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Calculation of magnetic force between magnetite particles in a low-intensity magnetic separator: A 3D finite element method

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Abstract: In this study, we address the fundamental problem of magnetic interaction between two magnetite particles in a low-intensity magnetic separator. The accuracy of the magnetic force between two adjacent particles was compared based on the magnetic dipole theories in a 3D model. The results indicate that compared with calculation results of 3D finite element method (FEM), the relative error of the formulas is about 31% when the inter-particle distance ranges from 1.1 to 5 times of the particle diameter. Furthermore, we came up with a more accurate and faster method (formula combined with finite element integration) for calculating the magnetic force throughout the whole range of the inter-particle distance. The results of FEM calculation agreed well with the new method, which can reduce the relative errors of magnetic dipole force to less than 3% in the two magnetite particles system.

***Keywords*:** magnetic dipole model, magnetite particles, attractive force, interaction, 3D FEM

1. Introduction

The complex interaction between magnetite particles is an important research content in the fields of functional materials (Bossis et al., 2002), magnetic fluids (Ekwebelem and See, 2009), mineral processing (Shang et al., 2022) and so on. Especially, in the field of mineral processing, the interaction force between magnetite particles has always been a large error due to the neglect of the magnetization of the particles themselves. Recent advances in mineral processing have demonstrated the critical role of precise magnetic control in enhancing the efficiency of iron recovery from complex secondary resources, such as the application of suspension magnetization roasting to optimize the magnetic separation of iron from sulfuric acid slag (Li et al., 2025). Typically, in a low-intensity magnetic separator, two magnetic forces are acting on magnetite particles: the first one is gradient magnetic force in a nonuniform magnetic field, and the second one is interaction magnetic force between particles in a uniform magnetic field. The gradient magnetic force and interaction magnetic force are essentially the same because they both originate from the external unified magnetic field. However, in a multi-particle system, magnetic interaction among magnetite particles is exceedingly complex because the magnetization of each particle is affected by the magnetic field produced by the other particles, and vice versa. At present, there is no accurate and adaptable formula for calculating the magnetic force.

A generally accepted magnetic dipole formula for calculating magnetic force is available in the literature. Because the far-field magnetic field is hardly affected by particle magnetization, when the magnetite particles are far apart, the influence caused by mutual magnetization between particles can be ignored. Consequently, when the measuring distance is much larger than the length of the magnet, the magnet can be approximately treated as a magnetic dipole or a plurality of magnetic dipoles (Dai et al., 2017). Zhang et al. (2010) studied magnetic objects of regular shapes, and the results indicated that at a distance of more than 2.5 times the length of the object, a magnetic object can be treated as a magnetic dipole. It can meet the requirements of general engineering applications. However, when the magnetite particles are at a much closer distance, it will cause a large error in calculating magnetic

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attraction force with existing magnetic dipole formulas (Zhao and Peng, 2012; Lemaire and Bossis, 1991; Tao et al., 1995; Biswal and Du, 2014). The nearer apart they are, the stronger the magnetic force is, the greater the relative error is and vice versa. However, to the best of the authors' knowledge, a general, tested and accurate formula is insufficient to predict the magnetic force between two adjacent magnetite particles or the force for the whole range. Based on various magnetic dipole models, Tian et al. (2012) used finite element software to calculate the magnetic field intensity at the particle surface under different particle permeabilities, different inter-particle distances and different applied magnetic fields. It was found that the maximum relative errors between the calculation results of the magnetic dipole model and the FEM could reach 70%, 62%, and 30%, respectively.

The FEM is a numerical technique for solving approximate solutions of boundary value problems for partial differential equations. The FEM has the advantages of high computing accuracy to obtain various forces and motion behaviors, as well as the ability to accommodate particles of different shapes (Kees et al., 2022; Ku et al., 2021; Su et al., 2001). The effectiveness of FEM in predicting magnetic field distributions has been corroborated by studies combining numerical simulation with experimental measurement, such as the analysis of magnetic flux density in energy harvester systems using Hall-effect sensors and 2D magnetostatic FE models (Bijak et al., 2023). However, calculating large-scale particles is extremely expensive, and the result is presented in the form of tensors, which do not intuitively reflect the influence of particle parameters and magnetic field parameters on the force. For this reason, using the simple simulated field combined with classic formulas to study the acting force and particle motion behaviors not only avoids expensive calculation, but also contributes to the thorough elucidation of physical process.

