

The spectrum behaviors of light waves on scattering from PT-symmetric semi-soft boundary medium

YINGYING XIAO, CHANG LIU, HAO WU, TAO WANG*

Department of Physics, Sichuan Normal University, Chengdu 610068, China

*Corresponding author: towerwang@126.com

The spectrum behaviors of light waves on scattering from a novel kind of medium, *i.e.*, the PT-symmetric semi-soft boundary medium, were discussed, and the dependence of the presentation of the far-zone scattered spectrum on the characteristics of the scattering medium was investigated. The results show that spectral shifts and spectral switches can be discovered in the scattered field, and the direction of spectral switch is closely related to the non-Hermitian parameter, the boundary characteristic, and the effective width of the scattering medium. These results will further enrich the scattering theory and may have some applications in the reconstruction of the structural information of scattering medium.

Keywords: light waves scattering, PT-symmetric medium, spectrum.

1. Introduction

The scattering of light waves has been the focus of researchers for decades because of its great development potential in optical communication, imaging, and detection. Recently, great progress has been made in the study of light wave scattering. For example, researchers have discussed the far-zone scattered fields of various scatterers, including deterministic media [1], quasi-homogeneous media [2], collection of particles [3], semi-soft boundary media [4,5], hollow media [6], and PT-symmetric media [7,8], *etc.* Among them, the PT-symmetric medium is a new type of scattering medium with the characteristics of balanced gain and loss. It is shown that the PT symmetry in the potential correlation function of the medium will destroy the central symmetry of the far-zone scattered field [9]. The PT symmetry means that the potential function satisfies the equal relationship under the action of the parity operator (P) and the time reversal operator (T). In 2010, the induced-experimental observation of the PT symmetry extended the research to the optical field [10]. After that, BRANDÃO proposed the deterministic medium with PT-symmetric scattering potential by combining PT symmetry theory and light waves scattering theory [11,12], which triggered a lot of discussion on PT-symmetric medium by researchers [13,14]. On the other hand, the spectrum var-

iation of light waves in the process of transmission and scattering has also been one of the hot spots for researchers. In 1989, WOLF and his collaborators found that the spectrum of light may change as it is scattered by a random medium [15]. After that, lots of researches on the spectral shifts and spectral switches in the process of light wave scattering have emerged [16-20]. In addition, the spectrum changes of the scattered far field are also very important for the inverse problem in scattering, and some application studies have been carried out. For example, ZHAO *et al.* have proved that correlation-induced spectral changes can be used to determine the correlation function of the scattering potential of an unknown random medium [21]. Wang discussed the influence of the boundary of the semi-soft boundary medium on the scattered spectrum, and found that the direction of the spectral switch is closely related to the boundary of the scatterer [22]. This phenomenon can provide a method to determine the boundary of the scatterer from the measurement of the far-zone scattered spectrum.

However, the previous researches on the spectrum changes in light wave scattering mainly focused on the traditional scatterers. As far as we know, there are few studies on the spectral shift of the scattered field of the medium with PT symmetry. However, PT-symmetric media have unique optical properties; so is there any novel and interesting relationship between the structural properties of the PT-symmetric scatterer and the scattered spectrum? This is an important problem, especially in the inverse problem. In this manuscript, we will study the far-zone scattered spectrum of a polychromatic light wave on scattering from a semi-soft boundary medium with PT symmetry, and discuss the influence of the non-Hermitian parameter, the boundary, and the effective width of the scatterer and the scattering angle on the scattered spectrum.

2. Distribution of the far-zone scattered spectrum

As shown in Fig. 1, consider that a polychromatic plane light wave is incident on a scatterer along the direction of the real unit vector $\mathbf{s}_0(0, 0, 1)$. The cross-spectral density function of incident light can be expressed as [23]

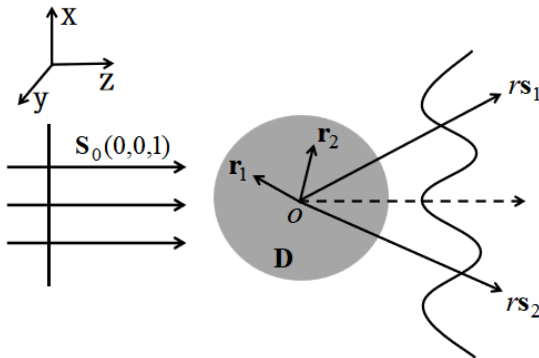


Fig. 1. Illustration of the notations.

$$W^{(i)}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{s}_0, \omega) = S^{(i)}(\omega) \exp \left[i k \mathbf{s}_0 \cdot (\mathbf{r}_2 - \mathbf{r}_1) \right] \quad (1)$$

where ω is the angular frequency of the incident field, $S^{(i)}(\omega)$ represents the spectrum of the incident field, and $k = \omega/c$ is the free space wave number with c being the light speed in vacuum.

