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FISHER EFFECT – CASE OF SLOVAKIA*

1. Theoretical framework

The difference between nominal and real rates of interest was first stressed by Irving Fisher [5] as a part of his theory of fluctuations in investment. The Fisher equation can be derived in the following way:

Let the initial nominal value of capital be C_0, the number of interest periods is n, nominal interest rate with the fixed interest period is i, then the future nominal value of capital after n interest bearing periods in the case of compound interest with an annual accrediting of interests will be

$$C_n = C_0\left(1+i\right)^n . \tag{1}$$

Assuming that the interest bearing period will be equal to one year, we have

$$C_1 = C_0\left(1+i\right). \tag{2}$$

Supposing that while calculating the future value of capital C_1 we will take price level change into account, deflator in the following form will be used:

$$\frac{1}{1+\pi_t}, \tag{3}$$

where π_t is an annual inflation rate (in form of decimal number) in time t.

The future real value of capital (C_{r1}) we can then define as:

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$$C_{r1} = C_0(1+i)\left(\frac{1}{1+\pi_t}\right). \tag{4}$$

At the same time the future value of the capital C_{r1} could be derived from (2), when replacing the nominal interest rate i by real interest rate i_r. Then

$$C_{r1} = C_0(1+i_r). \tag{5}$$

Substituting relations (4) into (5) we get:

$$C_0(1+i)\left(\frac{1}{1+\pi_t}\right) = C_0(1+i_r), \tag{6}$$

and from this:

$$(1+i) = (1+i_r)(1+\pi_t),$$
$$1+i = 1+i_r+\pi_t+i_r\pi_t.$$

Finally we get relation known as the Fisher equation:

$$i = i_r+\pi_t+i_r\pi_t. \tag{7}$$

In the case that due to the low inflation rate and real interest rate the $(i_r\pi_t)$ is negligible, we can omit it and modify the relation (7) as:

$$i = i_r+\pi_t. \tag{8}$$

We can name the relation (8) as Fisher equation in the short form. The relation (8) results in:

$$i_r = i-\pi_t, \tag{9}$$

e.c.
real interest rate equals to nominal interest rate reduced by annual inflation rate.

In the case, we cannot omit $(i_r\pi_t)$ conjunction we take from the relation (6):

$$\frac{1+i}{1+\pi_t}-1 = i_r,$$

from which follows:

$$i_r = \frac{i-\pi_t}{1+\pi_t}. \tag{10}$$

If expectations are rational as in [4], the realised future inflation rate can be written as (see [10]):

$$\pi_t = E_{t-1}(\pi_t) + \varepsilon_t , \tag{11}$$

where $E_{t-1}(\pi_t)$ is the inflation rate from t–1 to t, expected by the bond market at time t–1.

Another authors are of the opinion that if estimate of the inflation $E_{t-1}(\pi_t)^* = \pi_t^e$ is corrected in each period for the difference between real inflation in time t and its expected value in time t–1, the following holds:

$$\pi_t^e = g\pi_t + (1-g)\pi_{t-1}^e , \tag{12}$$

where g is an coefficient of adaptive expectations. When the value of g coefficient is close to 1, the expected inflation from t–1 period is nearly fully transmitted to real inflation in time t and *vice versa*.

From relation (12) results that expected rate of inflation π_t^e is weighted arithmetic average of π_t and expected inflation in the previous period π_{t-1}^e.

In empirical analyses we used coefficient of adaptive expectations $g = 0.8$ as it was presented in similar research of the Czech authors [7]. We took into consideration persistence of inflationary expectations in Slovak conditions in 2000-2005.

Taking previous statements into account relation (9), the real rate can be interpreted as the difference between the nominal rate and expected inflation. For the real rate to be affected only by transitory disturbances, any permanent shocks to expected inflation must be reflected in the nominal interest rate. That implies a one-for-one movement of inflation and interest rate in the long-run.

2. Is there evidence of the Fisher effect in Slovakia?

In empirical studies, if inflation is used as a dependent variable, usually the following equation is estimated:

$$E_{t-1}(\pi_t) = \alpha + \beta i_t + u_t . \tag{13}$$

Replacing expression for $E_{t-1}(\pi_t)$ in the equation (11) the Fisher equation can be written as:

$$\pi_t = \alpha + \beta i_t + \eta_t , \tag{14}$$

where following formula holds for regression residuals: $\eta_t = u_t + \varepsilon_t$.