In this study, the FEM is applied to calculate the interaction force between two magnetized magnetite particles. Simultaneously, the calculation results of FEM are compared with the magnetic dipole model to explore the effects of certain parameters on the accuracy of the magnetic dipole model, including different angles between particles, inter-particle distance and particle diameter. Moreover, the term of magnetic field in the magnetic dipole formula is modified by using the method of finite element integration. The relative error of modified magnetic dipole model compared with the whole finite element calculation results (hereinafter referred to as "relative error" or "error") can be reduced to 3% or less. The new method not only greatly improves the calculated accuracy of magnetic force between magnetite particles, but also can meet the needs of the magnetic force calculation between large-scale particles in the low magnetic field separator.

2. Finite element method

2.1. Finite element analysis

Based on the Maxwell equations, the electromagnetic field analysis module in the Finite Element Analysis (FEA) Software (COMSOL) can calculate magnetic position or potential, and then utilize the vector/scalar method to obtain magnetic field strength by calculating each element.

In this study, magnetite particle magnetization is assumed to be homogeneous and vary depending on the local magnetic field. In our baseline model for error evaluation, we specifically assume a constant magnetization defined by H_0 and μ_1 = 5. The correction factor Q_0 in our new method is introduced precisely to account for the discrepancy between this simplified assumption and the actual coupled magnetization physics.The uniformly magnetized particle is viewed as a magnetic dipole. By solving the magnetic flux conservation equation, the magnetic field intensity and magnetic induction intensity can be obtained, then the interaction force between the two magnetic dipoles can be calculated.
The Table 1 lists some notations essential for understanding the subsequent results.

2.2. Model construction

In this study, the interaction force between two magnetite particles which are uniformly magnetized is analyzed. The calculation parameters include particle position, magnetic field vector, etc. The specific setting methods in modeling include:

1. Set the 3D spherical domain as the analysis domain.
2. Set infinite element domain.

3. Set the element size for the spherical domain to fine, with a maximum size of 0.032 cm, a minimum size of 0.00576 cm, and a maximum growth rate of 1.5.
4. Set a uniform magnetizing field in two magnetite particles.
5. The parameter settings of magnetite particles include:
6. Two particles are placed at the symmetrical center of the spherical vacuum region.
7. Considering the calculation accuracy and efficiency, the fixed number of elements in edge distribution is 50, the maximum element size is 0.007 cm, the minimum element size is 0.0001 cm, the maximum element growth rate is 1.03, and the settings of other parameters are extra fine.
8. To facilitate the study, the magnetite particles are set as uniform material. In order to simulate a realistic magnetization behavior of magnetite, the relative permeability μ_1 of magnetite particles is 5 and the density ρ is 5.16 g/cm^3.
9. The magnetization direction of particles is normal, the magnetization $\boldsymbol{M}_0$ is set as 0.06 T/(4π×10^{-7}) (H/m).

Values and expressions of some parameters in the model are recorded in Table 2.

Table 1. Parameters in force calculation and model simulations

Sign	Meaning	Sign	Meaning
R	Radius of the 3D spherical vacuum domain, mm	θ	Rotation angle with respect to the center of two magnetite particles, °
r	Radius of spherical magnetite particles, mm	n	Unit vector along the normal line of surface
d	Diameter of spherical magnetite particles, mm	ρ	Density of magnetite particles, g/cm^3
μ_1	Relative magnetic permeability of magnetite particles	I_0	Distance between the center of two particles, mm
μ_2	Relative magnetic permeability of environmental media	H_0	Environmental magnetic field intensity, A/m
H	Magnetic field strength, A/m	m	Magnetic moment, A·m^2
B	Magnetic induction intensity, T	χ	Magnetic susceptibility
M	Magnetization intensity, A/m		

Note: In the text, "inter-particle distance" is the distance between the center of two particles. Unless specifically mentioned, the particle diameter of magnetite particles is 0.1 mm.

Table 2**.** Initial value or expression of part of the parameters in the model

Parameter	Value/expression	Formulation
r_0	0.05[mm]	Initial radius of particles
R	2[mm]	Radius of spherical vacuum domain
R_0	D/20	Thickness of infinite element domain
$\boldsymbol{M}_0$	0.06T/ mu0_const	Preset value of particle magnetization
mu0_const	4π×10^{-7}[H/m]	Conversion unit
x_1	sinθ·I_0/2	Coordinate of the center x of the first particle
y_1	cosθ·I_0/2	Coordinate of the center y of the first particle
x_2	-sinθ·I_0/2	Coordinate of the center x of the second particle
y_2	-cosθ·I_0/2	Coordinate of the center y of the second particle

Fig. 1 shows some parameters of the finite element calculation model of two uniformly magnetized particles (a) and magnetic induction field distribution (b). The two slightly smaller spheres in Fig. 1 (a) are two magnetite particles of uniform material, in which some diagrams of parameters are marked. The colors and arrows in Fig. 1 (b) represent the intensity and direction of the magnetic induction field, respectively. Since the two particles are always in the same plane and there is no force link in the z-direction, the physical quantities in the z direction are not discussed in this study.