Assuming that the scattering process is weak, *i.e.*, the scattering field is much smaller than the incident field, the scattering can be analyzed within the accuracy range of the first-order Born approximation [23]. For the deterministic medium, its optical properties are described by the scattering potential. The cross-spectral density function of the far-zone scattered field then can be given by the following formula [23]

$$W^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2, \mathbf{s}_0, \omega) = \frac{S^{(i)}(\omega)}{r^2} \tilde{F}^*(\mathbf{K}_1, \omega) \tilde{F}(\mathbf{K}_2, \omega) \quad (2)$$

where

$$\mathbf{K}_1 = k(\mathbf{s}_1 - \mathbf{s}_0) \quad (3a)$$

$$\mathbf{K}_2 = k(\mathbf{s}_2 - \mathbf{s}_0) \quad (3b)$$

and the asterisk “*” represents complex conjugation, $\mathbf{s}_{1,2}$ is the unit vector of scattering direction, $\tilde{F}(\mathbf{K}, \omega)$ is the Fourier transform of the scattering potential of the deterministic medium $F(\mathbf{r}, \omega)$,

$$\tilde{F}(\mathbf{K}, \omega) = \int F(\mathbf{r}, \omega) \exp(-i \mathbf{K} \cdot \mathbf{r}) d^3 \mathbf{r} \quad (4)$$

In addition, there is a quantitative relationship between the scattering potential and the refractive index as follows [23]

$$F(\mathbf{r}, \omega) = \frac{k^2}{4\pi} [n^2(\mathbf{r}, \omega) - 1] \quad (5)$$

When the two scattering directions coincide (*i.e.*, $\mathbf{s}_1 = \mathbf{s}_2 = \mathbf{s}$), the cross-spectral density function reduces to the spectral density of the far-zone scattered field, with the form of [23]

$$S^{(s)}(r\mathbf{s}, \mathbf{s}_0, \omega) = \frac{S^{(i)}(\omega)}{r^2} \left| \tilde{F} \left[k(\mathbf{s} - \mathbf{s}_0), \omega \right] \right|^2 \quad (6)$$

In this manuscript, we will study the far-zone scattered spectrum as a polychromatic light wave is scattered from a PT-symmetric semi-soft boundary medium. Due to the analogy between the Schrödinger equation for electrons and Maxwell's theory of light, the complex potential function of quantum theory can be viewed to be equivalent to a complex refractive index in optics. Therefore, in the field of physical optics, the

PT symmetry of a deterministic medium can be transformed into the Hermiticity of its complex refractive index, or similarly, into the Hermiticity of its scattering potential [24]. The imaginary part of the refractive index can be positive or negative, representing the optical gain or loss of the medium, respectively. For classical media, the real and imaginary parts of the refractive index are even (*i.e.*, $n_r(-\mathbf{r}, \omega) = n_r(\mathbf{r}, \omega)$ and $n_i(-\mathbf{r}, \omega) = n_i(\mathbf{r}, \omega)$). For a PT-symmetric medium, the complex refractive index needs to meet the following conditions [7]

$$n_r(-\mathbf{r}, \omega) = n_r(\mathbf{r}, \omega) \quad (7a)$$

$$n_i(-\mathbf{r}, \omega) = -n_i(\mathbf{r}, \omega) \quad (7b)$$

Since there is a relationship between the refractive index and the scattering potential in Eq. (5), the scattering potential of a PT-symmetric medium must satisfy the following condition [7]

$$F_{PT}(-\mathbf{r}, \omega) = F_{PT}^*(\mathbf{r}, \omega) \quad (8)$$

Next, we generalize the model of PT-symmetric semi-soft boundary medium on the basis of the traditional semi-soft boundary medium model [22]. The square of the real part and the square of the imaginary part of the complex refractive index can be written as

$$\text{Re}[n^2(\mathbf{r}, \omega)] = 1 + 4\pi B \sum_{m=1}^M (-1)^{m-1} C_M^m \exp\left(-m\beta_M \frac{\mathbf{r}^2}{2\sigma^2}\right) \quad (9)$$

$$\text{Im}[n^2(\mathbf{r}, \omega)] = 4\pi B \sum_{m=1}^M (-1)^{m-1} C_M^m \exp\left(-m\beta_M \frac{\mathbf{r}^2}{2\sigma^2}\right) (\boldsymbol{\beta} \cdot \mathbf{r}) \quad (10)$$

where

$$C_M^m = \frac{M!}{m!(M-m)!} \quad (11a)$$

$$\beta_M = -\ln\left[1 - (1 - e^{-1})^{1/M}\right] \quad (11b)$$

and B is a constant, σ denotes the effective width, M represents the boundary of the scatterer and $\boldsymbol{\beta}$ is the vector that controls the gain and loss characteristics of the scatterer. From Eq. (5), the scattering potential of PT-symmetric semi-soft boundary medium can be obtained as

$$F(\mathbf{r}, \omega) = (\omega/c)^2 B \sum_{m=1}^M (-1)^{m-1} C_M^m \exp\left(-m\beta_M \frac{\mathbf{r}^2}{2\sigma^2}\right) (1 + i \boldsymbol{\beta} \cdot \mathbf{r}) \quad (12)$$

