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Jaroslav Sixta

University of Economics, Prague, Czech Republic

THE ESTIMATES OF CONSUMPTION OF FIXED CAPITAL

1. Foreword

Although consumption of fixed capital (CFC) is an important indicator of national accounts, its sense and importance is not generally very well known. Economists know gross domestic product (GDP), gross national income (GNI) and they also know net domestic product and net national income. Then should appear an important question, what we deduct from gross aggregates to obtain net aggregates. There are presented here some relevant facts about consumption of fixed capital and moreover about capital stocks because the estimates of CFC can be hardly done without capital stocks. It is presented here also a short illustrative example of calculation of CFC to show the core of computation.

2. Consumption of fixed capital in national accounts

Statistical offices of member states of the EU have to keep the rules given by the European System of Accounts (ESA 1995) – Council Regulation (EC) No 2223/96. Among other items, ESA 1995 defines consumption of fixed capital as:

Consumption of fixed capital (K.1) represents the amount of fixed assets used up, during the period under consideration, as a result of normal wear and tear and foreseeable obsolescence, including a provision for losses of fixed assets as a result of accidental damage which can be insured against [1].

This definition is clear but there are some other obstacles. According to ESA 1995, CFC and capital stocks must be valued at current prices and CFC must respect the "probable average economic life of the different categories of those goods" [1]. These two assumptions represent the most important differences between depreciation (as it is recorded in bookkeeping) and CFC in national ac-

counts. CFC should be calculated on the basis of capital stocks by straight-line method (sometimes geometric method is allowed). These requirements lead statistical offices into unpleasant situation because it is very difficult to acquire capital stocks at current prices from companies. That is why ESA 1995 recommends some alternative approaches to the estimates of CFC and stocks. There are several possible ways but mostly recommended is perpetual inventory method (PIM) and I would like to illustrate it in paragraph 3.1.

Consumption of fixed capital plays an important role in the sequence of national accounts. It is recorded directly on the production account and on the capital account. But in some cases it is also a part of output and final consumption because it is used for so called cost method. It means that for other non-market producers (e.g. government units) the output is estimated by summing the costs (intermediate consumption, compensation of employees, consumption of fixed capital and other taxes on production). Moreover it is a part of output in the sector of households because it is included in imputed rent. The relevance of this indicator becomes clear when we consider that the share of CFC on GDP is about 19.5% and the share of CFC that is included in output is about 6.7% in the Czech Republic in 2003.

3. Estimates of consumption of fixed capital

There are several methods for estimating capital stocks and CFC but I would like to describe here very popular standard approach to perpetual inventory method (PIM). For example PIM is used in the Czech Republic (also price-quantity method is used), France, the Netherlands and in some other countries.

At first, it is necessary to acquire following data for PIM:

- long time series of gross fixed capital formation (GCF),
- suitable price indices for different types of assets,
- service lives.

It is also necessary to acquire information on:

- mortality pattern (e.g. linear, bell-shaped, simultaneous exit, etc.),
- the way of depreciating (straight-line or geometric).

The most difficult issue is data availability; it is especially in the case of countries, which has adopted ESA 1995 recently. The estimates of CFC and capital stocks are based on time series of gross fixed capital formation (GCF). GCF is also defined by ESA 1995 and the previous system of recording investment was different. This lays excessive demands on national statistical offices. Other problems can be rather easily solved. There are several possible ways such as discussion with experts, producers, users, etc. But my target is to show some practical computations and there is no scope for detailed descriptions of these problems and possible solutions because it would take enough space.

The following formulas for computations are described in [2]:

(1) gross capital stock $$GCS_t = \sum_{i=1}^{\infty} GCF_{t-i} \sum_{j=i+1}^{\infty} p_j,$$

(2) consumption of fixed capital $$D_t = \sum_{i=1}^{\infty} GCF_{t-i} \sum_{j=i}^{\infty} \frac{p_i}{j},$$

(3) net capital stock $$NCS_t = \sum_{i=1}^{\infty} GCF_{t-i} \sum_{j=i+1}^{\infty} \frac{j-i}{j} p_j,$$

where p_j are the probabilities of retirement, t refers to the year of calculation and i, j are indices which practically range from 1 to $2L$ (L – average service life). All computations have to be done in constant prices.

There is illustrated here a short example of PIM that is closely related to real situation. Suppose vehicles in an industry of banks (NACE 65 – Financial intermediation, except insurance and pension funding) and we would like to estimate CFC in 1995. We can take over parameters from the Czech Statistical Office, thus industry-specific average service-life is 4.7 years and mortality pattern is based on log-normal probability distribution. The probabilities derived from the mortality pattern are in the following graph.