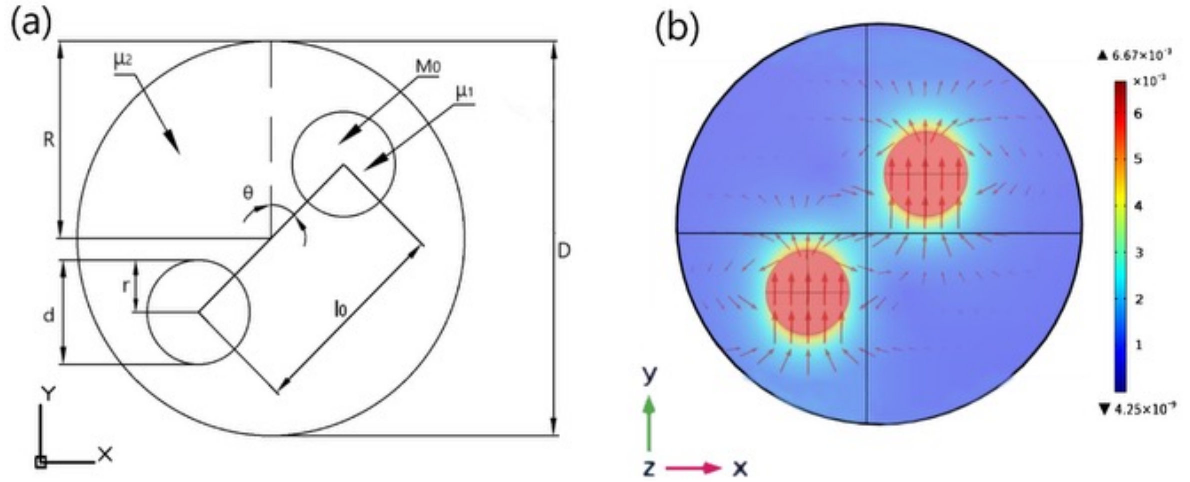

Fig. 1. (a) Schematic diagram of some parameters; (b) Distribution diagram of magnetic induction field *B* (T)

Fig. 2 Relative direction (a) and dimensionless magnetic force (b) between two magnetite particles in the background magnetic field under ideal conditions. In order to test the validity of the FEM, the magnetic force obtained with FEM was thoroughly validated against results obtained by the Stokesian dynamics method (Sand et al., 2016), and excellent agreement was found in all cases.

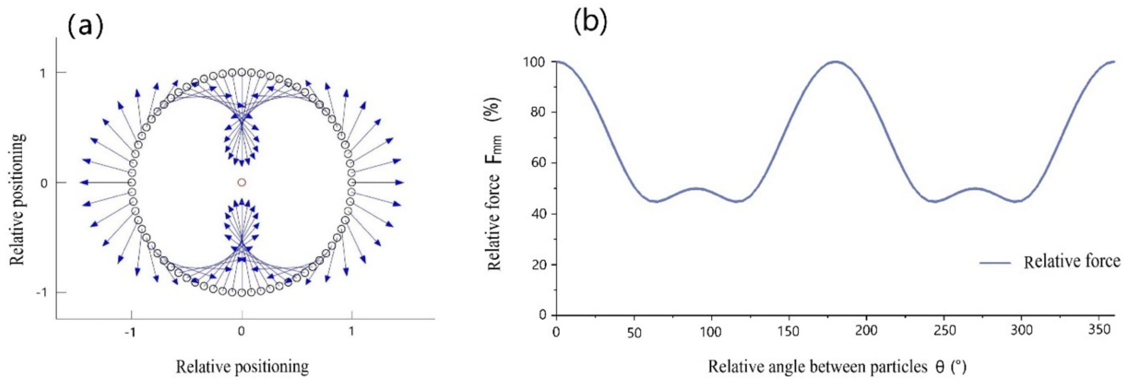

Fig. 2. The relative direction (a) and dimensionless relative force (b) between two magnetite particles in the background magnetic field in ideal situation

2.3. Mesh sensitivity analysis

To ensure the accuracy and mesh-independence of the FEM results and to enhance the reproducibility of the study, a systematic mesh sensitivity analysis was conducted. While keeping the model geometry and boundary conditions unchanged, the global mesh density was progressively refined, particularly in the region near the particle surfaces and the gap between them. Key parameters, including the average magnetic flux density within the computational domain and the total magnetic interaction force between the two particles, were monitored.

