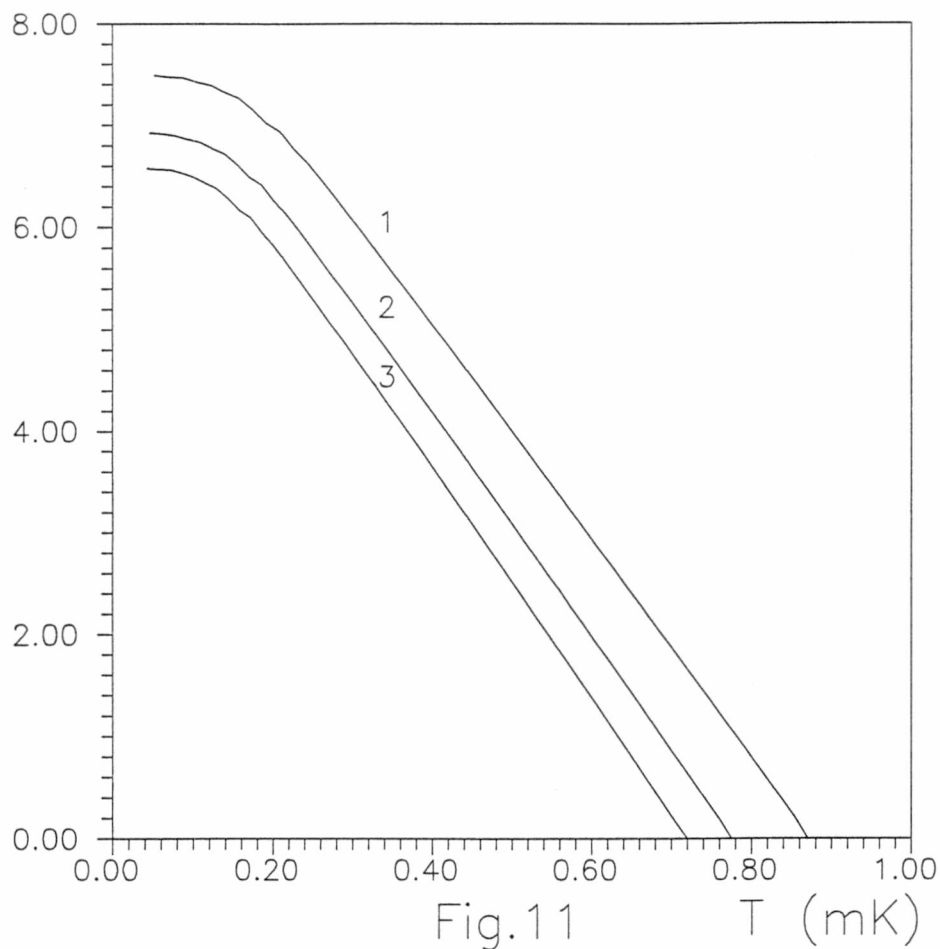


Fig.11

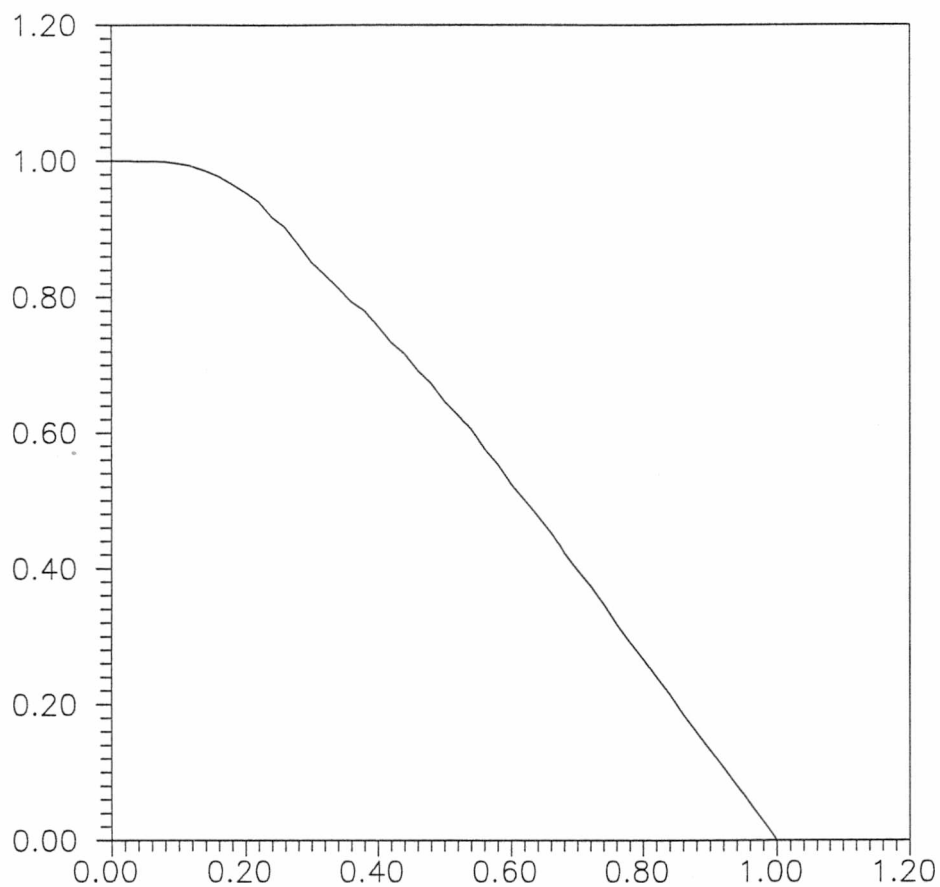
ρ_s/ρ 

Fig.12

 T/T_c^F

Flow in Thin Films of Superfluid ^3He on Rough Surfaces

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Abstract

Superflow of ^3He in thin films on rough surfaces is studied using the formalism of the microscopic theory. The gap equation is solved and the free energy and the superflow are calculated for three two-dimensional states that may exist in films with thickness $d \sim \xi_0$. In the absence of flow the two-dimensional B state and the two-dimensional A state are degenerate and energetically more favorable than other two-dimensional states. In the presence of flow the two-dimensional A state has the lowest energy. For increasing superflow, there occurs a transition from the two-dimensional A to the two-dimensional polar state. The order of this transition depends on temperature and the strength of pair-breaking on the surface. The theory is compared to recent experimental results.

PACS numbers: 67.50, 67.70

1. Introduction

Superfluid ^3He in restricted geometries is a subject of great interest. Experimental results point to nontrivial boundary effects for this system [1-6]. It is well known that ^3He , being a p-wave superfluid is very sensitive to scattering off irregularities of the interface superfluid-solid [7]. The superfluid transition temperature is significantly reduced when at least one dimension of the system is of the order of the zero-temperature coherence length ξ_0 . It turns out, however, that the coating of the interface with up to several layers of ^4He changes the character of quasiparticle scattering from diffusive to specular.[6] In experiments [3,4] addition of ^4He brought the critical temperature of the film close to T_c for the bulk liquid. Since ^4He is preferentially adsorbed on the solid surface one is led to the conclusion that even small amount of ^4He adsorbed on the surface, one or more layers, considerably reduces the pair-breaking of the surface. One possible explanation for this effect is that the adsorbed atoms of ^4He smooth out irregularities of the surface. Nevertheless, it is clear that surface roughness on larger length scales remains almost unchanged since no more than a few layers of ^4He were present in experiments of [3] and [4]. This effect remains puzzling although one might expect that spin relaxation at the boundary in pure ^3He contributes to the breaking of Cooper pairs, and consequently, replacing solidified atoms of ^3He with ^4He cuts off the spin-scattering channel.

There were efforts to study the problem near T_c using the Ginzburg-Landau formalism.[8-13] The obtained critical current [9] was found to follow the $(1 - T/T_c^F)^{3/2}$ dependence, T_c^F being the critical temperature of the film, characteristic of current limited by pair-breaking. However, the most interesting experiments are currently being performed [2] beyond the GL regime. The quasiclassical method was

also used to calculate the order parameter near surfaces [14], the critical current [15] and the density of states.[16,17] The reader may consult any of these papers to find more references to the quasiclassical studies.

Here we use an alternative scheme, based on the Abrikosov-Gorkov method. Numerical calculations for the case of a thin film present no special difficulties, if one assumes constant order parameter in the film. This should be a good approximation for two-dimensional phases. Our scheme can be used easily in any regular restricted geometry.

As described in [1], there are three length scales of the substrate roughness. We use a model of rough surface that should be appropriate for small, uncorrelated scattering centers. Since $\xi_0 \sim 60nm$, this should be a good approximation for 'bumps' up to $\sim 50 \text{ \AA}$ in diameter. The interaction of fermions with surface inhomogeneities is represented as 'white noise', changing the spectrum of single-particle excitations. This random process is called diffusive surface scattering.

In films with thickness $p_0^{-1} \ll d \sim \xi_0$, p_0 being the Fermi momentum of the bulk liquid, the likely candidates for the order parameter are the two-dimensional states considered by several authors.[18,19] In the preceding paper [20] (to be referred to as I) we have derived the gap equations, the free energy and the superflow for two-dimensional superfluid states in an arbitrary potential well. Here we apply this formalism to the case of a thin film on a rough substrate. In section 2 we present the model of this system, in section 3 we solve the gap equations and calculate the free energy for each of the states and section 4 contains results for the superflow and its comparison with available experimental data. We consider three states: two-dimensional polar, two-dimensional B and two-dimensional A, all three being two-

dimensional versions of their three-dimensional counterparts. We denote them 2-P, 2-B and 2-A, respectively. In [18] 2-A and 2-B are referred to as a and b. The 2-A state, with $\Delta_1 \neq \Delta_2$, becomes more stable than the 2-B state when the symmetry in the plane of the film is broken by the superflow. The relative stability of the states 2-P and 2-A depends on temperature T , the superfluid velocity v and the pair-breaking strength Γ . Since we expect the interaction with the substrate to be the dominant factor determining the behavior of the liquid in the thin film, we neglect the Fermi-liquid interaction between quasiparticles and treat the system as a Fermi gas. The pairing interaction is assumed [21] to be the same as in the bulk liquid. We assume $\hbar = k_B = 1$ throughout the calculation.