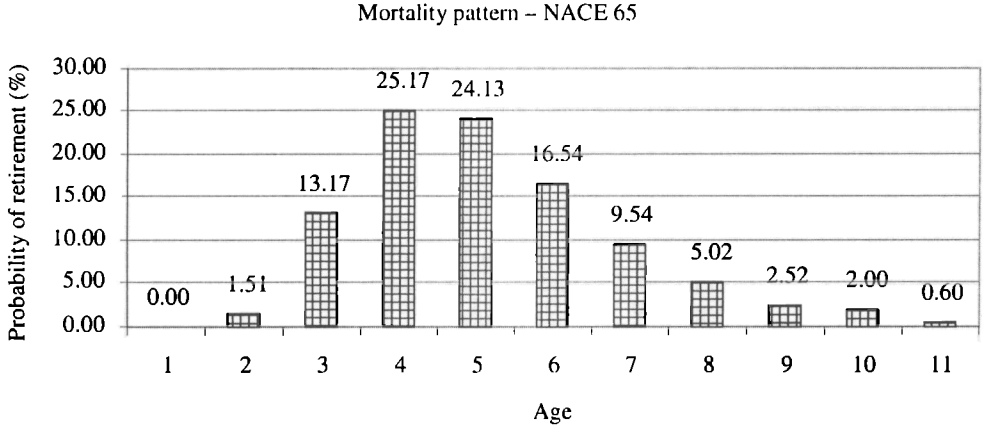

Fig. 1. Probabilities of retirement in NACE 65

Suppose that units in this industry had no vehicles before 1985 and thus for next computation we have to acquire gross capital formation from 1985. The first step is to revaluate GCF into constant prices of a base year (Table 1). CFC and stocks are calculated in Tables 2, 3 and 4.

Table 1. Gross fixed capital formation, price index*

Item\Year	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995
GCF (current prices), mil. CZK	24.6	78.0	86.8	89.4	964.0	1216.0	1972.4	2872.5	2685.2	2566.4	2856.3
Price index (1995 = 1.00)	0.471	0.466	0.461	0.465	0.448	0.450	0.694	0.796	0.892	0.973	1.000
GCF (constant prices), mil. CZK	52.3	167.4	188.2	192.5	2153.1	2701.6	2843.8	3609.8	3009.0	2638.3	2856.3

* All figures are rounded.

Table 2. Computation of gross capital stock*

GCF 1995:	52.3	167.4	188.2	192.5	2153.1	2701.6	2843.8	3609.8	3009.0	2638.3	2856.3
Age\Year	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995
1	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1	**0.1**
2	0.8	2.5	2.8	2.9	32.4	40.7	42.8	54.3	45.3	**39.7**	**43.0**
3	6.9	22.0	24.8	25.4	283.6	355.9	374.6	475.5	**396.4**	**347.5**	**376.2**
4	13.2	42.1	47.4	48.5	542.0	680.1	715.9	**908.7**	**757.5**	**664.1**	**719.0**
5	12.6	40.4	45.4	46.4	519.5	651.9	**686.2**	**871.1**	**726.1**	**636.6**	**689.2**
6	8.6	27.7	31.1	31.8	356.2	**447.0**	**470.5**	**597.2**	**497.8**	**436.5**	**472.6**
7	5.0	16.0	18.0	18.4	**205.5**	**257.8**	**271.4**	**344.5**	**287.1**	**251.8**	**272.6**
8	2.6	8.4	9.4	**9.7**	**108.1**	**135.6**	**142.8**	**181.2**	**151.0**	**132.4**	**143.4**
9	1.3	4.2	**4.7**	**4.8**	**54.2**	**67.9**	**71.5**	**90.8**	**75.7**	**66.4**	**71.8**
10	1.0	**3.3**	**3.8**	**3.8**	**43.1**	**54.0**	**56.9**	**72.2**	**60.2**	**52.8**	**57.1**
11	**0.3**	**1.0**	**1.1**	**1.1**	**12.8**	**16.1**	**17.0**	**21.5**	**17.9**	**15.7**	**17.0**
14 715	0.3	4.3	9.6	19.5	423.6	978.5	1716.2	3087.2	2969.7	2643.5	2862.1

* All figures are rounded.

Table 3. Computation of consumption of fixed capital*

Age (service lives)\Year	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995
1	0.00	0.00	0.00	0.00	0.05	0.06	0.06	0.08	0.07	0.06	**0.06**
2	0.39	1.26	1.42	1.45	16.21	20.34	21.41	27.17	22.65	**19.86**	**21.50**
3	2.29	7.35	8.26	8.45	94.54	118.62	124.87	158.50	**132.12**	**115.84**	**125.42**
4	3.29	10.53	11.84	12.11	135.50	170.02	178.96	**227.17**	**189.36**	**166.03**	**179.75**
5	2.52	8.08	9.08	9.29	103.91	130.38	**137.24**	**174.21**	**145.22**	**127.33**	**137.85**
6	1.44	4.61	5.19	5.31	59.37	**74.50**	**78.42**	**99.54**	**82.97**	**72.75**	**78.76**
7	0.71	2.28	2.57	2.62	**29.35**	**36.83**	**38.77**	**49.21**	**41.02**	**35.97**	**38.94**
8	0.33	1.05	1.18	**1.21**	**13.51**	**16.95**	**17.84**	**22.65**	**18.88**	**16.55**	**17.92**
9	0.15	0.47	**0.53**	**0.54**	**6.02**	**7.55**	**7.95**	**10.09**	**8.41**	**7.37**	**7.98**
10	0.10	**0.33**	**0.38**	**0.38**	**4.31**	**5.40**	**5.69**	**7.22**	**6.02**	**5.28**	**5.71**
11	**0.03**	**0.09**	**0.10**	**0.10**	**1.17**	**1.46**	**1.54**	**1.96**	**1.63**	**1.43**	**1.55**
2 890	0.03	0.43	1.00	2.23	54.35	142.69	287.44	592.05	625.64	568.41	615.45

* All figures are rounded.