The analysis revealed that when the total number of mesh elements exceeded approximately 50,000 (corresponding to the "fine" mesh settings described in Section 2.2), further mesh refinement (by about 20%) resulted in changes of less than 0.5% in the calculated magnetic force and less than 0.3% in the average magnetic flux density. This confirms that the mesh scheme selected in this paper (as detailed in Table 1 and Section 2.2) yields sufficiently converged results, and the computational accuracy is not significantly affected by mesh density. Therefore, all subsequent simulations were performed based on this mesh-independent verified scheme, ensuring reliable results while maintaining computational efficiency.

3. Calculation model of magnetic dipole

When there is only one single magnetite particle which is a uniform material in the uniform magnetic field, the particle will be magnetized. The magnetic field produced by the particle is equivalent to the magnetic field produced by a magnetic dipole which is located in the center of the particle, the magnetic dipole moment is m (Tian, 2012).

When multiple magnetite particles are exposed to a magnetic field, each magnetized particle influences the external magnetic field. With the change of external magnetic field, the particles are magnetized again and the process repeats many times until the magnetic field is completely stable. During this process, the internal magnetization of each magnetite particle also undergoes changes.

At this time, the magnetic field produced by one particle is equivalent to a magnetic field generated by a magnetic dipole which is located in a certain position around the particle, the magnetic dipole moment is m_e. The interaction between particles can be expressed by the force between two magnetic dipoles (Xia et al., 2022).

The commonly used magnetic dipole model does not consider the mutual magnetization between magnetic dipoles. The magnetic dipole moment formula is (Jolly et al., 1996; Peng and Li, 2007):

$$m = 4\pi\mu_0\mu_1 kr^3 H \tag{1}$$

where μ_0 is permeability of vacuum, r is the radius of magnetite particle, H is applied magnetic field, and k is a parameter related to relative permeability of medium and magnetite particle, $k=(\mu_1-\mu_2)/(2\mu_1+\mu_2)$, among which μ_1 and μ_2 are particle and environment permeabilities, respectively. In this study, the environment is vacuum, so μ_2 =1.

3.2. Force

In magnetic field, interaction energy between two magnetic dipoles (m_1, m_2) can be expressed as (Ku et al., 2015):

$$E = \frac{1}{4\pi\mu_0\mu_2}\left[\frac{m_1 m_2 - 3(m_1 \cdot e_r)(m_2 \cdot e_r)}{I_0^3}\right] \tag{2}$$

where e_r is the unit vector of the connecting line between the centers of two magnetic dipoles, I_0 is the center distance between two particles.

When magnetic moments (magnitude and direction) of two magnetic dipoles are the same, the above formula can be simplified as:

$$E = \frac{|m|^2 (1-3\cos^2\theta)}{4\pi\mu_0\mu_2 I_0^3} \tag{3}$$

$\sin\theta = y/\sqrt{x^2+y^2}$, $\cos\theta = x/\sqrt{x^2+y^2}$, $I_0=\sqrt{x^2+y^2}$ were substituted into the above formula. Then take the derivative of the result with respect to x and y. The component force of inter-particle interaction forces along the respective direction can be obtained (Tao et al., 1995):

$$F_x = \frac{3|m|^2 \sin\theta(5\cos^2\theta-1)}{4\pi\mu_0\mu_2 I_0^4} \tag{4}$$

$$F_y = \frac{3|m|^2 \cos\theta(5\sin^2\theta-3)}{4\pi\mu_0\mu_2 I_0^4} \tag{5}$$

Then calculate the resultant force on F_x and F_y, which is the magnetic force between the two magnetite particles:

$$F_{mm} = \frac{3|m|^2 \sqrt{1-2\cos^2\theta+5\cos^4\theta}}{4\pi\mu_0\mu_2 I_0^4} \tag{6}$$

4. Error analysis

Finite element models and magnetic dipole models are developed for precise calculation and analysis of the interaction force between two particles. Furthermore, relationships between particle spacing (I_0), relative angle between particles (θ), particle size (d) and magnetic force are calculated and plotted.

4.1. Inter-particle distance

The change of interaction force between particles in x-direction (tangential direction) and y-direction (normal direction) was studied. Fig. 3 shows changes in the forces when gradually expanding the inter-particle distance at range of 1.1-3.0 times and 1.8-2.4 times of the particle diameter along y-direction.