Substituting Eq. (12) into Eq. (4), the Fourier transform of the scattering potential can be obtained as

$$\begin{aligned} \tilde{F}(\mathbf{K}, \omega) = & \left(\frac{2\pi}{m\beta_M} \right)^{3/2} \sigma^3 (\omega/c)^2 B \sum_{m=1}^M (-1)^{m-1} C_M^m \\ & \times \exp\left(-\frac{\sigma^2 |\mathbf{K}|^2}{2m\beta_M}\right) \left(1 + \frac{\sigma^2}{m\beta_M} \boldsymbol{\beta} \cdot \mathbf{K}\right) \end{aligned} \quad (13)$$

After substituting Eq. (13) into Eq. (6), one can find that the expression of the far-zone scattered spectrum is

$$\begin{aligned} S^{(s)}(r\mathbf{s}, \mathbf{s}_0, \omega) = & \frac{B^2 S^{(i)}(\omega)}{r^2} \left(\frac{2\pi}{m\beta_M} \right)^3 \sigma^6 (\omega/c)^4 \\ & \times \left| \sum_{m=1}^M (-1)^{m-1} C_M^m \exp\left(-\frac{\sigma^2 |\mathbf{K}|^2}{2m\beta_M}\right) \left(1 + \frac{\sigma^2}{m\beta_M} \boldsymbol{\beta} \cdot \mathbf{K}\right) \right|^2 \end{aligned} \quad (14)$$

Since the medium is isotropic, a pair of symmetric vectors can be selected to highlight the statistical properties of the scattering field. We define the angle between $\mathbf{s}_0$ and $\mathbf{s}$ is θ and consider that this kind of medium can be described by $\boldsymbol{\beta} = \beta \mathbf{s}_0$, where β is the non-Hermitian parameter. In this case, we can get the following expression

$$|\mathbf{K}|^2 = |k(\mathbf{s} - \mathbf{s}_0)|^2 = 4k^2 \sin^2\left(\frac{\theta}{2}\right) \quad (15)$$

$$\boldsymbol{\beta} \cdot \mathbf{K} = -2k\beta \sin^2\left(\frac{\theta}{2}\right) \quad (16)$$

Then the spectral density of the far-zone scattered field of Eq. (14) can be rewritten as

$$\begin{aligned} S^{(s)}(\theta, \omega) = & \frac{B^2 S^{(i)}(\omega)}{r^2} \left(\frac{2\pi}{m\beta_M} \right)^3 \sigma^6 (\omega/c)^4 \\ & \times \left| \sum_{m=1}^M (-1)^{m-1} C_M^m \exp\left[-\frac{2\sigma^2 (\omega/c)^2 \sin^2\left(\frac{\theta}{2}\right)}{m\beta_M}\right] \left[1 - \frac{2\omega}{c} \beta \sigma^2 \sin^2\left(\frac{\theta}{2}\right)\right] \right|^2 \end{aligned} \quad (17)$$

3. Numerical results and discussions

In the following numerical simulations, let us assume that the spectrum of the incident field has a form of Gaussian distribution [15], *i.e.*

$$S^{(i)}(\omega) = S_0 \exp \left[-\frac{(\omega - \omega_0)^2}{2\Gamma_0^2} \right] \quad (18)$$

where S_0 is a positive constant, Γ_0 is the spectral width, and ω_0 denotes the central frequency. By substituting Eq. (18) into Eq. (17), the far-zone scattered spectrum can be expressed as

$$S^{(s)}(\theta, \omega) = \frac{B^2 S_0}{r^2} \left(\frac{2\pi}{m\beta_M} \right)^3 \sigma^6 (\omega/c)^4 \exp \left[-\frac{(\omega - \omega_0)^2}{2\Gamma_0^2} \right] \times \left| \sum_{m=1}^M (-1)^{m-1} C_M^m \exp \left[-\frac{2\sigma^2 (\omega/c)^2 \sin^2(\frac{\theta}{2})}{m\beta_M} \right] \left[1 - \frac{2\frac{\omega}{c}\beta\sigma^2}{m\beta_M} \sin^2(\frac{\theta}{2}) \right] \right|^2 \quad (19)$$

In order to make the expression look more concise, we can make ω , Γ_0 , β , and σ dimensionless through the following transformations:

$$\omega' = \frac{\omega}{\omega_0}, \quad \Gamma'_0 = \frac{\Gamma_0}{\omega_0}, \quad \beta' = \frac{c}{\omega_0} \beta, \quad \sigma' = \frac{\omega_0}{c} \sigma \quad (20)$$

and defining the normalized spectral density

$$S_\beta(\theta, \omega') = \frac{\omega_0^2 r^2}{B^2 S_0 c^2 \sigma^6} \left(\frac{m\beta_M}{2\pi} \right)^3 S^{(s)}(\theta, \omega) \quad (21)$$

Eq. (19) can be rewritten as

$$S_\beta(\theta, \omega') = \omega'^4 \exp \left[-\frac{(\omega' - 1)^2}{2\Gamma_0'^2} \right] \times \left| \sum_{m=1}^M (-1)^{m-1} C_M^m \exp \left[-\frac{2\sigma'^2 \omega'^2 \sin^2(\frac{\theta}{2})}{m\beta_M} \right] \left[1 - \frac{2\omega' \beta' \sigma'^2}{m\beta_M} \sin^2(\frac{\theta}{2}) \right] \right|^2 \quad (22)$$

It can be seen from the expression that the far-zone scattered spectrum is generally different from the incident spectrum. The non-Hermitian parameter β' , the boundary constant M , and the effective width σ' all play a role in the scattered spectrum (for convenience, we still call the transformed β' and σ' as the non-Hermitian parameter and

the effective width in the following). In order to intuitively explain the changes in the spectrum, we will give some numerical results of the far-zone scattered spectrum.

