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Agnieszka Marciniuk
Wrocław University of Economic, Poland

PRICE OF ZERO-COUPON BONDS IN LIFE INSURANCE

1. Introduction

In traditional actuarial literature a constant rate of interest i to actuarial calculations is used. Due to it the discounting factor v is also constant because $v = (1 + i)^{-1}$. In a new actuarial literature a financial stochastic discounting factor is applied, that is a price of zero-coupon bonds. In many actuarial definitions there is used the expected value of different random or stochastic variables dependent on the discounting factor. When the discounting factor is stochastic it's the expected value must be used. This expected value is just price of zero-coupon bond. In the article it is applied just this way.

There is described in the paper two discrete stochastic models of interest rate used by Bühlmann. Some examples of price of zero-coupon bonds are given, and applied to calculation of premiums and reserves and insurer's losses (gains).

2. Price of zero-coupon bond

Zero-coupon bonds are securities paying to their holders one unit at a date m in the future [10; 11]. The holder of these bonds does not receive interest. The profit is equal to difference between nominal price and selling price. Therefore zero-coupon bonds are also called discount bonds.

Let v_t (or $v(0,t)$) denotes the stochastic discounting factor from time 0 to $t > 0$. This stochastic process is adapted to the history (σ-algebra) $\mathcal{F}_t$. This σ-algebra has increasing sequence that is

$$\mathcal{F}_0 \subset \mathcal{F}_1 \subset \ldots \subset \mathcal{F}_{t-1} \subset \mathcal{F}_t \subset \ldots \subset \mathcal{F}_n.$$

The price of zero-coupon riskless bond at time t ($t \geqslant 0$) with a maturity m ($m \geqslant 0$) is denoted by $P(t,m)$. It is obvious that $P(t,t) = 1$. The price of zero-coupon bond is given by the following equation [2; 5]:

$$P(t,m) = E\left[v(t,m) \middle| \mathcal{F}_t\right], \tag{1}$$

where $v(t,m)$ means the discounting factor m back to t.

If the interest rate is defined as a continuous process then the discounting factor is as follows:

$$v(t,m) = \exp\left(-\int_t^m r_s ds\right),$$

where r_t denotes the instantaneous short rate at time $t \geqslant 0$.

Denote the discounting factor from time $j-1$ to j by symbol Y_j. Then the structure is discrete and the v_t is given by the following form:

$$v_t = \prod_{j=1}^{t} Y_j$$

and $v_0 = 1$.

The value of zero-coupon bond at time $k < m$ is as follows:

$$P(t,m) = E\left(Y_{t+1} \cdot \ldots \cdot Y_m \middle| \mathcal{F}_t\right).$$

The recursion form is the following:

$$P(t-1,m) = E\left(Y_t \cdot P(t,m) \middle| \mathcal{F}_{t-1}\right).$$

Obviously $P(0,m)$ is nominal price of zero-coupon bond. It must have

$$P(t,m) = \frac{P(0,m)}{P(0,t)}.$$

It can be seen from the fact that the payment of $P(0,t) \cdot P(t,m)$ and of $P(0,m)$ at time 0 both give one unit at time m.

3. Models of the discrete discounting factor

The modelling of the discounting factor consists in the construction of a probability distribution of Y_t. Two models of the discounting factor are described. Both of them are from Bühlmann. Next the price of zero-coupon bond at time $t \geqslant 0$ is derived.

3.1. Uncertainty of interest rates – model I

The first model of discounting factor has the following form [4; 8]:

$$Y_t = \varepsilon \cdot (1 - Z_t) + \delta \cdot Z_t = \varepsilon + (\delta - \varepsilon) \cdot Z_t, \tag{2}$$

where $0 < \varepsilon < \delta \leqslant 1$ and Z_t are stochastic weights. The Y_t are weighted averages of ε and δ. In the article only the binary model is considered. To derive the probability distribution of Y_t a pure Bayes construction is used. This construction is the following:

1) for all $t = 1, 2, \ldots, n$ the Z_t are identically independent random variables with binomial distribution with parameters 1 and p, i.e. $Z_t \sim B(1, p)$,

2) p has a beta distribution with parameters α and β, i.e.