2. The Model

We assume an infinite potential square well in the direction perpendicular to the surface. The substrate's roughness is represented by a Gaussian random height function $u(x, y)$ [21]:

$$\langle u(x, y) \rangle = 0, \quad (1)$$

$$\langle u(x, y), u(x', y') \rangle = w^2 \delta(x - x') \delta(y - y'). \quad (2)$$

The parameter w describes the absolute magnitude of the surface irregularities. The interaction with the substrate in this model is a consequence of the boundary condition for the single-particle wave function $\Psi(x, y, d + u(x, y)) = 0$, and $\Psi(x, y, 0) = 0$. For sufficiently small $u(x, y)$ this problem is essentially that of dirty superconductors and can be solved in an analogous way. The total number of Fermi circles below the chemical potential is ν_c , the nearest integer less than ν_0 , where $\nu_0 = [2md^2\mu/\pi^2]^{1/2}$. More detailed description of the model can be found in [21] and [22]. For each of the

Fermi circles we can now write the Abrikosov-Gorkov equations (the spin indices are suppressed here)

$$G_\nu(i\omega_n, \mathbf{p}) = G_{0\nu}(i\omega_n, \mathbf{p}) + G_{0\nu}(i\omega_n, \mathbf{p})\Delta_\nu(\mathbf{p})F_\nu^\dagger(i\omega_n, \mathbf{p}), \quad (3)$$

$$F_\nu^\dagger(i\omega_n, \mathbf{p}) = -G_{0\nu}^-(i\omega_n, \mathbf{p})\Delta_\nu^\dagger(\mathbf{p})G_\nu(i\omega_n, \mathbf{p}). \quad (4)$$

The superscript '-' denotes the time inversion. The Green's function in presence of surface scattering and superflow is

$$G_{0\nu}^{-1} = i\omega_n - \xi_p - \lambda_\nu - \Sigma_\nu(i\omega_n) - \mathbf{v}\mathbf{p}_F, \quad (5)$$

where

$$\xi_p = \frac{p^2}{2m} - \mu, \quad \lambda_\nu = \frac{1}{2m} \left(\frac{\nu\pi}{d} \right)^2. \quad (6)$$

λ_ν is the single-particle spectrum of states in the potential well, p is momentum in the plane of the film and μ is the chemical potential of the system. The self-energy Σ_ν is due to the scattering at the surface and for each of the Fermi circles is given by [21],[22]

$$\Sigma_\nu(i\omega_n) = \frac{w^2}{2m^2d^2} \left(\frac{\nu\pi}{d} \right)^2 \times \frac{1}{S} \sum_{\mathbf{p}, \nu'} \left(\frac{\nu'\pi}{d} \right)^2 G_{\nu'}(i\omega_n, \mathbf{p}), \quad (7)$$

S is the surface area of the film. It is convenient to introduce a function $U(i\omega_n)$ and to write the self-energy in the following form

$$\Sigma_\nu(i\omega_n) = -i\Gamma U(i\omega_n) \cos^2 \theta_\nu, \quad (8)$$

Γ , being a function of w , is a measure of surface roughness:

$$\Gamma = \frac{\pi^4}{2md^2} \frac{w^2}{d^4} \nu_0^5. \quad (9)$$

$U(i\omega_n)$ turns out to be the solution of the following equation:

$$U(i\omega_n) = \sum_{\cos \theta_\nu} \cos^2 \theta_\nu \int_{-\pi/2}^{\pi/2} \frac{d\phi}{\pi} |z_\nu|^{-1/2} \tilde{\omega}_n \sin \frac{\phi(n, \nu)}{2} \\ + \text{sign}(\tilde{\omega}_n) v p_F \sin \theta_\nu \cos \phi \cos \frac{\phi(n, \nu)}{2}, \quad (10)$$

where

$$|z_\nu| = \left[\left(\tilde{\omega}_n^2 + \frac{3}{2} \Delta^2(\phi) \sin^2 \theta_\nu - v^2 p_F^2 \sin^2 \theta_\nu \cos^2 \phi \right)^2 \right. \\ \left. + 4v^2 p_F^2 \tilde{\omega}_n^2 \sin^2 \theta_\nu \cos^2 \phi \right]^{1/2}, \quad (11)$$

and

$$\cos \phi(n, \nu) = |z_\nu|^{-1} \left[-\tilde{\omega}_n^2 - \frac{3}{2} \Delta^2(\phi) \sin^2 \theta_\nu + v^2 p_F^2 \sin^2 \theta_\nu \cos^2 \phi \right], \quad (12)$$

$\cos \theta_\nu$ and $\sum_{\cos \theta_\nu}$ are defined as

$$\cos^2 \theta_\nu = \left(\frac{\nu}{\nu_0} \right)^2, \quad \sum_{\cos \theta_\nu} = \frac{1}{\nu_0} \sum_\nu,$$

and

$$\tilde{\omega}_n = \omega_n + \Gamma U(i\omega_n) \cos^2 \theta_\nu. \quad (13)$$

$$\Delta^2(\phi) = \Delta_1^2 \cos^2 \phi + \Delta_2^2 \sin^2 \phi. \quad (14)$$

3. The Gap Equations and the Free Energy

The order parameter can be written as

$$\Delta_\nu(\hat{\mathbf{p}}) = \sqrt{3/2} \sin \theta_\nu (\Delta_1 \hat{p}_x + i \Delta_2 \hat{p}_y) \sigma_z i \sigma_y, \quad (15)$$

for the 2-A state, and

$$\Delta_\nu(\hat{p}) = \sqrt{3/2} \sin \theta_\nu ((\Delta_1 \hat{p}_x - \Delta_2 \hat{p}_y) \sigma_x + (\Delta_2 \hat{p}_x + \Delta_1 \hat{p}_y) \sigma_y) i \sigma_y, \quad (16)$$

for the 2-B state. In the 2-P state $\Delta_1 = 0$ in equation (15). Δ_1 is chosen to be the gap in the direction of the flow. The gap equation for the 2-A state is the following:

$$\begin{Bmatrix} \Delta_1 \\ \Delta_2 \end{Bmatrix} = \begin{Bmatrix} \Delta_1 \\ \Delta_2 \end{Bmatrix} 6gN_0 \frac{\pi^2}{p_0 d} T \sum_{\nu, \omega_n \geq 0} \sin^2 \theta_\nu \int_{-\pi/2}^{\pi/2} \frac{d\phi}{\pi} \begin{Bmatrix} \cos^2 \phi \\ \sin^2 \phi \end{Bmatrix} \sin \frac{\phi(n, \nu)}{2} |z_\nu|^{-1/2}. \quad (17)$$

Above, g and N_0 are the superfluid coupling constant and the density of states at the Fermi level, and p_0 is defined in I. For $\Delta_1 = 0$ the above equations apply to the 2-P state. The form of the gap equation used in numerical calculations is given in the Appendix. Examples of solutions of the gap equations (17) and (21) at $T = 0.3T_{c0}$, T_{c0} being the critical temperature of the film with no pair-breaking, are shown in figure 1. Δ_1 and Δ_2 are double-valued functions of vp_F for lower temperatures. At $\Gamma = 0$ this double-valuedness is removed above $T \simeq 0.518T_{c0}$. The free energy can be found through integration of equation (17) over self-energies Δ_1 and Δ_2 . In a system with some form of disorder, equation (17) should also be integrated over the self-energy due to impurities. In order to do that the equation determining $U(i\omega_n)$, i.e. equation (10) must be integrated together with the gap equation. Then the free energy of the superfluid system with disorder is expressed as

$$\begin{aligned} F_S^A(\Delta_1, \Delta_2) - F_N &= \int_0^{\Delta_1} f_1(\Delta_1', \Delta_2, U(i\omega_n)) d\Delta_1' + \int_0^{\Delta_2} f_2(0, \Delta_2', U(i\omega_n)) d\Delta_2' \\ &+ \sum_n \int_{U_0}^{U(i\omega_n)} f_3(0, 0, U_n') dU_n'. \end{aligned} \quad (18)$$

U_0 denotes $U(i\omega_n)$ in the normal phase

$$U_0 \equiv U(i\omega_n)|_{\Delta_1=\Delta_2=0} = \frac{1}{\nu_0} \sum_\nu \cos^2 \theta_\nu = \frac{\nu_c(\nu_c + 1/2)(\nu_c + 1)}{3\nu_0^3}, \quad (19)$$

and the superscript A stands for 2-A. Functions f_1 , f_2 and f_3 are given in the Appendix. After some algebra we arrive at the final result for the free energy:

$$\begin{aligned}
F_S^A(\Delta_1, \Delta_2) - F_N = & \\
& 3gN_0 \frac{\pi^2}{p_0 d} T \left\{ \sum_{\nu, \omega_n \geq 0} \int_{-\pi/2}^{\pi/2} \frac{d\phi}{\pi} |z_\nu|^{-1/2} \Delta^2(\phi) \sin \frac{\phi(n, \nu)}{2} \sin^2 \theta_\nu \right. \\
& - \frac{4}{3} \sum_{\nu, \omega_n \geq 0} \int_{-\pi/2}^{\pi/2} \frac{d\phi}{\pi} (|z_\nu|^{1/2} \sin \frac{\phi(n, \nu)}{2}) \\
& \left. + \frac{4}{3} \nu_0 \Gamma \sum_{\omega \geq 0} (U(i\omega_n) U_0 + (U(i\omega_n) - U_0)^2) \right\}. \quad (20)
\end{aligned}$$

Now we can analyze the relative stability of the different states. For small vp_F the 2-A state is always more stable than the 2-P state. However, for increasing vp_F there is a transition to the 2-P state. This transition is of first or second order, depending on temperature and the strength of surface scattering Γ , see figure 2. For smooth walls, $\Gamma = 0$, the transition is always of second order above $T \simeq 0.518T_{c0}$. For larger Γ , the interval of temperatures for which the first-order transition takes place is reduced. The 2-P to normal transition is always of second order.