First, let us consider the demonstration of the scattered spectrum with increase of the scattering angle when the non-Hermitian parameter $\beta' = 0.23$. In order to facilitate the comparison, we also draw the spectrum of the incident field in the figure (as shown by the black solid line). As shown in Fig. 2, when the light wave with Gaussian spectrum is scattered from the PT-symmetric semi-soft boundary medium, the scattered spectrum will be different from that of the incident light, which is the phenomenon of spectral shift. It is well-known that if the central frequency of the scattering spectrum is smaller than that of the incident spectrum, the red shift occurs (as shown in Fig. 2(b)). On the contrary, one can find from Fig. 2(d) that if the central frequency of the scattering spectrum is greater than that of the incident spectrum, the blue shift occurs [25, 26]. This is because the spectrum of the scattered field will split into two peaks, and we always take the location of the larger peak as the central frequency. As the scattering angle increases, the maximum peak and the secondary peak exchange positions, which induces the spectrum to jump from red shift to blue shift. It should be noted from

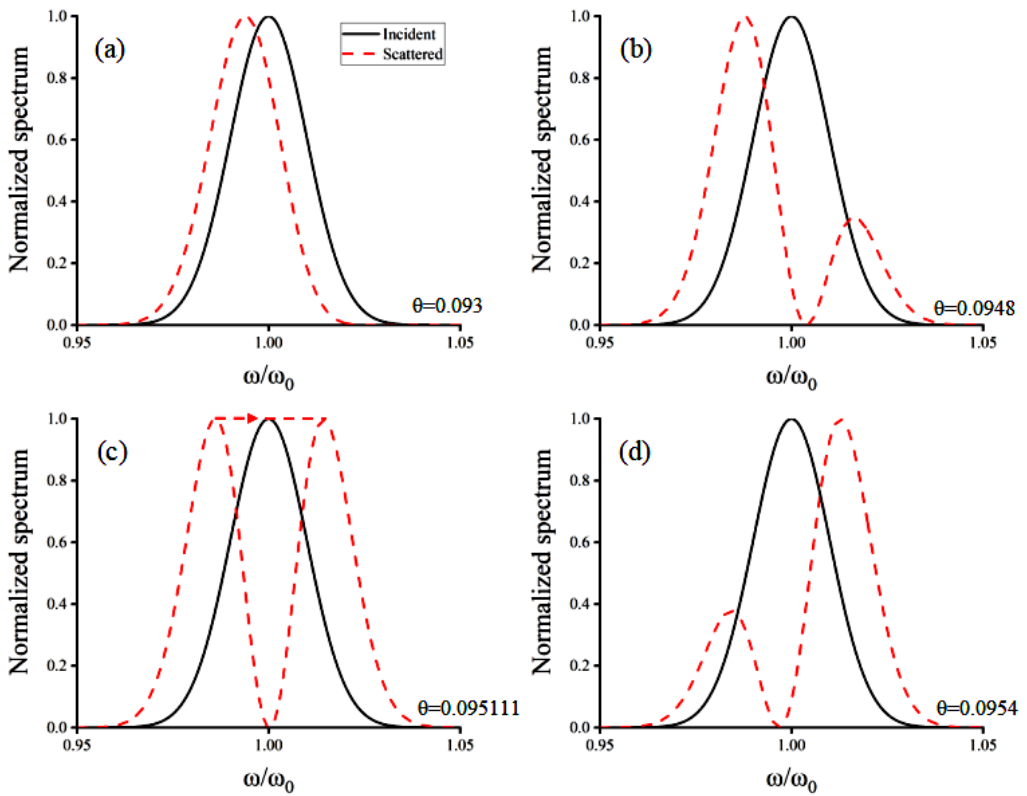


Fig. 2. The far-zone scattered spectrum with different scattering angles. The calculation parameters are selected as follows: $\beta' = 0.23$, $M = 4$, $\Gamma'_0 = 0.01$ and $\sigma' = 18$.

Fig. 2(c) that there is a critical direction, where the two peaks of the spectrum reach the same height, such spectrum is called a spectral switch [27].