Fisher hypothesis involves testing that α in equation (14) is a constant and $\beta = 1$. Phillips [12] showed that in least squares regressions with nonstationary regressors, coefficient estimates do not converge in probability, as the sample increases, and the distribution of t-statistics diverges. Optimizing behaviour in asset markets implies that the nominal rate of interest and the expected rate of inflation will move in the same direction. In the long run the nominal rate of interest cannot stay far from expected rate of inflation, and assuming market efficiency, it cannot stay far from the actual rate of inflation.

In case of a long run relationship, ordinary least squares estimates do not have the best features. Using these estimates to generate forecasts would lead to increasing variance. If there is intention to test the long run relationship, we should examine cointegration. Two variables are cointegrated, if they are nonstationary and their linear combination is stationary. If time series of inflation (π_t) and interest rates (i) are nonstationary (one differencing is needed to get stationary time series $\sim I(1)$), then examining the impact of current interest rates on current, or future inflation plays no role. It was proved that changes in macroeconomic policy have an important influence on Fisher effect validity. Therefore it is of great importance to choose the period with stable macroeconomic, especially monetary policy, for testing the relevance of Fisher equation. Because of turbulent political changes in the last decade of 20th century, only data starting from January 2000 are used in this paper. We also use a monthly data on y-o-y inflation, y-o-y core inflation and three months BRIBOR (Bratislava Interbank Offering Rate). The sample takes 66 monthly observations, starting from January 2000, as mentioned.

We tested the null hypothesis of no cointegration (against the alternative of cointegration) using the Engle–Granger two-step procedure. In first step of examining the relationship between inflation and nominal interest rates, we estimate the regression of these two variables (14) using ordinary least squares method. In the second step we test the stationarity of the residuals, making use of ADF test.

Table 1. OLS Estimates of $\pi_t = \alpha + \beta \cdot i_t + u_t$

Dependent Variable: INFL
Method: Least Squares
Date: 08/27/05 Time: 15:03
Sample: 2000:01 2005:06
Included observations: 66

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.201752	1.453376	1.514923	0.1347
BRIBOR	0.774620	0.211278	3.666356	0.0005

R-squared	0.173577	Mean dependent var	7.327273
Adjusted R-squared	0.160664	S.D. dependent var	3.524008
S.E. of regression	3.228530	Akaike info criterion	5.211765
Sum squared resid	667.0978	Schwarz criterion	5.278118
Log likelihood	-169.9882	F-statistic	13.44216
Durbin-Watson stat	0.128454	Prob(F-statistic)	0.000502

2.1. Testing the Fisher effect using least squares method

As a first step in testing of inflation and nominal interest rate relationship we estimated the regression given by equation (14) using OLS (Table 1). Coefficient $\hat{\beta} \doteq 0.775$ is statistically significant, but according to the theory, the value of coefficient should be practically one or close to one.

2.2. Co-integration tests

It was pointed out by Granger and Newbold [6] and Philips [12] that if the variables in a regression contain stochastic trends the results may be spurious. In this case standard inference procedures are inappropriate.

In the case that time series contain unit roots ($\sim I(1)$), the framework for the analysis of nonstationary variables is the cointegration analysis. To test for cointegration two main procedures are used. One is the residual-based Engle and Granger [3] method and the other is Johansen's maximum likelihood approach.

Johansen's test of cointegration is appropriate only when there is evidence of nonstationarity of time series. In the next step we examine whether the variables are nonstationary.

Stationarity of the y-o-y inflation time series (INFL) and interest rates (BRIBOR) we will test by ADF test (Augmented Dickey–Fuller test). Testing statistics of this stationarity test can be written as:

$$\Delta Y_t = \beta_1 + \beta_2 t + \delta Y_{t-1} + \sum_{k=1}^{m} \alpha_k \Delta Y_{t-k} + \varepsilon_t , \tag{15}$$

where: Y_t account for value of testing time series in time t,

ΔY_{t-k} k – period lagged difference,

β_1, β_2 intercept and trend slope parameters.