Table 4. Computation of net capital stock*

(1) GCS	0.3	4.3	9.6	19.5	423.6	978.5	1716.2	3087.2	2969.7	2643.5	2862.1
(2) CFC	0.03	0.43	1.00	2.23	54.35	142.69	287.44	592.05	625.64	568.41	615.45
(3) AGE	11	10	9	8	7	6	5	4	3	2	1
(4) NCS (1 – 2 × 3)											
6012	0	0	1	2	43	122	279	719	1093	1507	2247

* All figures are rounded.

These tables are more illustrative than the formulas above. Figures in Table 2 are obtained by multiplying GCF by probabilities of retirement for individual age (see Fig. 1). For example, the value of the property with service-life of 11 years in 1985 (0.3 mil. CZK) is GCF (52.39 mil. CZK) × p_{11}(0.006). When we sum the values of the property, which "survive" to 1995 (bold figures), we obtain gross capital stock in 1995 (14 715 mil. CZK) in current prices (because we use constant prices of 1995). Figures in Table 3 are the figures from Table 2 divided by the service lives for each category. Thus the depreciation of the property with service-life of 11 years in 1985 (0.03 mil. CZK) is GCS (0.3 mil. CZK)/11. Consumption of fixed capital in 1995 (2890 mil. CZK) is also obtained by summing bold figures (the depreciation of the part of property that "survives" to 1995). As it is shown in Table 4, net capital stock (6012 mil. CZK) is obtained indirectly by deducting cumulative CFC from GCS (sums in CZK are rounded).

Although this example is very short, it shows the core of capital consumption estimates. The real estimates differ only by the volume of figures because statistical offices usually need time series. Practical computation cannot be solved without computers and some cases are not solvable without special programmes. The bulk of the figures is so large that the spreadsheet's tables and sheets are insufficient. There are 10 types of fixed assets, 13 sectors (CZ) and 60 industries (NACE). The procedure causes that for the estimate of consumption of fixed capital in a particular year is about 10 (types of assets) × 13 (sectors) × 60 (industries) × 20 (data on gross fixed capital formation of previous years) = 156 000 figures needed. Although the computation is quite difficult, the worst problem (especially for transitive countries) is data availability. Thus the revisions of consumption of fixed capital in transitive countries took lots of time and resources because it had to be done before EU enlargement. If it had not been completed, GDP in these countries would not have been comparable.

The rules for national accounting are still developing and we can also expect some changes in near future. Nowadays experts from different parts of the world are discussing proposed changes on special meetings and accepted proposals will be probably used for improvement of the current system of national accounts. Some proposals are quite fundamental like including water into fixed assets and the

concept of capital services, which deal with future incomes from assets. It is possible that some new proposals will be included in a new system of national accounts and therefore in new ESA.

References

[1] *European system of accounts (ESA 1995)*, Eurostat, Luxembourg 1996.
[2] *Links between Business Accounting and National Accounting*, United Nations 2000.
[3] *Measuring Capital. A Manual of the Measurement of Capital Stocks, Consumption of Fixed Capital and Capital Services*, OECD, Paris 2001.
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OSZACOWANIA ZUŻYCIA ŚRODKÓW TRWAŁYCH

Streszczenie

Celem artykułu jest przedstawienie pewnych zagadnień dotyczących zużycia środków trwałych. Oszacowania zużycia środków trwałych reprezentują dość trudny obszar rachunkowości narodowej, a poprawne liczby są ważne dla poprawnych oszacowań produktu krajowego brutto. Istnieją różne metody estymowania zużycia środków trwałych, a jedną z nich jest krótko opisana tu metoda nieustannego spisu inwentarza PIM (*perpetual inventory method*). Powszechnie stosowana metoda PIM jest narzędziem, które wspomaga statystyków w tym trudnym zadaniu. Metoda ta jest oparta na modelowaniu statystycznym i występuje w kilku modyfikacjach. Klasyczne podejście do PIM, opisane tutaj, składa się z bezpośrednich obliczeń zapasów brutto i zużycia środków trwałych. Następnie zapasy netto są otrzymane z zapasów brutto po potrąceniu zakumulowanej amortyzacji. Zastosowanie PIM zależy od pewnych założeń i jest warunkowane dostępnością danych.

Słowa kluczowe: zużycie środków trwałych, PKB.