Figure 3 shows the spectral shifts and spectral switches produced by scattering from the PT-symmetric semi-soft boundary media with different non-Hermitian parameters β' . The solid line, dashed line and dotted line represent the normalized scattered spectrum when the polychromatic light wave is scattered from the PT-symmetric

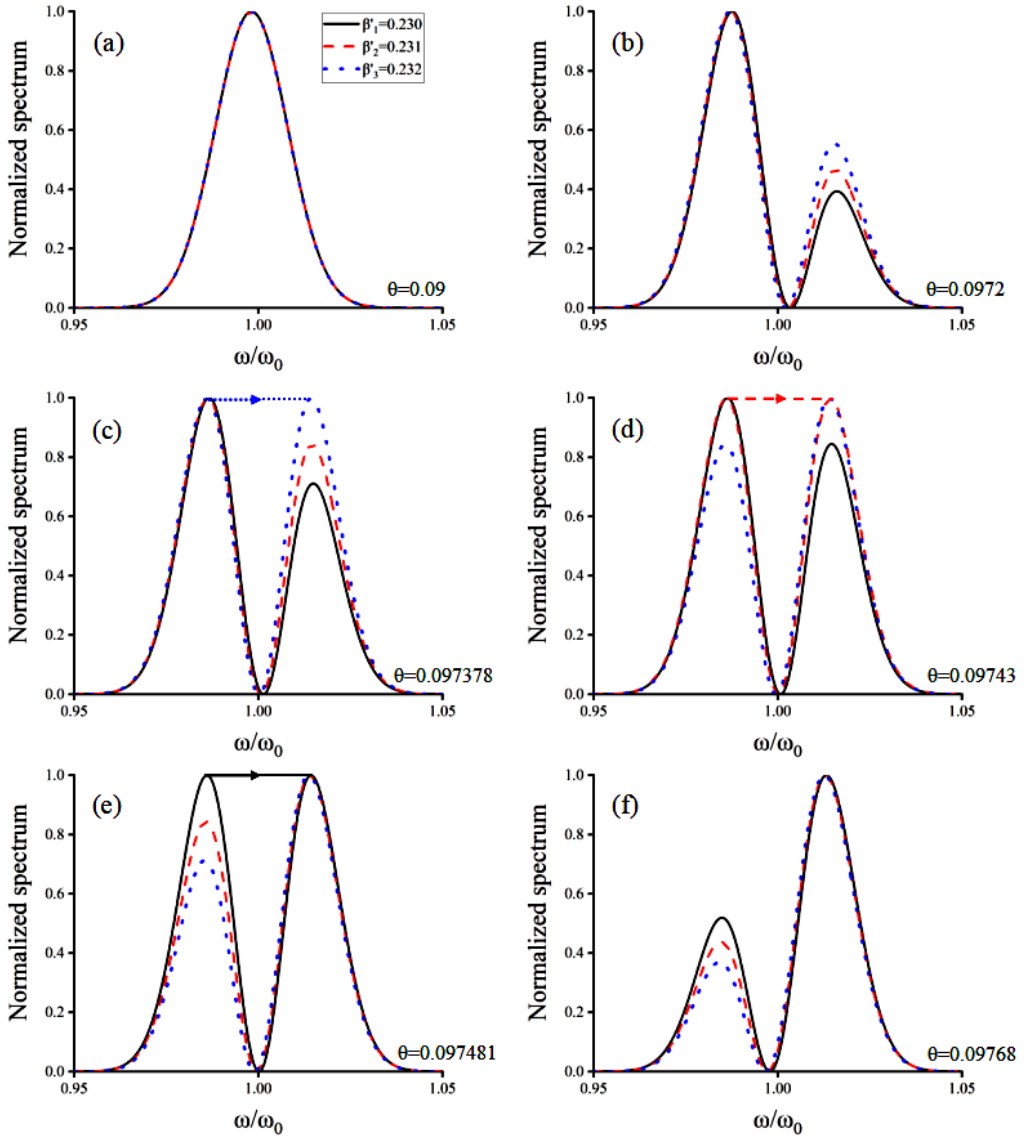


Fig. 3. The influence of the non-Hermitian parameter on the spectral switch of the far-zone scattered field. The calculation parameters are selected as follows: $M = 3$, $\Gamma'_0 = 0.01$ and $\sigma' = 18$.

semi-soft boundary medium with $\beta' = 0.230$, $\beta' = 0.231$ and $\beta' = 0.232$, respectively. From Figs. 3(c)-3(e), we find that the non-Hermitian parameter β' of the scatterer has an important influence on the location of the spectral switch in the scattered far field. For the case of $\beta' = 0.232$, the spectral switch occurs at $\theta = 0.097378$ (see Fig. 3(c)); as the scattering angle continues to increase, the spectral switch of the scatterer with $\beta' = 0.231$ will appear at $\theta = 0.09743$ (see Fig. 3(d)); while the spectrum corresponding