It holds that time series are nonstationary, if $\delta = 0$. We are testing hypothesis that:

$$\begin{aligned} H_0 &: \delta = 0, \\ H_1 &: \delta < 0. \end{aligned} \tag{16}$$

Consequently, applying ADF test we can select the inclusion of constant and linear trend, or neither. We also have to specify the number of lagged difference terms to be added to the test regression. The usual practice is to include a number of lags sufficient to remove serial correlation in the residuals of the equation (15). Generally, if the value of the test statistics is greater than the critical values, we do not reject the null hypothesis at conventional level of significance.

Table 2. ADF stationarity test: the inflation time series

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(INFL)
Method: Least Squares
Date: 08/27/05 Time: 22:57
Sample(adjusted): 2000:02 2005:06
Included observations: 65 after adjusting endpoints

ADF Test Statistic	-1.636454	1% Critical Value*	-4.1035
		5% Critical Value	-3.4790
		10% Critical Value	-3.1669

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(INFL,2)
Method: Least Squares
Date: 08/27/05 Time: 23:01
Sample(adjusted): 2000:03 2005:06
Included observations: 64 after adjusting endpoints

ADF Test Statistic	-7.017881	1% Critical Value*	-4.1059
		5% Critical Value	-3.4801
		10% Critical Value	-3.1675

*MacKinnon critical values for rejection of hypothesis of a unit root.

Table 2 shows that we cannot reject the null hypothesis of unit root (nonstationarity) of inflation (INFL) time series at all conventional levels of significance. But we reject the null hypothesis of inflation first differences unit root at all conventional levels of significance. We also conclude that the y-o-y inflation time series is nonstationary (INFL ~ I (1)). The form of ADF statistic contained constant and deterministic trend, but no lagged difference terms.

Unit root test results in Table 3 for three months BRIBOR interest rate are similar to previous ones. We cannot reject the null of unit root (nonstationarity) of three months interbank interest rate (BRIBOR) time series at all conventional level of significance. But we reject the null hypothesis of BRIBOR first differences unit root at all conventional level of significance. We again conclude that the nonstationarity (BRIBOR ~ I (1)). Applied ADF test statistic contained constant and deterministic trend term and included no lagged differences. Including the constant and deterministic trend did not influence the results.

Table 3. ADF stationarity test: the interbank interest rates time series

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(BRIBOR)
Method: Least Squares
Date: 08/27/05 Time: 23:40
Sample(adjusted): 2000:02 2005:06
Included observations: 65 after adjusting endpoints

ADF Test Statistic	-1.825451	1% Critical Value*	-4.1035
		5% Critical Value	-3.4790
		10% Critical Value	-3.1669

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(BRIBOR,2)
Method: Least Squares
Date: 08/27/05 Time: 23:42
Sample(adjusted): 2000:03 2005:06
Included observations: 64 after adjusting endpoints

ADF Test Statistic	-5.033527	1% Critical Value*	-4.1059
		5% Critical Value	-3.4801
		10% Critical Value	-3.1675

*MacKinnon critical values for rejection of hypothesis of a unit root.

Most simple way to test long-time relationship of two time series is testing residuals stationarity u_t in the inflation linear regression equation on interest rate (14).

Table 4. ADF stationarity test: the residuals of inflation and interest rates regression

ADF Test Statistic	-2.882822	1% Critical Value*	-2.6000
		5% Critical Value	-1.9457
		10% Critical Value	-1.6185

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(E)
Method: Least Squares
Date: 08/28/05 Time: 00:23
Sample(adjusted): 2000:05 2005:06
Included observations: 62 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
E(-1)	-0.132161	0.045844	-2.882822	0.0055
D(E(-1))	0.195924	0.122206	1.603225	0.1143
D(E(-2))	0.079264	0.124910	0.634571	0.5282
D(E(-3))	0.137479	0.118482	1.160332	0.2507

R-squared	0.151179	Mean dependent var	-0.145539
Adjusted R-squared	0.107275	S.D. dependent var	1.090277
S.E. of regression	1.030139	Akaike info criterion	2.959606
Sum squared resid	61.54885	Schwarz criterion	3.096841
Log likelihood	-87.74779	Durbin-Watson stat	2.010010

The best value of DW statistic (nearest to 2) was reached by inclusion of three lagged differences into the ADF test statistic. This number of lagged differences assures the best residuals (ε_t) distribution with least remaining serial correlation. We did not include the constant and deterministic value in the statistic, because of no significance and small explanation ability. The results effect the rejection of the null of unit root in regression residuals series at all conventional levels of significance. We state the stationarity of regression residuals ($u_t \sim I\,(0)$) and also the cointegration presence between nominal interest rates and inflation. Because of often incorrect null rejection of no cointegration we used additional cointegration tests.