$$f_{\alpha,\beta}(p) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\cdot\Gamma(\beta)} p^{\alpha-1}(1-p)^{\beta-1}, \; 0 \leqslant p \leqslant 1.$$

Under the above assumptions the price of zero-coupon bond at time 0 is derived. The price has the following form [4]:

$$P_{\alpha,\beta}(0,m) = E(Y_1 \cdot Y_2 \cdot \ldots \cdot Y_m | \mathcal{F}_0) = E[(\varepsilon + (\delta - \varepsilon)p)^m] = \sum_{t=0}^{m} \binom{m}{t} \cdot \varepsilon^t \cdot (\delta - \varepsilon)^{m-t} \frac{\alpha^{[m-t]}}{(\alpha+\beta)^{[m-t]}}, \tag{3}$$

where $\alpha^{[j]} = \alpha \cdot (\alpha+1) \cdot \ldots \cdot (\alpha + j - 1) = (\alpha + j - 1)!/(\alpha - 1)!$.

The price of zero-coupon bond at time $t > 0$ is as follows:

$$P_{\alpha,\beta}(t,m) = E(Y_{t+1} \cdot Y_{t+2} \cdot \ldots \cdot Y_m | \mathcal{F}_t) = P_{\alpha_t,\beta_t}(0, m-t), \tag{4}$$

$$\alpha_t = \alpha_{t-1} + Z_t, \; \alpha_0 = \alpha,$$

$$\beta_t = \beta_{t-1} + (1 - Z_t), \; \beta_0 = \beta.$$

We can see that the point (α_t, β_t) is stochastic. It is also noticeable that all posterior distributions of p are again beta distributions. In this case the form of price of zero-coupon bond is explicit for all t.

3.2. Ehrenfest model – model II

The second model is called the Ehrenfest model. In two urns there are $2s$ balls. We pick one ball at random and shift it to the other urn. In first urn is y balls. The homogeneous Markov transition probabilities is as follows [7]:

$$p_{y,y+1} = 1 - \frac{y}{2s},$$

$$p_{y,y-1} = \frac{y}{2s}.$$

If the step of the movement is $\pm 1/M$ then the Markov transition probabilities are calculated by the use of the following form:

$$p_{y,y\pm\frac{1}{M}} = 0{,}5 \pm a(b-y), \tag{5}$$

$$y_{\max} = b + \frac{1}{2a},$$

$$y_{\min} = b - \frac{1}{2a}.$$

This model is called generalized Ehrenfest model.
Using that model leads to the following recursion formula [4]:

$$P_y(t,m) = E\left(Y_{t+1} \cdot P_{Y_{t+1}}(t+1,m) \middle| Y_t = y\right)$$

which becomes by homogeneity

$$P_y(0,m-t) = E\left(Y_1 \cdot P_{Y_1}(0,m-t-1) \middle| Y_0 = y\right).$$

From (5) we have

$$P_y(0,m) = \left(y + \frac{1}{M}\right) \cdot P_{y+\frac{1}{M}}(0,m) \cdot (0{,}5 + a(b-y)) + \left(y - \frac{1}{M}\right) \cdot P_{y-\frac{1}{M}}(0,m) \cdot (0{,}5 - a(b-y)). \tag{6}$$

This model is analogous to the term structures obtained in the continuous case from the Cox-Ingersoll-Ross model [10; 12].

4. Stochastic payment streams in insurances

4.1. Valuation of payment streams at time *t*

In life insurance there are defined different streams of cash payments. One of them could be a loss or a gain of insurer. Let us define stochastic net cash payments:

$$X_t = \text{benefits in } (t-1, t] - \text{premiums in } t,\ t = 1, 2, \ldots$$

and

$$X_0 = -\text{ premiums in } 0.$$

These payments are defined at the end of interval $(t-1, t]$. If the insurer's costs are added to net payments, the gross cash payments will be defined.

Assume that the time horizon is finite, i.e. $t = 0, 1, \ldots, n$. Even if the insurance policy is whole life, the insurer limits the time horizon, e.g. to 100 or 125 years. Then n could be equal to e.g. 100.