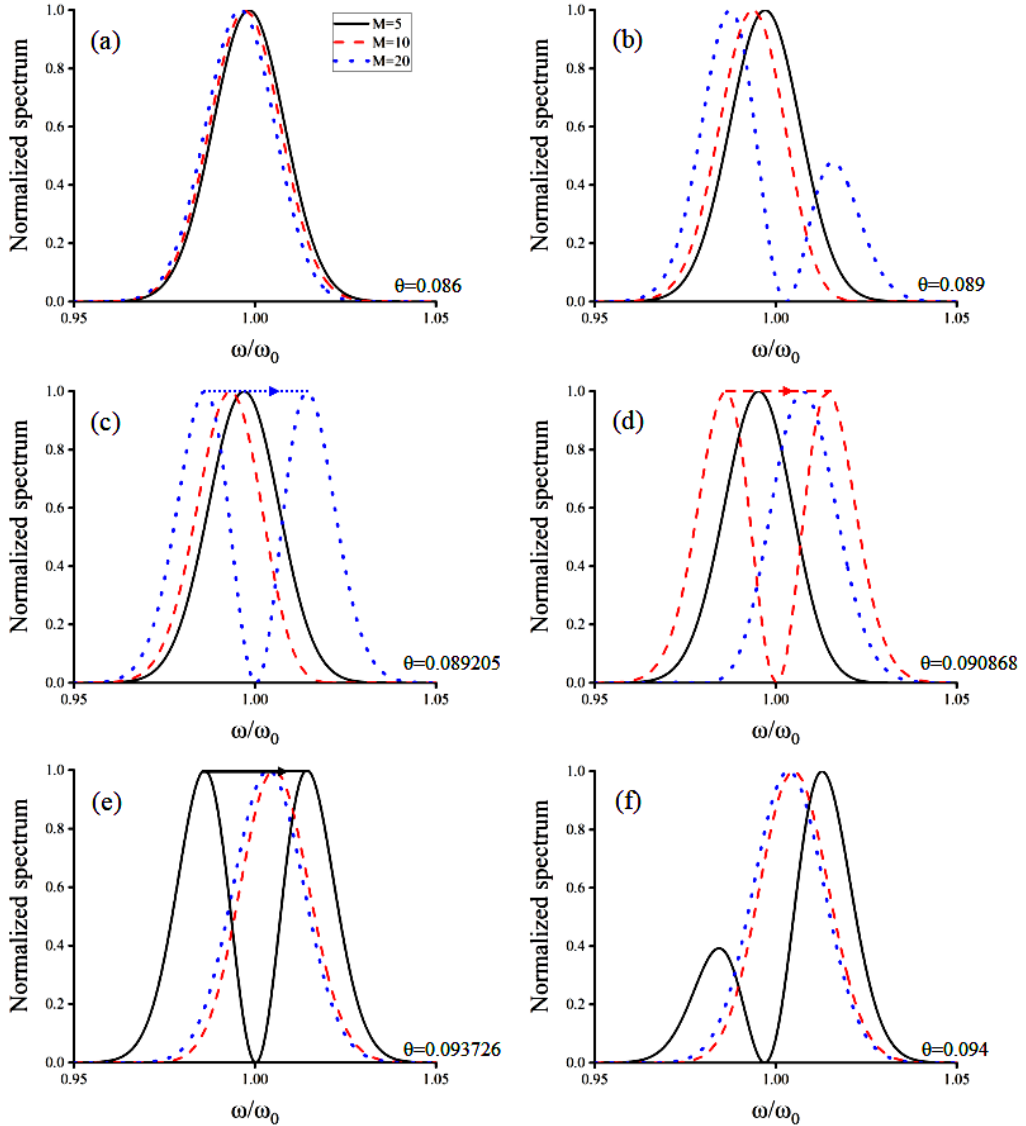


Fig. 4. The influence of the boundary of the scatterer on the spectral switch of the far-zone scattered field. The calculation parameters are selected as follows: $\beta' = 0.23$, $\Gamma'_0 = 0.01$ and $\sigma' = 18$.

to the scatterer with $\beta' = 0.230$ will jump at $\theta = 0.097481$ (see Fig. 3(e)). We summarize the influence of the non-Hermitian parameter on the scattered spectrum by stating that with a larger value of β' of the scatterer, the appearance of the spectral switch will be at a smaller scattering angle as θ increases.

Figure 4 discusses the influence of the scatterer boundary M on the spectral switch of the far-zone scattered field. The solid line, dashed line and dotted line represent the normalized scattered spectrum of the PT-symmetric semi-soft boundary medium with $M = 5$, $M = 10$ and $M = 20$, respectively. As shown in Figs. 4(c)-4(e), the direction of the spectral switch is closely related to the boundary of the scatterer (*i.e.* M). When $M = 20$, the medium has a spectral switch at $\theta = 0.089205$; while for PT-symmetric semi-soft boundary medium with $M = 10$ and $M = 5$, the corresponding scattered spectrum jumps at $\theta = 0.090868$ and $\theta = 0.093726$, respectively. So in general, the larger the value of M is, the earlier the spectral switch occurs as θ increases.