The gap equation for the 2-B state is similar to equation (17):

$$\Delta = 3\Delta g N_0 \frac{\pi^2}{p_0 d} T \sum_{\nu, \omega_n \geq 0} \sin^2 \theta_\nu \int_{-\pi/2}^{\pi/2} \frac{d\phi}{\pi} \sin \frac{\phi(n, \nu)}{2} |z_\nu|^{-1/2}. \quad (21)$$

Here $|z_\nu|$ and $\sin \frac{\phi(n, \nu)}{2}$ are obtained from equations (11) and (12) by changing $\Delta_1^2 \cos^2 \phi + \Delta_2^2 \sin^2 \phi \rightarrow \Delta^2$. Equation for $U(i\omega_n)$ is modified accordingly. The

free energy of the 2-B state is found in the same way as in equation (20):

$$\begin{aligned}
F_S^B(\Delta) - F_N = & 3gN_0 \frac{\pi^2}{p_0 d} T \left\{ \sum_{\nu, \omega_n \geq 0} \sin^2 \theta_\nu \int_{-\pi/2}^{\pi/2} \frac{d\phi}{\pi} \Delta^2 \sin \frac{\phi(n, \nu)}{2} |z_\nu|^{1/2} \right. \\
& - \frac{4}{3} \sum_{\nu, \omega_n \geq 0} \int_{-\pi/2}^{\pi/2} \frac{d\phi}{\pi} \left(|z_\nu|^{-1/2} \sin \frac{\phi(n, \nu)}{2} - \omega_n \right) \\
& \left. + \frac{4}{3} \nu_0 \Gamma \sum_{\omega_n \geq 0} (U(i\omega_n)U_0 + (U(i\omega_n) - U_0)^2) \right\}. \quad (22)
\end{aligned}$$

The superscript B in equation (22) refers to 2-B. For $v = 0$ the 2-A and 2-B state are degenerate, but for any $v > 0$ the 2-A state is more stable although the energy difference between these two state is always small. Figure 3 shows details of the phase diagram near the first-order transition from the 2-A to the 2-P state. In general, one cannot exclude the possibility that the relative stability of states in other theoretical calculations will change when additional features, which influence the self-energy are introduced into the model.

4. The Superflow

The superflow is given by

$$\mathbf{j}_s = \frac{1}{V} T \sum_{\omega_n, \nu} \int \frac{d^2 p}{(2\pi)^2} \frac{\mathbf{p} + m\mathbf{v}}{m} G_\nu(i\omega_n, \mathbf{p}). \quad (23)$$

Using the Green's function derived in section 2, we arrive at

$$\begin{aligned}
j_s = & \frac{2(24\pi^5)^{1/6}}{(p_0 d \sum_\nu \sin^2 \theta_\nu)^{1/2}} \rho^{2/3} T \sum_{\nu, \omega_n \geq 0} \sin \theta_\nu \int_{-\pi/2}^{\pi/2} \frac{d\phi}{\pi} \cos \phi \\
& \times |z_\nu|^{-1/2} \left(v p_F \sin \theta_\nu \cos \phi \sin \frac{\phi(n, \nu)}{2} - \tilde{\omega}_n \cos \frac{\phi(n, \nu)}{2} \right). \quad (24)
\end{aligned}$$

This equation holds for all three states. To apply it to the 2-B state, equation (24) should be used with modifications of equation (10) for $U(i\omega_n)$ in the same way as

described following equation (20). To find the maximum current with increasing vp_F one has to solve the gap equations and calculate the free energy. The maximum flow is always attained in the 2-A state. However, one must know the value of vp_F at the transition of the 2-A into 2-P, since for a certain range of T and Γ the transition occurs *before* the current in the 2-A state would reach its absolute maximum. To illustrate this point, we plot in figure 4 the current for the same parameter set as in figures 1 and 2. Also, on comparing figures 2 and 4 we see that although the largest flow might be achieved in the 2-B state, this state is not stable. Figure 5 shows the depression of the critical current density j_{sc} with increasing surface roughness. The dependence of j_{sc} on temperature is presented in figure 6. With exception of very small temperatures, it follows the $(1 - T/T_c^F)^{3/2}$ behavior as expected when the flow is limited by pair-breaking. Comparing the magnitude of j_{sc} from our calculations to experimental results in figure 10 of [1] we find a significant difference. Although the phases seen in experiments may not be the same as the ones studied here, we think it is fair to compare at least orders of magnitude of critical currents for a given critical temperature in order to test the mechanism limiting critical currents in thin films. The experimental current density is several times smaller than the one obtained from equation (24) for a given critical temperature of the film. This discrepancy also occurs [1] for j_{sc} in the GL theory of [9]. Another experimentally relevant quantity is the superfluid density ρ_s . Comparing our result, figure 7, and figure 2 in [2] we again see a significant difference. Finally, in figure 8 we show the dependence of the superfluid density on Γ .

5. Conclusion

We have studied the properties of two-dimensional states of superfluid ^3He in films on rough surfaces including effects of superflow.

We found that the 2-A and the 2-P states are always more stable than the 2-B state for nonzero superflow. Apart from reducing the critical temperature of the film and the superflow, both facts already well known, surface roughness has an important effect on the order of the phase transition between the 2-A and the 2-P state. Increasing strength of pair-breaking at the surface moves the critical point between the first- and second-order transition line to lower temperatures. However, the critical point is not removed from the phase diagram. With increasing Γ it approaches $T = 0$, but does not disappear from the phase diagram until superfluidity is destroyed entirely.

It becomes apparent that the pair-breaking alone does not suffice to explain the flow experiments in superfluid ^3He films. The next logical step towards a satisfying theory of this system is inclusion of more realistic boundary conditions for the super-normal interface in restricted geometry, perhaps along the lines suggested recently by two groups of authors.[23,24] Creation of vortices may also substantially reduce critical currents.[25] We have not considered large surface irregularities which may have significant effect on j_{sc} . The liquid over large bumps on the surface may be thinner and these areas may act as 'bottlenecks' for the critical current. A related question is the proper description of scattering processes at the interface superfluid-solid. The addition of small amount of ^4He , the equivalent of only about 2 layers, to superfluid ^3He films [3,4] was enough to increase T_c to almost its bulk value. Since the roughness of the liquid-solid boundary was not removed, except perhaps for bumps on the atomic scale, it may seem that the effect of the surface in the absence of superflow

is mostly to break pairs through spin relaxation at the walls. So far the spin-flip scattering at interfaces of ^3He is not well characterized and the spin-relaxation times at the boundaries of experimental cells are largely unknown. This fact was particularly emphasized by Meyerovich [26] in the context of studies of spin-polarized normal ^3He . However, in a recent experimental study of the effect of ^4He at surfaces with roughness on the scale of 20 Å in normal ^3He [27] the authors suggest that increased specularity of surfaces coated by ^4He is not due to the replacement of the localized magnetic layer of ^3He . It is likely, that momentum transfer from the surface to the superfluid ^3He is reduced by the presence of the ^4He film,[27,28] perhaps due to the superfluidity of the ^4He film.[27]

Thus we can conclude that interpretation of experiments on superflow in thin films of ^3He is hindered by insufficient knowledge of processes occurring at the superfluid-solid interface.

We would like to thank Prof. M. Salomaa for helpful comments.

Appendix

To solve the gap equation numerically, we transform it into

$$\ln\left(\frac{T}{T_{c0}}\right) = 7.12\alpha \frac{T}{T_{c0}} \frac{1}{\sum_{\nu} \sin^2 \theta_{\nu}} \sum_{\nu, n \geq 0} \sin^2 \theta_{\nu} \int_{-\pi/2}^{\pi/2} \frac{d\phi}{\pi} \left\{ \frac{\cos^2 \phi}{\sin^2 \phi} \right\} |z_{\nu}|^{-1/2} \sin \frac{\phi(n, \nu)}{2} - \sum_{n \geq 0} \frac{1}{n + 1/2}, \quad (A1)$$

where the temperature and the order parameter are now scaled with respect to their magnitudes in a film with no pair-breaking, T_{c0} and Δ_0 , respectively. Δ_0 is the gap at $T = 0$ and $v = 0$. Equations (10)-(12) are then rewritten with the following changes:

$$\tilde{\omega}_n \rightarrow 1.78\alpha(2n + 1) \frac{T}{T_{c0}} + \frac{\Gamma}{\Delta_0} U(i\omega_n) \cos^2 \theta_{\nu}, \quad (A2)$$

$$\Delta^2(\phi) \rightarrow \Delta^2(\phi)/\Delta_0, \quad vp_F \rightarrow vp_F/\Delta_0, \quad (A3)$$

where

$$\alpha = \exp(< \ln \sin \theta_{\nu} >), \quad < \ln \sin \theta_{\nu} > \equiv \left(\sum_{\nu} \sin^2 \theta_{\nu} \right)^{-1} \sum_{\nu} \sin^2 \theta_{\nu} \ln \sin \theta_{\nu}. \quad (A4)$$

The integrands f_1 and f_2 in equation (17) are

$$\begin{aligned} \left\{ \begin{array}{l} f_1(\Delta_1, \Delta_2, U(i\omega_n)) \\ f_2(\Delta_1, \Delta_2, U(i\omega_n)) \end{array} \right\} &\equiv \left\{ \begin{array}{l} \Delta_1 \\ \Delta_2 \end{array} \right\} - 3gN_0 \frac{\pi^2}{p_0 d} T \sum_{\nu, \omega_n \geq 0} \sin^2 \theta_{\nu} \int_{-\pi/2}^{\pi/2} \frac{d\phi}{\pi} \left\{ \begin{array}{l} \Delta_1 \cos^2 \phi \\ \Delta_2 \sin^2 \phi \end{array} \right\} \\ &\times 2\text{Re}([(\tilde{\omega}_n + ivp_F \sin \theta_{\nu} \cos \phi)^2 + \frac{3}{2} \Delta^2(\phi) \sin^2 \theta_{\nu}]^{-\frac{1}{2}}), \end{aligned} \quad (A5)$$

At $f_1 = 0$ and $f_2 = 0$ these two equations are just the gap equations, written in a form more convenient for integration. f_3 is given by

$$\begin{aligned}
f_3(\Delta_1, \Delta_2, U(i\omega_n)) &\equiv 6gN_0T\nu_0\Gamma U(i\omega_n)\frac{\pi^2}{p_0d} - \frac{1}{2} \sum_{\cos\theta_\nu}^{\pi/2} \int_{-\pi/2}^{\pi/2} \frac{d\phi}{\pi} \\
&\times 2\text{Re} \frac{\tilde{\omega}_n + ivp_F \sin\theta_\nu \cos\phi}{[(\tilde{\omega}_n + ivp_F \sin\theta_\nu \cos\phi)^2 + \frac{3}{2}\Delta^2(\phi)\sin^2\phi]\sin^2\theta_\nu]^{1/2}}. \tag{A6}
\end{aligned}$$

The numerical calculations of j_s given by equation (23) are carried out using equations (A2)-(A4).