One of the newest cointegration tests is Johansen's test (1995) based on maximal likelihood method. Cointegration LR test statistic has not a usual χ^2 distribution, because the distribution depends on deterministic trend assumption. Therefore, in order to carry out the test, we need to make an assumption regarding the trend underlying the data. We should choose whether none of the series appears to have a trend, or if there is a deterministic trend, stochastic trend, or if some of the series are trend stationary. In the first step we performed the test under each of the mentioned trend assumptions to be able to see how sensitive the results of the test are to the assumption of trend and see the quality of the cointegrating equation. According to AIC, the best results are brought by the linear trend assumption. In the first step we tested the null of no cointegrating relation and if rejected, the hypothesis of one cointegrating relationship was tested.

Table 5. Johansen's test of inflation and interest rates cointegration

Date: 08/28/05 Time: 09:41
Sample(adjusted): 2000:02 2005:06
Included observations: 65 after adjusting endpoints
Trend assumption: Linear deterministic trend (restricted)
Series: INFL BRIBOR
Lags interval (in first differences): No lags

Unrestricted Cointegration Rank Test

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	5 Percent Critical Value	1 Percent Critical Value
None	0.102594	10.97794	25.32	30.45
At most 1	0.058843	3.941912	12.25	16.26

*(**) denotes rejection of the hypothesis at the 5%(1%) level
Trace test indicates no cointegration at both 5% and 1% levels

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	5 Percent Critical Value	1 Percent Critical Value
None	0.102594	7.036032	18.96	23.65
At most 1	0.058843	3.941912	12.25	16.26

*(**) denotes rejection of the hypothesis at the 5%(1%) level
Max-eigenvalue test indicates no cointegration at both 5% and 1% levels

None of the tests based on traced statistics or eigenvalue proved existence of cointegration of inflation and interest rate (Table 5). In the first step we could not reject the null hypothesis about nonexisting cointegration relation by utilising both tests.

Table 6. Coefficients of cointegrating equation

1 Cointegrating Equation(s): Log likelihood -112.7842

Normalized cointegrating coefficients (std.err. in parentheses)

INFL	BRIBOR	@TREND(00:02)
1.000000	4.285802	0.457257
	(2.02134)	(0.19791)

Even if cointegration would be proved, the value of normalized cointegrating coefficient of interest rate equal to 4.28 is in conflict with proposed one-to-one nominal interest rate and inflation relationship.

Generally, it is not specified whether to regress inflation on interest rates or interest rate on inflation. To distinguish between dependent and independent variable we perform the Granger causality test (or precisely two tests).

Table 7. Granger causality test

Pairwise Granger Causality Tests
Date: 08/28/05 Time: 11:11
Sample: 2000:01 2005:06
Lags: 1

Null Hypothesis:	Obs	F-Statistic	Probability
BRIBOR does not Granger Cause INFL	65	0.92365	0.34025
INFL does not Granger Cause BRIBOR		1.27453	0.26327

Granger causality tests in both cases failed to reject the null hypothesis of no dependence between interest rates and inflation. This test procedure confirmed the previous results of no causal relationship between examined time series.

We conclude that even when the Engle–Granger ADF residual test confirmed the cointegration, the value of regression coefficient equal to 0.775 does not confirm the proportional relation, which is proposed by Fisher equation. The problem is that the ADF test [3] belongs into the group of the null non-cointegration hypothesis tests[1]. Monte Carlo simulations showed that these tests relatively often reject the null hypothesis of non-cointegration. This is the reason, why we also utilised Phillips–Perron test [14] which belongs to the group of unit root tests. Phillips–Perron test estimates the single DF test equation, but modifies the statistic so that the serial correlation does not affect the asymptotic distribution of the test statistic. Phillips–Perron test could not reject the null of unit root. Johansen's test (1990) showed no cointegration relation between inflation and interest rates. These findings lead us to reject the Fisher equation validity in Slovak condition in the period from January 2000 till June 2005. Similar results were get using core inflation and net inflation time series instead of inflation measured by consumer prices changes.