The effects of the effective width of the scatterer on the scattered spectrum have been investigated in Fig. 5. The solid line, dashed line and dotted line represent the normalized scattered spectrum of the PT-symmetric semi-soft boundary medium with $\sigma' = 10$, $\sigma' = 15$ and $\sigma' = 18$, respectively. As shown in Figs. 5(b)-5(d), different σ' will also lead to different critical angles when the spectral switch occurs. It is obvious that

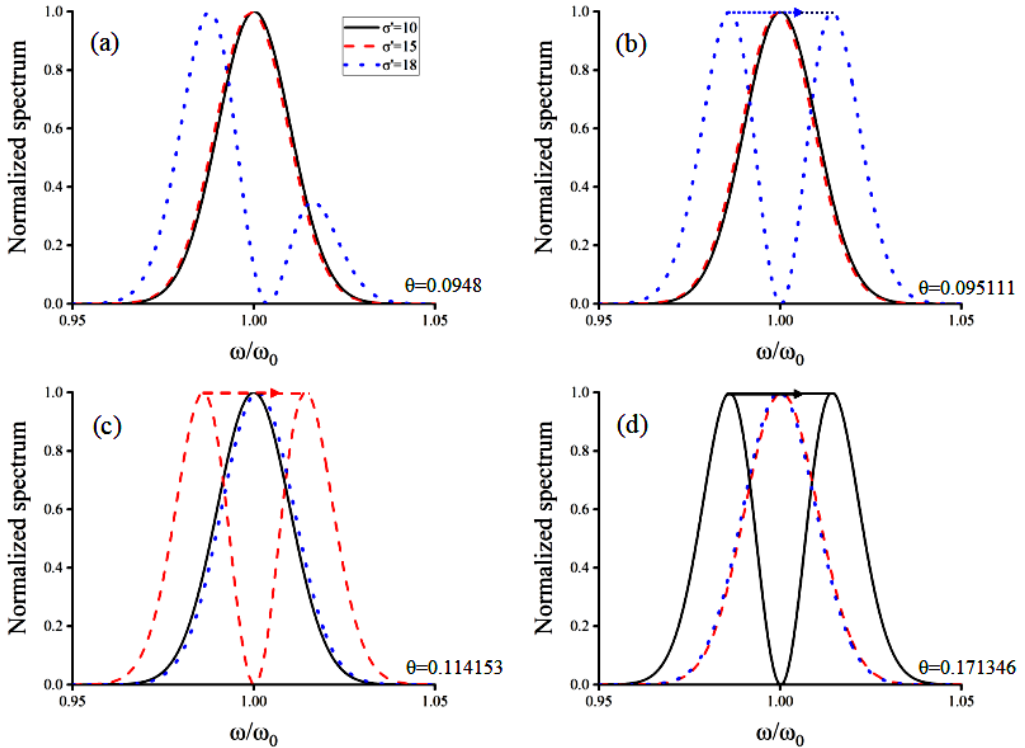


Fig. 5. The influence of the effective width of the scatterer on the spectral switch of the far-zone scattered field. The calculation parameters are selected as follows: $\beta' = 0.23$, $M = 4$, and $\Gamma'_0 = 0.01$.

the critical angle when the spectral switch occurs becomes smaller with the increase of σ' .

4. Conclusion

In this manuscript, the far-zone scattered spectrum of a polychromatic light wave on scattering from a PT-symmetric semi-soft boundary medium is discussed. The analytical expression of the far-zone scattered spectrum is derived, and the influence of the scattering angle and the properties of the scatterer on the scattered spectrum is discussed. The results show that with the increase of the scattering angle, the spectral shifts and spectral switches can be found in the scattered field. The scattering angle where the spectral switch occurs is closely related to the non-Hermitian parameter, the boundary characteristic and the effective width of the medium. This phenomenon further enriches the scattering theory, and may provide some applications in the reconstruction of the structure information of the scattering medium from the measurements of the far-zone scattered spectrum.

Acknowledgement

This work was supported by the National Natural Science Foundation of China (NSFC) under grants 11404231, 12304343.

References

- [1] LAHIRI M., WOLF E., *Spectral changes of stochastic beams scattered on a deterministic medium*, Optics Letters **37**(13), 2012: 2517-2519. <https://doi.org/10.1364/OL.37.002517>
- [2] LI J., CHANG L., *Spectral shifts and spectral switches of light generated by scattering of arbitrary coherent waves from a quasi-homogeneous media*, Optics Express **23**, 2015: 16602-16616. <https://doi.org/10.1364/OE.23.016602>
- [3] DU X., ZHAO D., *Scattering of light by a system of anisotropic particles*, Optics Letters **35**(10), 2010: 1518-1520. <https://doi.org/10.1364/OL.35.001518>
- [4] SAHIN S., GBUR G., KOROTKOVA O., *Scattering of light from particles with semisoft boundaries*, Optics Letters **36**(20), 2011: 3957-3959. <https://doi.org/10.1364/OL.36.003957>
- [5] WANG T., JIANG Z., JI X., ZHAO D., *Spectrum of an electromagnetic light wave on scattering from an anisotropic semisoft boundary medium*, Journal of the Optical Society of America A **33**(4), 2016: 625-629. <https://doi.org/10.1364/JOSAA.33.000625>
- [6] ZHOU J., ZHAO D., *Spectral shifts and spectral switches produced by scattering from a random hollow scatterer with adjustable shell thickness*, Journal of Optics **19**(5), 2017: 055609. <https://doi.org/10.1088/2040-8986/aa6565>
- [7] BRANDÃO P.A., KOROTKOVA O., *Scattering theory for stationary materials with PT symmetry*, Physical Review A **103**, 2021: 013502. <https://doi.org/10.1103/PhysRevA.103.013502>
- [8] KOROTKOVA O., BRANDÃO P.A., *Light scattering from stationary PT-symmetric collections of particles*, Optics Letters **46**(6), 2021: 1417-1420. <https://doi.org/10.1364/OL.418537>
- [9] ZHANG X., LIU Y., CHEN Y., WANG F., CAI Y., *Noncentrosymmetric far-zone spectral density induced by light scattering with random media having parity-time symmetry*, Physical Review A **105**, 2022: 023510. <https://doi.org/10.1103/PhysRevA.105.023510>
- [10] RÜTER C.E., MAKRIKIS K.G., EL-GANAINY R., CHRISTODOULIDES D.N., SEGEV M., KIP D., *Observation of parity-time symmetry in optics*, Nature Physics **6**, 2010: 192-195. <https://doi.org/10.1038/nphys1515>