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Figure Captions

1. Solutions of the gap equation for phases: deformed planar (solid line), planar (dotted line) and polar (broken line) at $T = 0.3T_{c0}$ for several values of the surface scattering strength Γ , (a) 0, (b) $0.1\Gamma_c$, (c) $0.2\Gamma_c$, (d) $0.4\Gamma_c$. Γ_c is the critical scattering strength at $T = 0$ and $vp_F = 0$. Δ_0 is the gap at $T = 0$ and $vp_F = 0$.
2. The free energy difference between the superfluid state and the normal state for all phases with the same parameter set as in fig. 1. The lines are denoted as in fig. 1. Here energy is scaled with respect to the energy difference between the 2-A phase and the normal state at zero temperature and no flow, $\Delta F_S^A(T = 0, v = 0) = 3gN_0(\pi^2/p_0d)\Delta_0T_c^F$. The scale was changed in fig. (d).
3. Details of the phase diagram near the first-order transition at $T = 0.3T_{c0}$ and $\Gamma = 0$.
4. The superflow for the same parameter set as in fig. 1. and with the same notation. Note the different scale in fig. (d).
5. Critical current density j_{sc} as a function of increasing surface roughness at temperatures: 0, $0.2T_{c0}$, $0.4T_{c0}$ and $0.6T_{c0}$.
6. Critical current density j_{sc} as a function of temperature for several values of pair-breaking strength at the surface: 0, $0.2\Gamma_c$, $0.4\Gamma_c$ and $0.6\Gamma_c$. The units for the current density are the same as those in figure 10 of [1]. The film thickness was also chosen to be in the range measured in [1].
7. Superfluid density vs. temperature. The temperature is scaled to the critical temperature in the film. The values of Γ are the same as in figure 6.