Now we can define the cash payment vector

$$X = (X_0, X_1, \ldots, X_n)$$

and the stochastic discounting vector

$$v = (v_0, v_1, \ldots, v_n).$$

The valuation of payment at time t is the expected discounted present value of cash payment stream. When $t = 0$ the valuation is the following:

$$Q(X) = E\left(\sum_{t=0}^{n} v_t \cdot X_t\right). \tag{7}$$

Of course the calculation has sense if the equivalence principle is kept. Then $Q(X) = 0$. This equation allows calculating the premium.

The valuation at time $t > 0$ is denoted as $Q(X|\mathcal{F}_t)$. At time zero this valuation is a random variable, but at time $t > 0$ $Q(X|\mathcal{F}_t)$ is observable. The $Q(X|\mathcal{F}_t)$ is defined as follows [4]:

$$Q(X|\mathcal{F}_t) = \frac{1}{v_t} E\left(\sum_{k=0}^{n} v_k \cdot X_k|\mathcal{F}_t\right). \tag{8}$$

The $Q(X|\mathcal{F}_t)$ could be described in another way:

$$Q(X|\mathcal{F}_t) = E\left(\sum_{k=0}^{n} \frac{v_k}{v_t} \cdot X_k|\mathcal{F}_t\right) = \sum_{k=0}^{t} \frac{v_k}{v_t} \cdot X_k + E\left(\sum_{k=t+1}^{n} \frac{v_k}{v_t} \cdot X_k|\mathcal{F}_t\right) \tag{9}$$

or

$$Q(X|\mathcal{F}_t) = \sum_{k=0}^{t} \frac{1}{Y_{k+1} \cdot \ldots \cdot Y_t} \cdot X_k + E\left(\sum_{k=t+1}^{n} Y_{t+1} \cdot \ldots \cdot Y_k \cdot X_k|\mathcal{F}_t\right), \tag{10}$$

where

$$v_t = \prod_{j=1}^{t} Y_j .$$

The first component of the above sums means the accumulated payments and the second one is the prospective reserve. The components are denoted by $A(X|\mathcal{F}_t)$ and $R(X|\mathcal{F}_t)$ adequately.

4.2. Decomposition of annual losses of insurer

The function $Q(X|\mathcal{F}_t)$ could be treated as accumulated loss (or gain) of insurer with interest. Consider the annual loss of insurer $L_t(X)$ in interval $(t-1, t]$ that is discounted to time $t-1$. This loss has the following form [3]:

$$L_t(X) = Y_t \cdot Q(X|\mathcal{F}_t) - Q(X|\mathcal{F}_{t-1}).$$

One can notice that

$$\sum_{k=1}^{n} v_{k-1} \cdot L_k(X) = v_n \cdot Q(X|\mathcal{F}_n) = E\left(\sum_{j=0}^{n} v_j \cdot X_j | \mathcal{F}_n \right).$$

The right-hand side is a martingale for any filtration.

The way of decomposition of annual losses is made in Bühlmann's paper. Here it is not quoted. Technical loss is defined by using the following form:

$$(LT)_t = Y_t(X_t + R(X|\mathcal{F}_t)) - Y_t R^+(X|\mathcal{G}_t), \tag{11}$$

where

$$R(X|\mathcal{F}_t) = \frac{1}{v_t} E\left(\sum_{k=t+1}^{n} v_k \cdot X_k | \mathcal{F}_t \right) \tag{12}$$

and

$$R^+(X|\mathcal{G}_t) = \frac{1}{v_t} E\left(\sum_{k=t}^{n} v_k \cdot X_k | \mathcal{G}_t \right). \tag{13}$$

This σ-algebra $\mathcal{G}_t$ expresses the fact that at time t the X-variables are only known up to and including $t-1$ whereas all v-variables are known up to and including t. The following filtration sequence is established:

$$\mathcal{F}_0 \subset \mathcal{G}_1 \subset \mathcal{F}_1 \subset \mathcal{G}_2 \subset \mathcal{F}_2 \subset \ldots$$

and

$$\mathcal{F}_t = \sigma(X_0, X_1, \ldots, X_t; v_0, v_1, \ldots, v_t),$$
$$\mathcal{G}_t = \sigma(X_0, X_1, \ldots, X_{t-1}; v_0, v_1, \ldots, v_t).$$