- [11] BRANDÃO P.A., CAVALCANTI S.B., *Scattering of partially coherent radiation by non-Hermitian localized structures having parity-time symmetry*, Physical Review A **100**, 2019: 043822. <https://doi.org/10.1103/PhysRevA.100.043822>
- [12] BRANDÃO P.A., CAVALCANTI S.B., *Non-Hermitian spectral changes in the scattering of partially coherent radiation by periodic structures*, Optics Letters **44**(17), 2019: 4363-4366. <https://doi.org/10.1364/OL.44.004363>
- [13] PINTO M.A., BRANDÃO P.A., *Asymmetrical splitting in the spectrum of stochastic radiation scattered by non-Hermitian materials having PT symmetry*, Physical Review A **101**, 2020: 053817. <https://doi.org/10.1103/PhysRevA.101.053817>
- [14] ZHANG X., CHEN Y., WANG F., CAI Y., *Scattering of partially coherent vector beams by a deterministic medium having parity-time symmetry*, Photonics **9**(3), 2022: 140. <https://doi.org/10.3390/photonics9030140>
- [15] WOLF E., FOLEY J.T., GORI F., *Frequency shifts of spectral lines produced by scattering from spatially random media*, Journal of the Optical Society of America A **6**(8), 1989: 1142-1149. <https://doi.org/10.1364/JOSAA.6.001142>
- [16] DOGARIU A., WOLF E., *Spectral changes produced by static scattering on a system of particles*, Optics Letters **23**(17), 1998: 1340-1342. <https://doi.org/10.1364/OL.23.001340>
- [17] GAO W., *Spectral changes of the light produced by scattering from tissue*, Optics Letters **35**(6), 2010: 862-864. <https://doi.org/10.1364/OL.35.000862>
- [18] ZHU R., SRIDHARAN S., TANGELLA K., BALLA A., POPESCU G., *Correlation-induced spectral changes in tissues*, Optics Letters **36**(21), 2011: 4209-4211. <https://doi.org/10.1364/OL.36.004209>
- [19] DU X., ZHAO D., *Spectral shifts produced by scattering from rotational quasi-homogeneous anisotropic media*, Optics Letters **36**(24), 2011: 4749-4751. <https://doi.org/10.1364/OL.36.004749>
- [20] DU X., ZHAO D., *Frequency shifts of spectral lines induced by scattering from a rotational anisotropic particle*, Optics Communications **285**(6), 2012: 934-936. <https://doi.org/10.1016/j.optcom.2011.10.073>
- [21] ZHAO D., KOROTKOVA O., WOLF E., *Application of correlation-induced spectral changes to inverse scattering*, Optics Letters **32**(21), 2007: 3483-3485. <https://doi.org/10.1364/OL.32.003483>
- [22] WANG T., LI X., JI X., ZHAO D., *Spectral changes and spectral switches of light waves on scattering from a semisoft boundary medium*, Optics Communications **324**, 2014: 152-156. <https://doi.org/10.1016/j.optcom.2014.03.054>
- [23] WOLF E., *Introduction to the Theory of Coherence and Polarization of Light*, Cambridge University Press, 2007.
- [24] LONGHI S., *Parity-time symmetry meets photonics: A new twist in non-Hermitian optics*, Europhysics Letters **120**, 2017: 64001. <https://doi.org/10.1209/0295-5075/120/64001>
- [25] WOLF E., *Non-cosmological redshifts of spectral lines*, Nature **326**, 1987: 363-365. <https://doi.org/10.1038/326363a0>
- [26] WOLF E., *Red shifts and blue shifts of spectral lines emitted by two correlated sources*, Physical Review Letters **58**, 1987: 2646-2648. <https://doi.org/10.1103/PhysRevLett.58.2646>
- [27] PU J., ZHANG H., NEMOTO S., *Spectral shifts and spectral switches of partially coherent light passing through an aperture*, Optics Communications **162**(1-3), 1999: 57-63. [https://doi.org/10.1016/S0030-4018\(99\)00051-6](https://doi.org/10.1016/S0030-4018(99)00051-6)

Received July 6, 2024
in revised form August 3, 2024