8. Superfluid density as a function of Γ at four temperatures: 0 , $0.2T_{c0}$, $0.4T_{c0}$ and $0.6T_{c0}$.

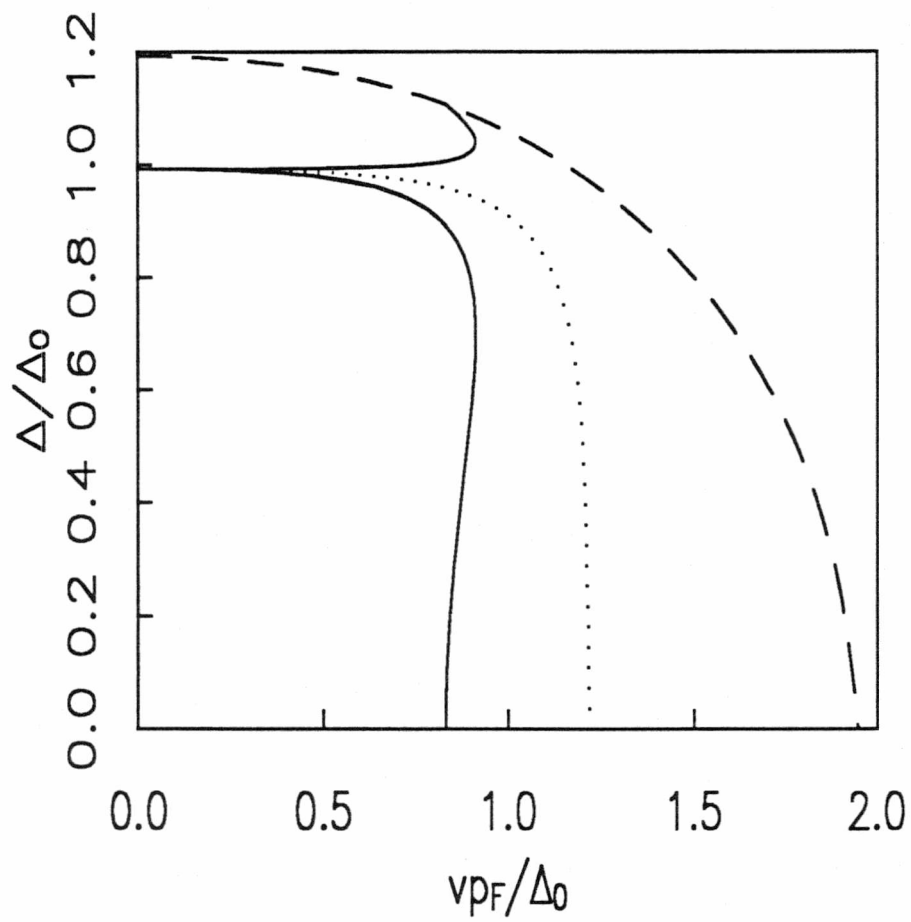


Fig. 1a

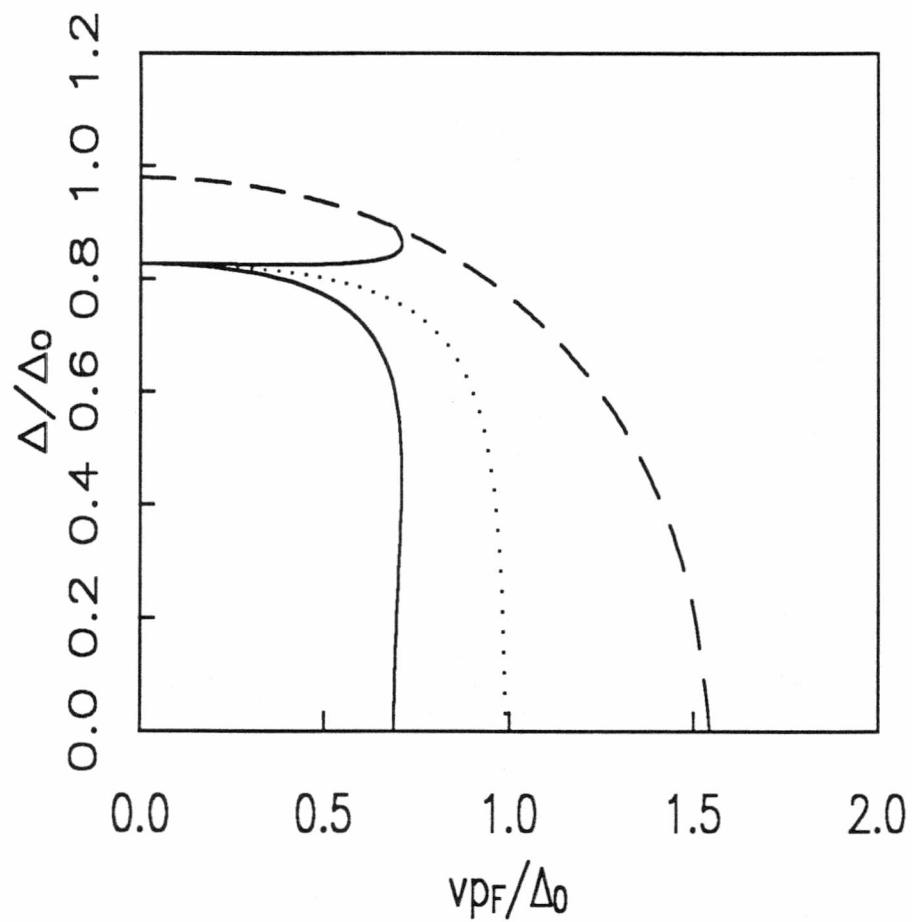


Fig. 1b

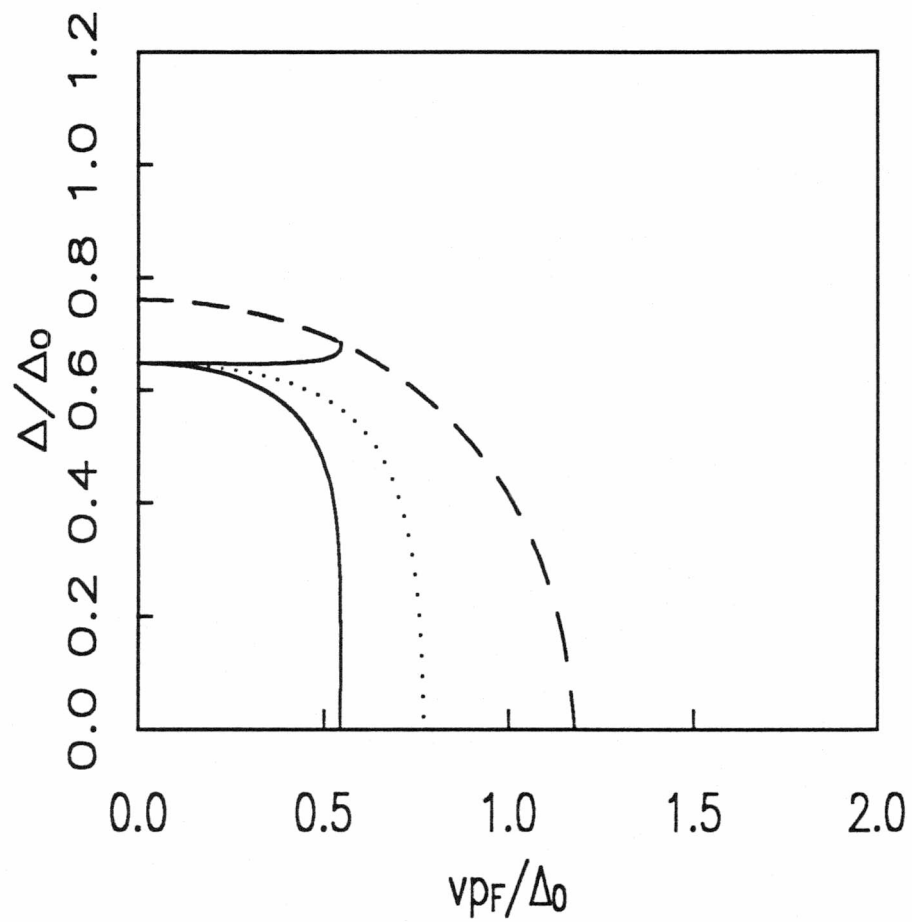