Financial loss has the following form [4]:

$$(LF)_t = Y_t \cdot R^+(X|\mathcal{G}_t) - R(X|\mathcal{F}_{t-1}). \tag{14}$$

Total loss is a sum of technical and financial losses, that is

$$L_t = (LT)_t + (LF)_t. \tag{15}$$

5. Applications of price of zero-coupon bonds in insurance

5.1. Term life insurance and assumptions

Assume the following interpretation of variables X and v. Let v be the financial variables and X the insurance variables. The vector (X, v) is a pair of two vectors. In our consideration it is assumed that X and v are independent variables. The mortality tables for X are used. They are Polish Life Tables 2000 from Ostasiewicz [9].

The calculations are made for n-years life insurance. Premiums P are due payable. Benefit in the amount of one unit is payable at the end of year of the insured's death, if this occurs during the first n years, otherwise the benefit would not pay. Insured is x years old at the moment of buying the insurance policy. The insurer's loss at the end of t-th year is as follows [3]:

$$X_0 = -P,$$

$$X_t = \begin{cases} -P & \text{if insured is alive at time } t, \\ 1 & \text{if insured is dead at time } t. \end{cases}$$

$$X_n = \begin{cases} 0 & \text{if insured is alive at time } n, \\ 1 & \text{if insured is dead at time } n. \end{cases}$$

It is assumed that

$$X_t^{(m)} = E(X_t|\mathcal{F}_m). \tag{16}$$

If $m \geqslant t$ then $X_t^{(m)} = X_t$.

Suppose that insured is a 28-years-old woman. Policy is bought for 5 years. Probability that a person at age x will be alive at least t years is denoted by ${}_t p_x$.

For $t = 1$ there is set ${}_1p_x = p_x$. From Polish Life Tables for women the probabilities are as follows:

$$\begin{aligned} p_{28} &= 0.99960, & q_{28} &= 1 - p_{28} = 0.00040, \\ p_{29} &= 0.99958, & q_{29} &= 1 - p_{29} = 0.00042, \\ p_{30} &= 0.99954, & q_{30} &= 1 - p_{30} = 0.00046, \\ p_{31} &= 0.99951, & q_{31} &= 1 - p_{31} = 0.00049, \\ p_{32} &= 0.99947, & q_{32} &= 1 - p_{32} = 0.00053. \end{aligned} \tag{17}$$

The calculations are made in following parts for two models for modeling of price of zero-coupon bond.

5.2. Model I applications

First price of zero-coupon bonds is calculated by using model I. The following parameters are used:

$$\varepsilon = 0.9, \qquad \delta = 1, \qquad \alpha = 3, \qquad \beta = 1.$$

Model of Y_t has the following form:

$$Y_t = 0.9 + 0.1 \cdot Z_t,$$

where $Z_t \sim B(1, p)$, $p \sim Beta(\alpha, \beta)$. Therefore Y_t takes values 0.9 or 1.
In Table 1 values of zero-coupon bond at time $t = 0$ are presented with different maturity of m ($m = 1, 2, \ldots, 20$). Formula (3) for $\alpha = 3$ and $\beta = 1$ was used.

Table 1. Price of zero-coupon bond $P_{\alpha,\beta}(0, m)$ – model I

m	$P_{\alpha,\beta}(0, m)$	m	$P_{\alpha,\beta}(0, m)$	m	$P_{\alpha,\beta}(0, m)$	m	$P_{\alpha,\beta}(0, m)$
1	0.97500	6	0.86404	11	0.77262	16	0.69651
2	0.95100	7	0.84435	12	0.75628	17	0.68283
3	0.92795	8	0.82539	13	0.74053	18	0.66961
4	0.90580	9	0.80714	14	0.72534	19	0.65683
5	0.88451	10	0.78956	15	0.71067	20	0.64448

Graph 1 shows the values of price of zero-coupon bond $P_{\alpha,\beta}(0, m)$ for $m = 1, 2, \ldots, 20$ (from Table 1).