Table 8 shows the results of Phillips–Perron residual test of core inflation regression on three months interest rates. We reject the cointegration on 5% and 1% levels of significance, we are not able to do so on 10% level of significance. The results of this test are nondistinct. Therefore we continue with Johansen's test.

[1] The theory distinguishes between a "strong" and a "weak" Fisher effect. If there is a relationship between the interest rate and inflation, but the cointegration vector does not correspond to the strict Fisher equation, the Fisher effect is said to be "weak". Alternatively, if the cointegration vector corresponding to the Fisher equation equals unity, the Fisher effect is said to be "strong".

Table 8. Phillips–Perron cointegration test of core inflation and interest rate

Phillips-Perron Test Equation
Dependent Variable: D(E3)
Method: Least Squares
Date: 08/28/05 Time: 22:24
Sample(adjusted): 2000:02 2005:06
Included observations: 65 after adjusting endpoints

PP Test Statistic	-1.847930	1% Critical Value*	-2.5983
		5% Critical Value	-1.9454
		10% Critical Value	-1.6184

*MacKinnon critical values for rejection of hypothesis of a unit root.

The assumption about trend we set on minimal AIC (Akaike Information Criteria) value base. Minimal value of AIC has in this case the assumption, where the level data have linear trends but the cointegrating equations have only intercepts. As you can see in Table 9, we reject the hypothesis about cointegration between core inflation and interest rates. Removing the nonmarket factors from inflation had also no impact on conclusion about cointegration between interest rates and inflation in Slovak condition.

Table 9. Johansen's cointegration test of core inflation and interest rate

Date: 08/29/05 Time: 00:02
Sample(adjusted): 2000:02 2005:06
Included observations: 65 after adjusting endpoints
Trend assumption: Linear deterministic trend (restricted)
Series: CORINFL BRIBOR
Lags interval (in first differences): No lags

Unrestricted Cointegration Rank Test

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	5 Percent Critical Value	1 Percent Critical Value
None	0.123698	10.67248	25.32	30.45
At most 1	0.031636	2.089588	12.25	16.26

*(**) denotes rejection of the hypothesis at the 5%(1%) level
Trace test indicates no cointegration at both 5% and 1% levels

3. Conclusion

The paper is an attempt to find empirical evidence for the Fisher effect in the Slovak conditions. The Fisher hypothesis says that a 1% increase in inflation will be accompanied by a 1% increase in interest rates. In other words, the nominal interest rate will be the sum of real interest rate and inflation rate. In the first part of contribution the Fisher equation was derived. In the second part the cointegration was tested by applying two main procedures that are widely used: Engle and Granger's residual based ADF method and Johansen's maximum likelihood approach. Although ADF test proved cointegration between interest rate and inflation, the value of regression coefficient 0.775 is not consistent with the theory (it should be 1). Consequently we have applied Phillips–Perron test which estimates the nonaugmented DF test equation, and modifies the t-ratio of the δ coefficient so that serial correlation does not affect the asymptotic distribution of the test statistic. Johansen's test also proved the previous results of no cointegration between nominal interest rates and inflation (we have examined both the inflation measured by CPI and core inflation). These results were expected, because of negative real interest rate presence during analysed period.

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EFEKT FISHERA - PRZYPADEK SŁOWACJI

Streszczenie

Efekt Fishera jest kamieniem węgielnym wielu modeli teoretycznych i jest ważny dla rozumienia ruchów nominalnych stóp procentowych. Hipoteza Fishera mówi, że rzeczywista stopa procentowa jest stała. Zatem nominalna stopa przesuwa się wraz z inflacją. Rzeczywiste stopy procentowe działają na wszystkie decyzje co do oszczędności i inwestycji w gospodarce. Z tego powodu zachowanie rzeczywistej stopy procentowej jest centralnym zagadnieniem w badaniu rynków finansowych. Artykuł składa się z dwu części. W pierwszej jest opisany aspekt teoretyczny i metodologia efektu Fishera. Druga część artykułu zawiera analizy empiryczne zastosowane do warunków słowackich.

Słowa kluczowe: nominalna stopa procentowa, rzeczywista stopa procentowa, stopa inflacji, równanie Fishera, metoda najmniejszych kwadratów, testy kointegracji, test stacjonarności ADF (Augmented Dickey-Fuller), test Johansena, test przyczynowości Grangera, test jednego pierwiastka Phillipsa–Perrona, metoda największej wiarygodności.