Fig. 1c

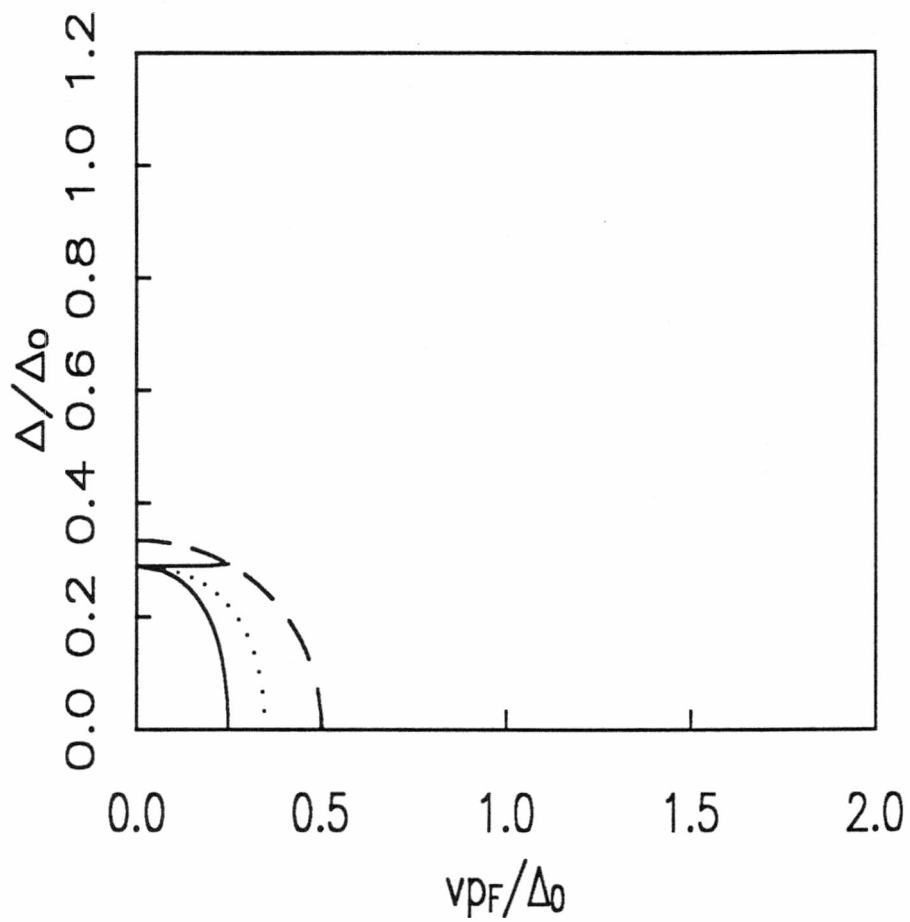


Fig. 1d

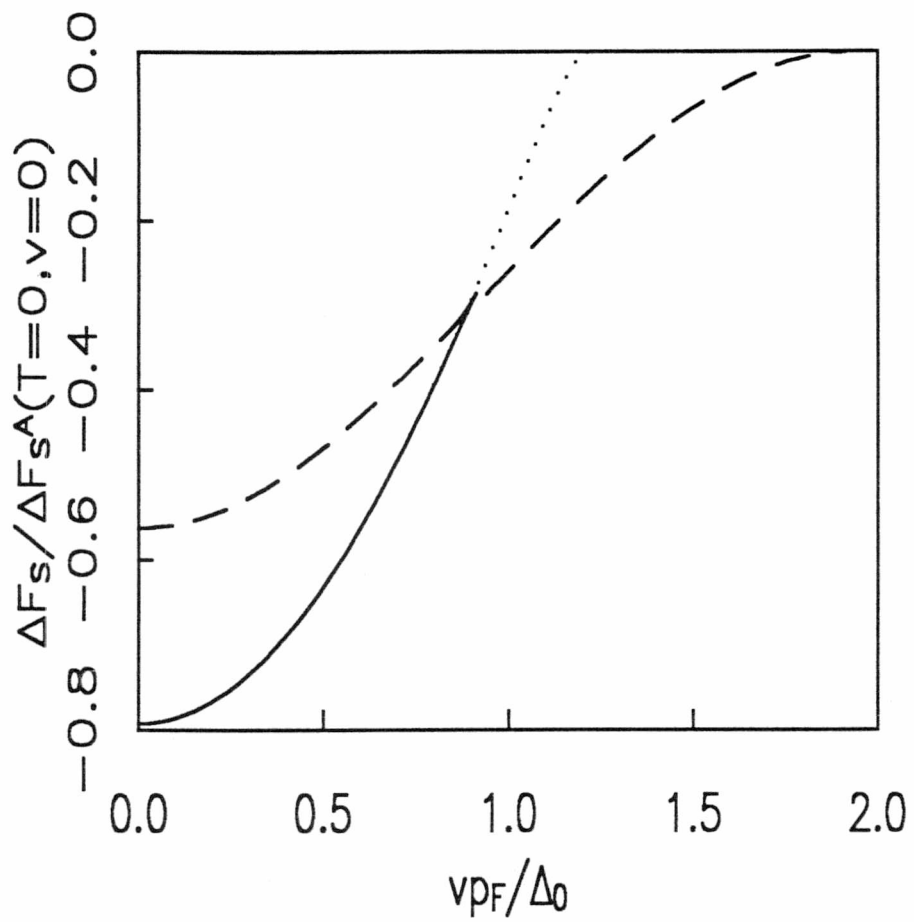


Fig. 2a

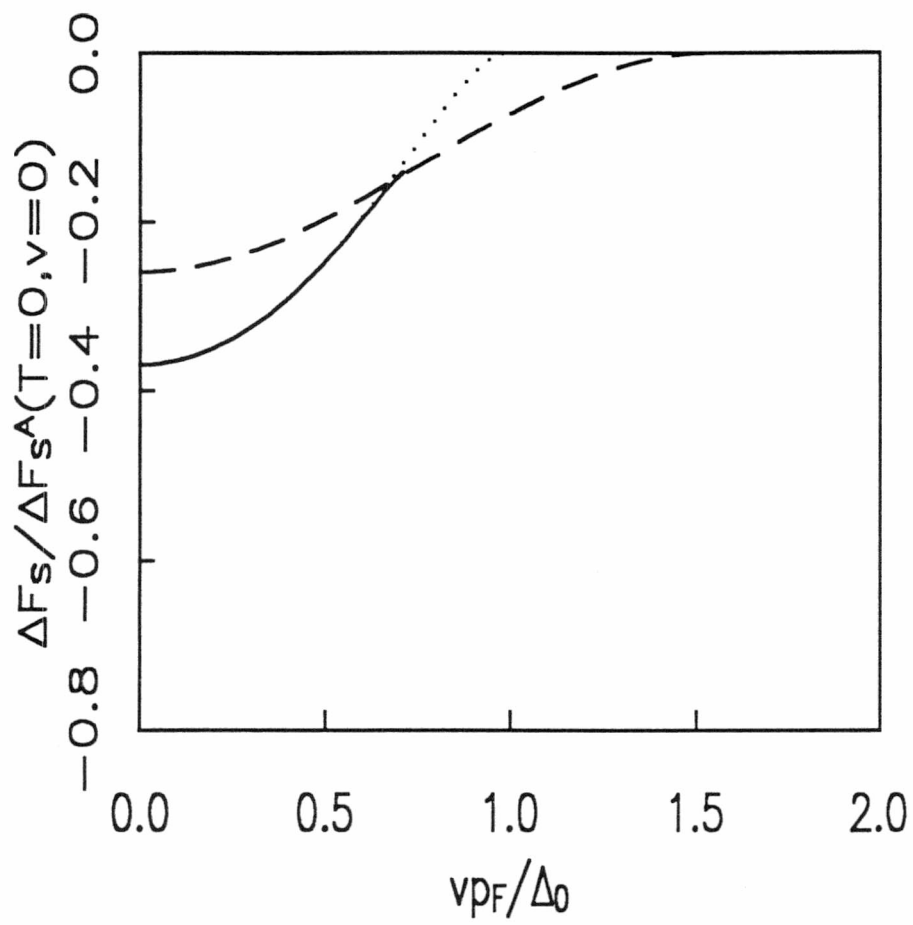


Fig. 2b

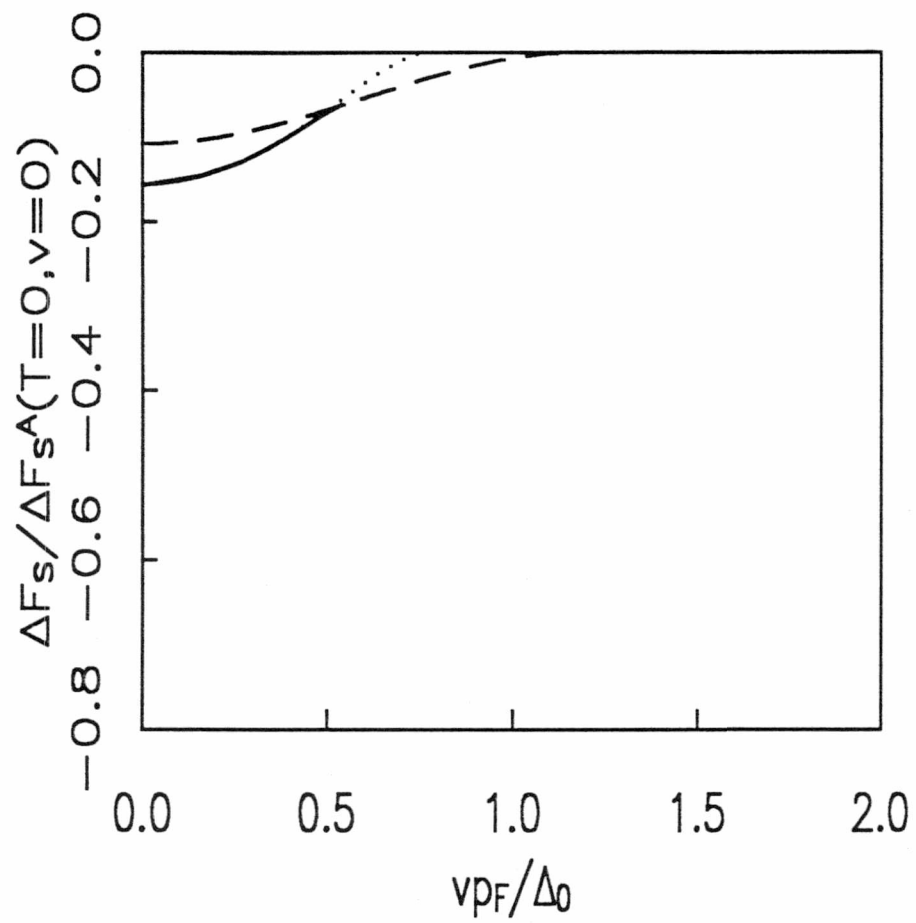


Fig. 2c