If we use relationship (4) we get price of zero-coupon bond $P_{\alpha,\beta}(t, m)$. On Fig. 2 there are presented 20 such trajectories for $m = 20$.

Received values are applied in the following insurance calculations. First we calculate premium. In general case we have

$$Q(X) \overset{(7)}{=} E\left(\sum_{t=0}^{n} v_t \cdot X_t\right) = \sum_{t=0}^{n} P_{\alpha,\beta}(0,t) \cdot E\left(X_t \middle| \mathcal{F}_0\right). \tag{18}$$

Second equality results from independence of variables X_t and v_t [1; 6].

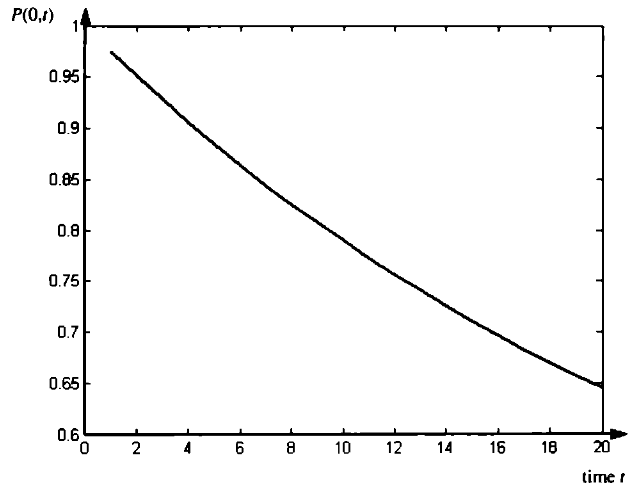

Fig. 1. Price of zero-coupon bond $P_{\alpha,\beta}(0, t)$ – model I

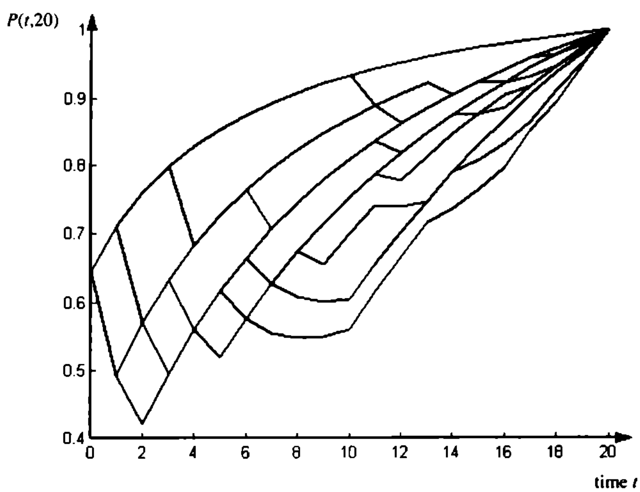

Fig. 2. 20 trajectories of price of zero-coupon bond $P(t, 20)$

From equivalence principle $Q(X)$ is equally zero. Hence

$$\sum_{t=0}^{5} P_{\alpha,\beta}(0,t)\cdot X_t^{(0)} = 0 .$$

The values of $P_{\alpha,\beta}(0,t)$ are taken from Table 1. $X_t^{(0)}$ is calculated by using (16). We get the following equation

$$-P + P_{\alpha,\beta}(0.1)\cdot(q_{28} - P\cdot p_{28}) + P_{\alpha,\beta}(0.2)\cdot(q_{29} - P\cdot p_{29})\cdot p_{28} +$$
$$+ P_{\alpha,\beta}(0.3)(q_{30} - P\cdot p_{30})\cdot p_{28}\cdot p_{29} + P_{\alpha,\beta}(0.4)\cdot(q_{31} - P\cdot p_{31})\cdot p_{28}\cdot p_{29}\cdot p_{30} +$$
$$+ P_{\alpha,\beta}(0.5)\cdot q_{32}\cdot p_{28}\cdot p_{29}\cdot p_{30}\cdot p_{31} = 0.$$

Hence

$$P = 0.00045 .$$

Now we calculate both of prospective reserves at time t, i.e. $R(X|\mathcal{F}_t)$ (without payment $t-1$) and $R^+(X|\mathcal{G}_t)$ (with the payment at time $t-1$). These reserves are calculated by the use of formulas (12) and (13). Remember also that

$$P_{\alpha,\beta}(t,m) = E(Y_{t+1}\cdot Y_{t+2}\cdot\ldots\cdot Y_m|\mathcal{F}_t) = P_{\alpha_t,\beta_t}(0,m-t)$$

where α_t and β_t are given by condition (4). The results are presented in Table 2.

Table 2. Prospective reserves – model I

$R(X\|\mathcal{F}_0)$	0.00045	$R^+(X\|\mathcal{G}_1)$	0.00047
$R(X\|\mathcal{F}_1)$	0.00049	$R^+(X\|\mathcal{G}_2)$	0.00053
$R(X\|\mathcal{F}_2)$	0.00056	$R^+(X\|\mathcal{G}_3)$	0.00057
$R(X\|\mathcal{F}_3)$	0.00056	$R^+(X\|\mathcal{G}_4)$	0.00057
$R(X\|\mathcal{F}_4)$	0.00052	$R^+(X\|\mathcal{G}_5)$	0.00053

Table 3. Insurance losses – model I

Technical loss		Financial loss		Total loss	
$(LT)_1$	–0.00042	$(LF)_1$	0.00002	L_1	–0.00040
$(LT)_2$	–0.00042	$(LF)_2$	0.00004	L_2	–0.00038
$(LT)_3$	–0.00046	$(LF)_3$	0.00001	L_3	–0.00045
$(LT)_4$	–0.00049	$(LF)_4$	0.00001	L_4	–0.00048
$(LT)_5$	–0.00053	$(LF)_5$	0.00001	L_5	–0.00052

Table 3 includes results of technical, financial and total losses. They are calculated by using formulas (11), (14) and (15) adequately. These results are got under the assumption that $Y_1 = Y_2 = Y_3 = Y_4 = Y_5 = 1$.

The minus means a gain of insurer. Notice that average gain is equally 0.00045. It is exactly the same as premium. The financial loss is very small in this case and it does not balance the technical gain. All results are received under the assumption that the woman is still alive.

5.3. Model II applications

For the second model the following parameters are used:

$$y_{\min} = 0.9, \qquad y_{\max} = 1, \qquad M = 20, \qquad a = 10, \qquad b = 0.95.$$

The parameters are selected in such a way that the discounting factor takes the values from the interval [0.9; 1]. The values are changing by 0.01.

The calculations of price of zero-coupon bond $P_y(0, m)$ are made by the use of formula (6). In Table 4 there are presented $P_y(0, m)$ for different maturity of m = 1, 2, …,10 and all y.

Table 4. Price of zero-coupon bond $P_y(0, m)$ – model II

	y										
	0.9	**0.91**	**0.92**	**0.93**	**0.94**	**0.95**	**0.96**	**0.97**	**0.98**	**0.99**	**1**
$P_y(0, 0)$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$P_y(0, 1)$	0.910	0.918	0.926	0.934	0.942	0.950	0.958	0.966	0.974	0.982	0.990
$P_y(0, 2)$	0.835	0.849	0.862	0.875	0.889	0.903	0.916	0.930	0.944	0.958	0.972
$P_y(0, 3)$	0.772	0.789	0.806	0.823	0.840	0.858	0.875	0.893	0.911	0.930	0.948
$P_y(0, 4)$	0.718	0.737	0.756	0.775	0.795	0.815	0.835	0.856	0.877	0.899	0.921
$P_y(0, 5)$	0.670	0.690	0.711	0.732	0.753	0.775	0.797	0.819	0.842	0.866	0.890
$P_y(0, 6)$	0.628	0.649	0.670	0.692	0.714	0.736	0.759	0.783	0.807	0.832	0.857
$P_y(0, 7)$	0.590	0.611	0.633	0.655	0.677	0.700	0.724	0.748	0.772	0.798	0.824
$P_y(0, 8)$	0.556	0.577	0.598	0.620	0.642	0.665	0.689	0.713	0.738	0.764	0.790
$P_y(0, 9)$	0.525	0.545	0.566	0.588	0.610	0.633	0.656	0.680	0.705	0.730	0.756
$P_y(0, 10)$	0.496	0.516	0.537	0.558	0.599	0.602	0.625	0.648	0.672	0.697	0.723

Notice that if y is larger than the price of zero-coupon bond is bigger. For example $P_1(0, 10)$ is higher by 0.227 than $P_{0.9}(0, 10)$.