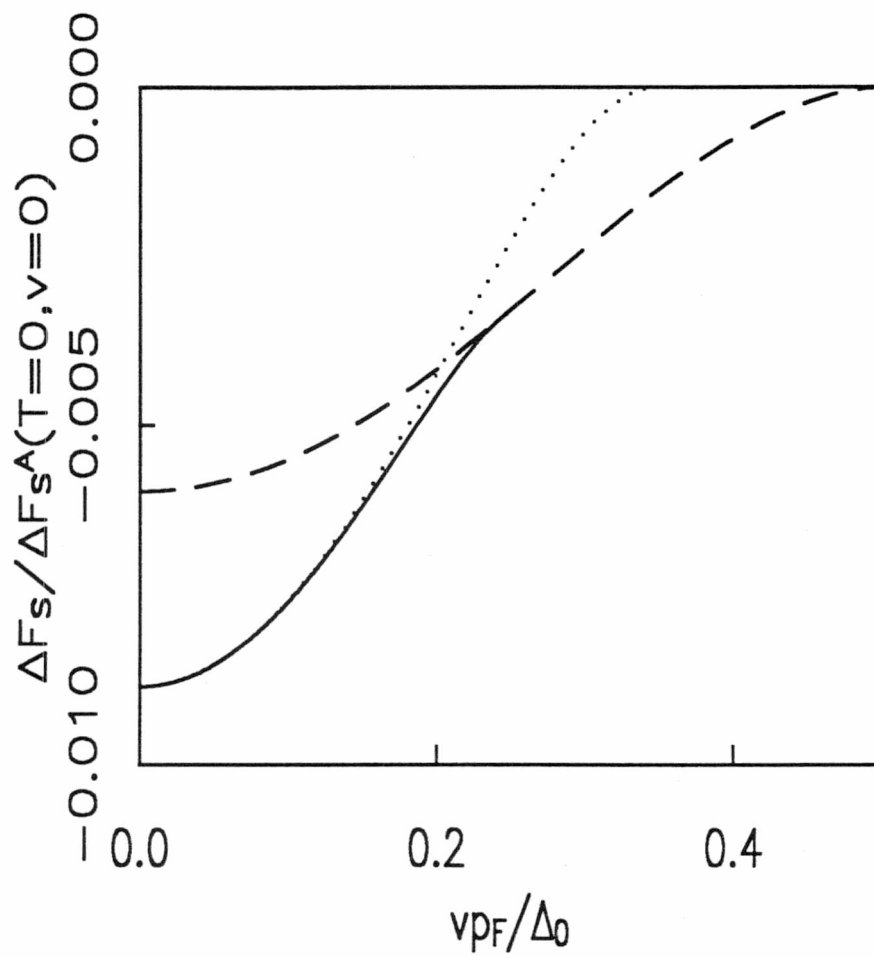


Fig. 2d

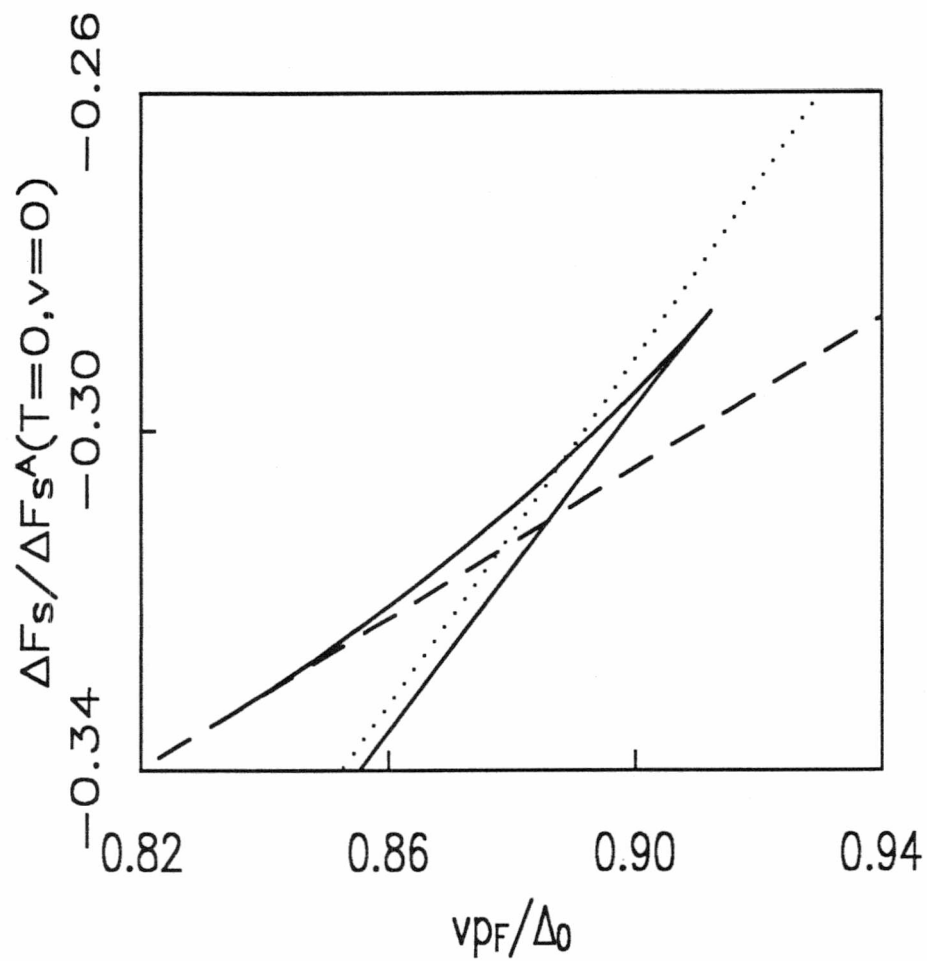


Fig. 3

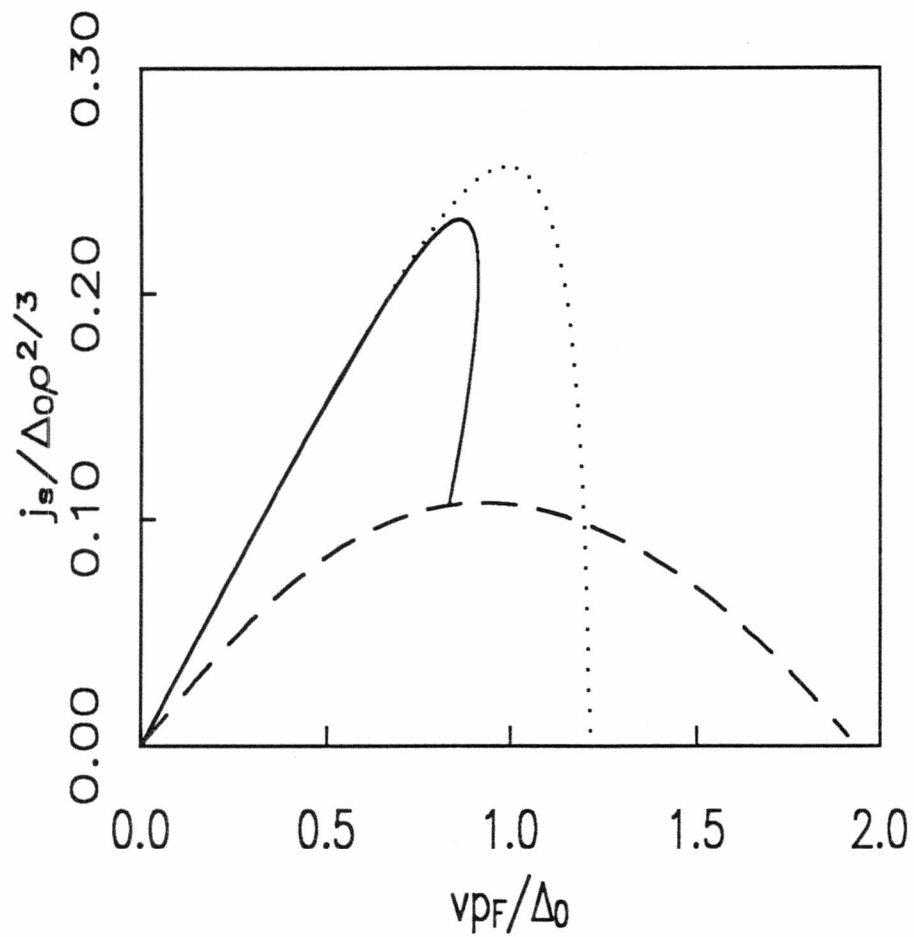


Fig. 4a

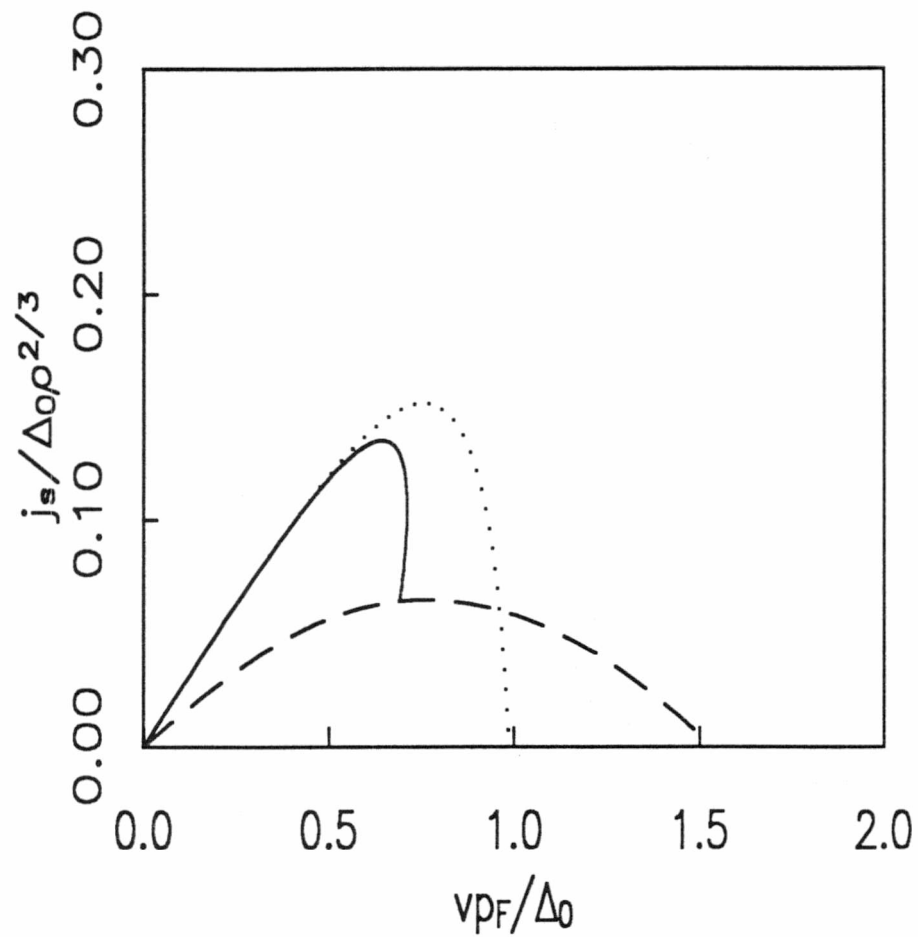


Fig. 4b

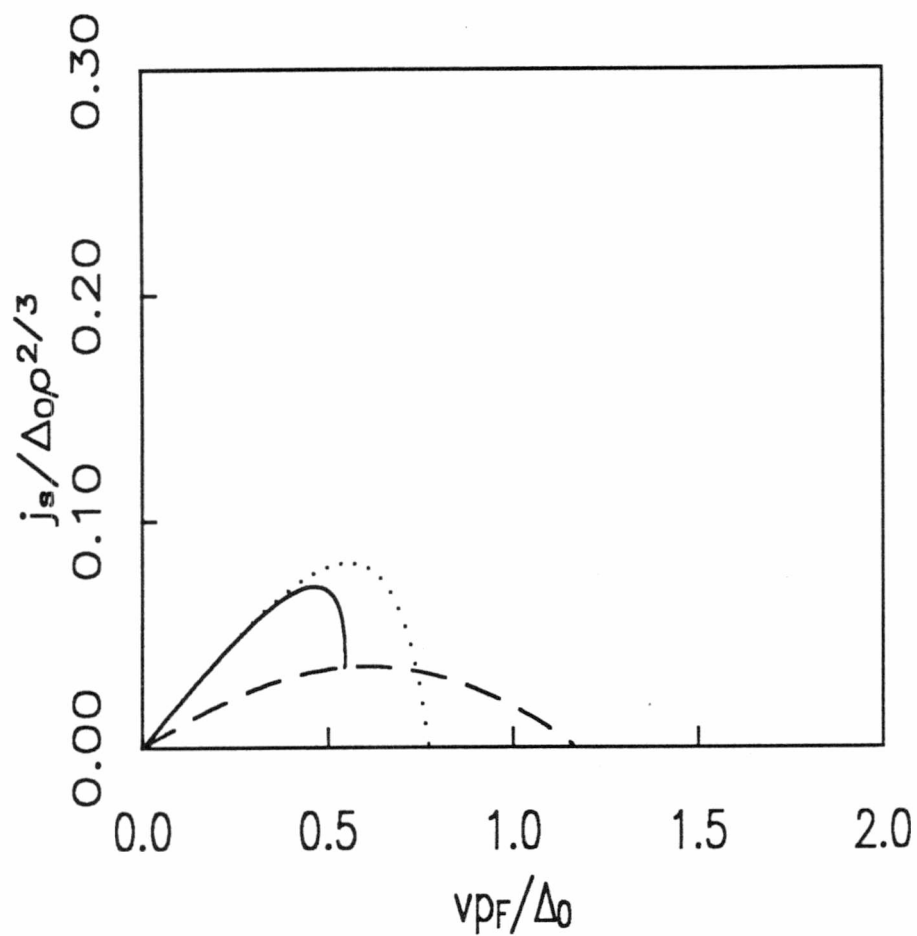


Fig. 4c