In Fig. 3 the received values of $P_y(0, m)$ are shown. Notice that the function of price could be concave as well as convex. The function of price of zero-coupon bond for first model is only concave.

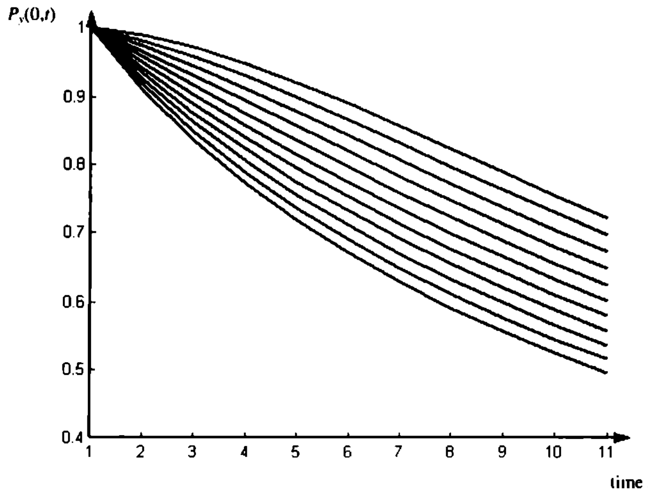

Fig. 3. Price of zero-coupon bond $P_y(0, t)$ – model II

We used this model to calculate only the premium. The received premium was the same as before. In the equation (18) $P_{\alpha,\beta}(0,t)$ was replaced with $P_y(0, t)$. The premium is calculated for all y. The results are in Table 5.

Table 5. Term life insurance's premium for different y – model II

y	P	y	P	y	P
0.90	0.000442	0.94	0.000453	0.98	0.000463
0.91	0.000445	0.95	0.000455	0.99	0.000465
0.92	0.000448	0.96	0.000458	1.00	0.000468
0.93	0.000450	0.97	0.000460		

The values of premiums are similar. That is also for model I. However, premium for most y is higher. Exactly value of premium for first model is 0.0004472. So it is smaller for all smaller y than 0.92.

6. Summary

In the article two models of price of zero-coupon bond are described. Price of zero-coupon bond is applied as the financial discounting factor in life insurance. The calculations show that the price for both models is different. Therefore the term life insurance's premiums are different but similar. Premium in the first model is smaller than the premium in the second model almost for all y. The structure of models of price of zero-coupon bond is different. The function of price could be concave as well as convex for second model. That is like the structure obtained in the continuous case from the Cox-Ingersoll-Ross model. The price in the first model is only concave.

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CENA OBLIGACJI ZEROKUPONOWYCH W UBEZPIECZENIACH ŻYCIOWYCH

Streszczenie

W artykule zastosowano finansowy stochastyczny czynnik dyskontujący do obliczeń aktuarialnych. W wielu aktuarialnych definicjach do obliczeń stosowana jest wartość oczekiwana różnych losowych lub stochastycznych zmiennych, które zależą od czynnika dyskontującego. Jeżeli dodatkowo czynnik dyskontujący jest stochastyczny, to do obliczeń musi być użyta jego wartość oczekiwana. Okazuje się, że ta wartość oczekiwana jest właśnie ceną obligacji zerokuponowej.

Przedstawione zostały 2 dyskretne modele stochastycznej stopy procentowej, zastosowane przez Bühlmanna, które zostały wykorzystane do wyznaczenia ceny obligacji zerokuponowych. Następnie cena obligacji zerokuponowych została zastosowana do obliczeń składek, rezerw oraz straty (zysku) ubezpieczyciela.

Słowa kluczowe: proces stochastyczny, stopa procentowa, czynnik dyskontujący, obligacja zerokuponowa, ubezpieczenie życiowe.