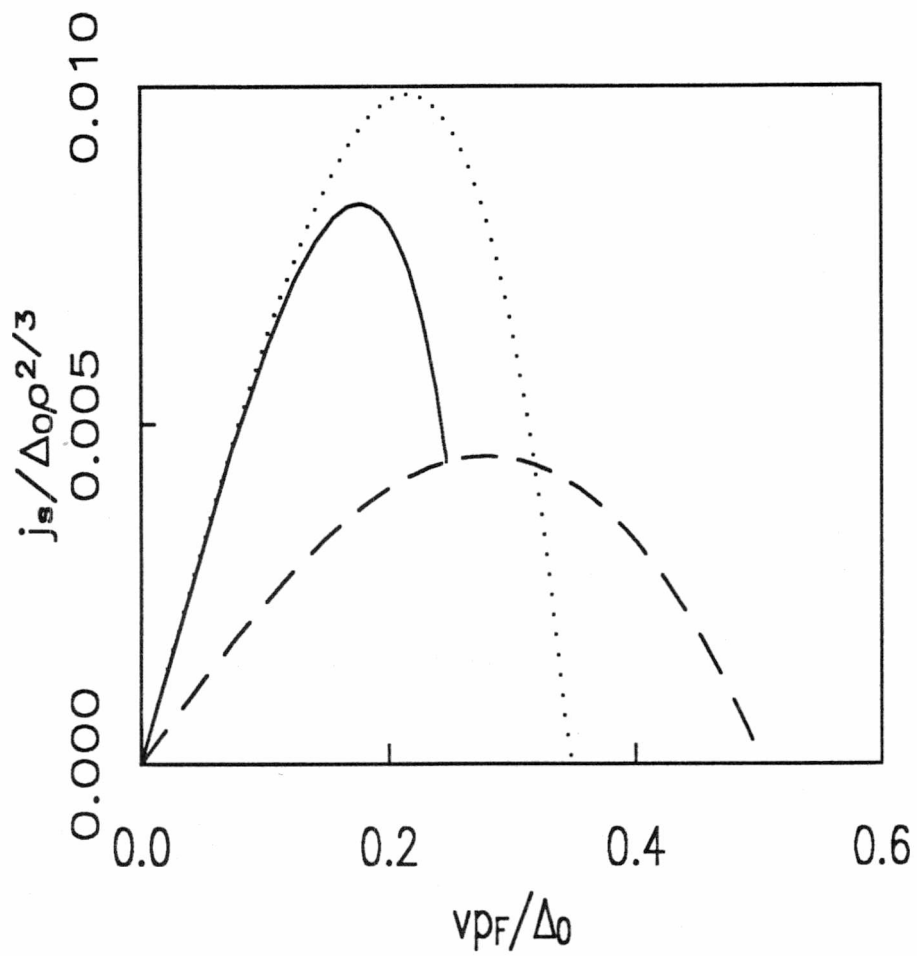


Fig. 4d

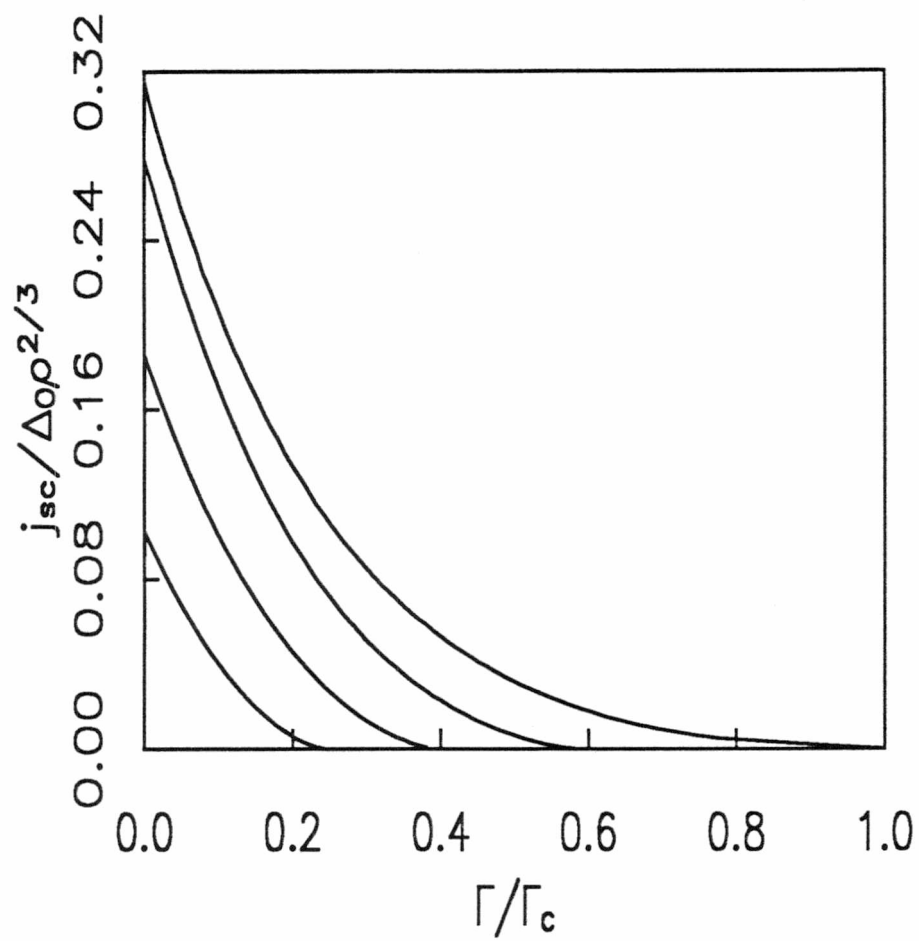


Fig. 5

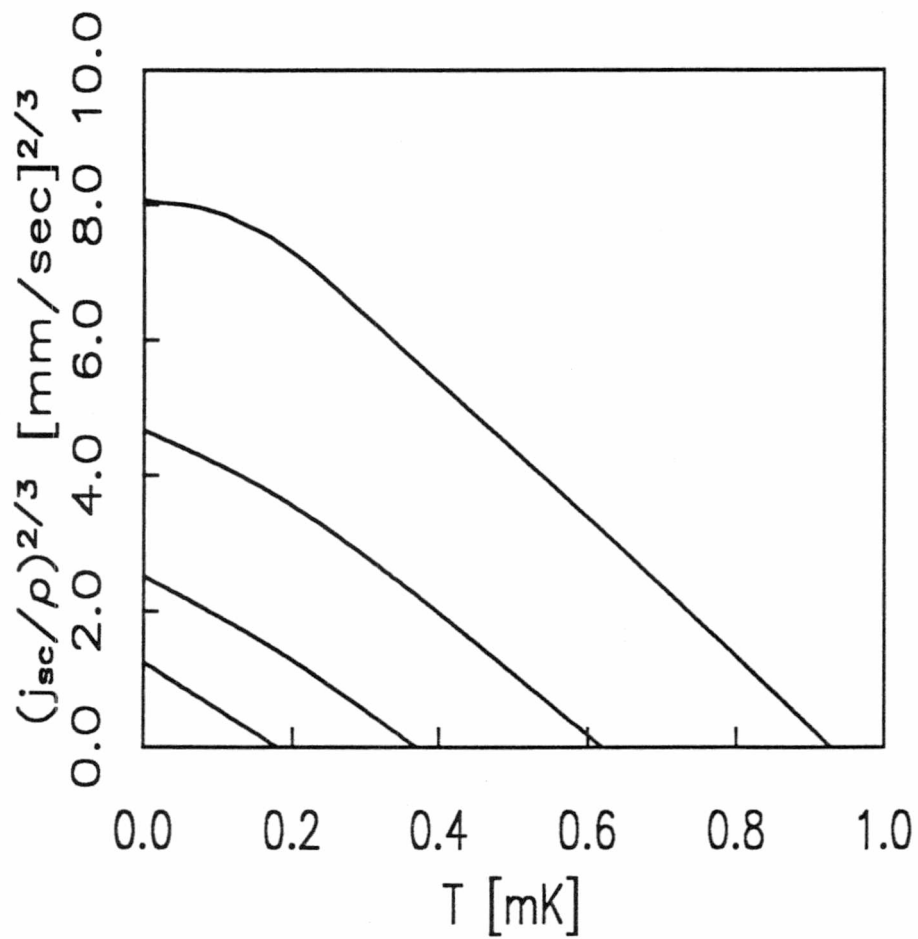


Fig. 6

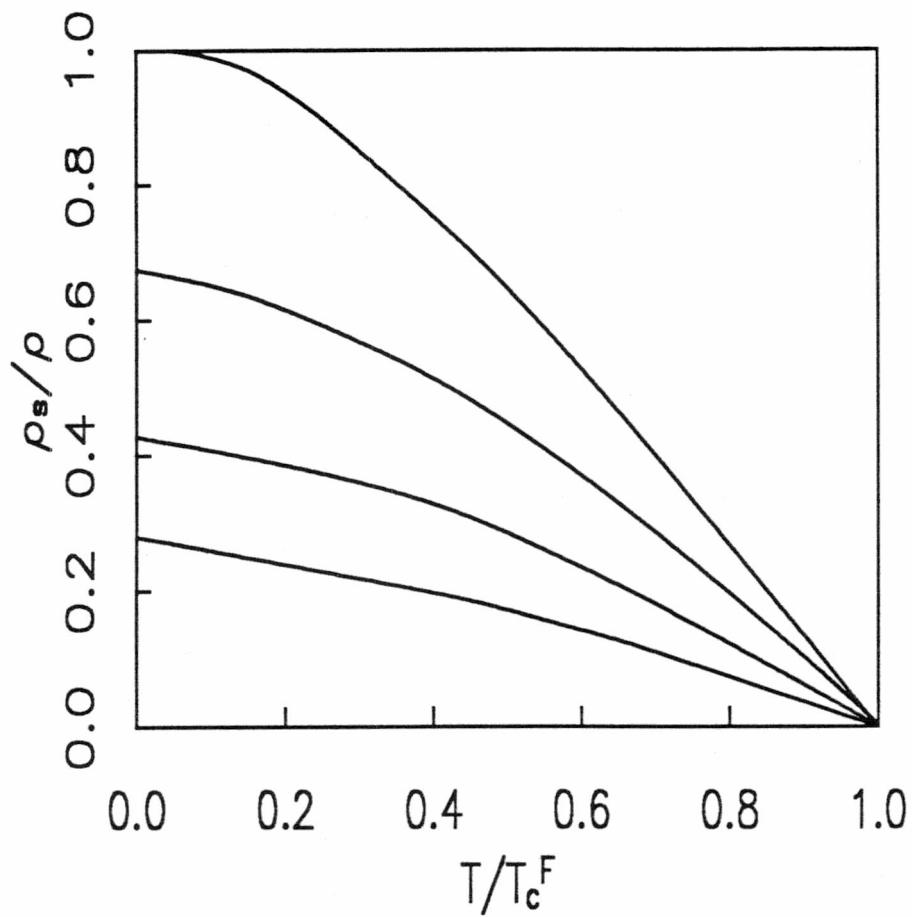


Fig. 7

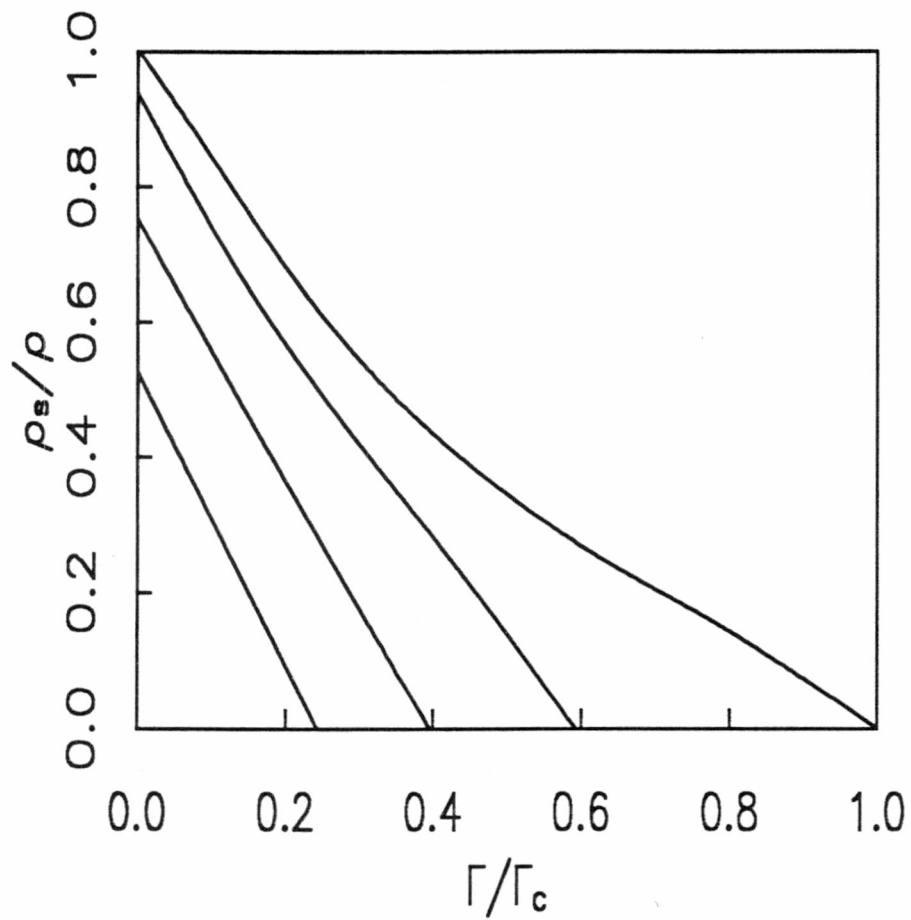


Fig. 8