The interaction force between the two particles increases rapidly as they get closer to each other. Moreover, with the increase of the inter-particle distance, the upward trend of the curve has gradually slowed down. When the inter-particle distance is 2.5~3.0 times of the particle diameter, the interaction force increases slowly, the upward trend has no significant change. When the inter-particle distance is 2.0~2.5 times of the particle diameter, the upward trend is more obvious. When the inter-particle distance is 1.2~2.0 times of the particle diameter, the interaction force increases sharply. When the inter-particle distance is less than 1.2 times of the particle diameter, the interaction force between the particles has decreased 200 times in magnitude. In Fig. 3, I_0/d is the multiple of inter-particle distance to particle diameter.

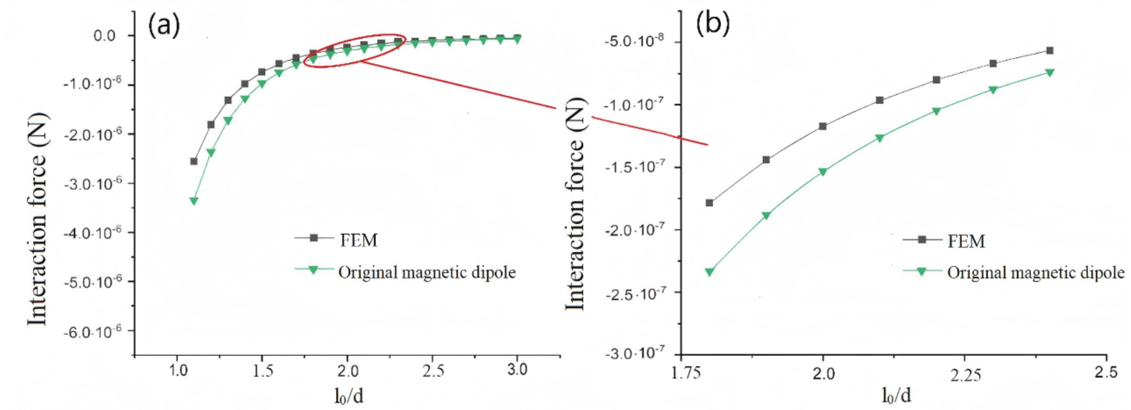

Fig. 3. The interaction between different inter-particle distances along y-direction. (a) Full range (l_0/d = 1.0-3.0); (b) Enlarged view of the dashed-box region in (a) (l_0/d = 1.75-2.5)

Fig. 4 further shows the changes in forces when gradually expanding the inter-particle distance at range of 1.1-3.0 times and 2.1-3.0 times of the particle diameter along x-direction. Similar to Fig. 3, as the two particles approach each other, the interaction force between them increases slowly and then rapidly, but the direction of interaction force changes from attraction to repulsion.

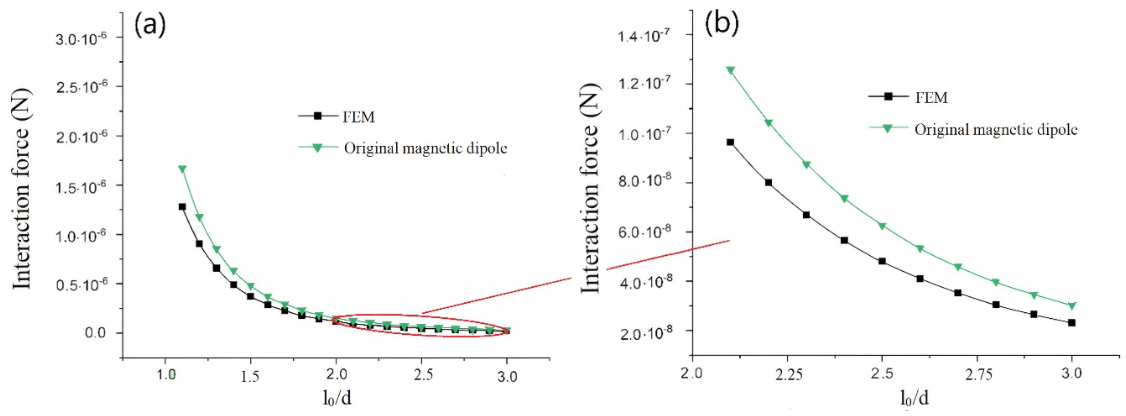

Fig. 4. The interaction between different inter-particle distances along x direction. (a) Full range (l_0/d = 1.1-3.0); (b) Enlarged view of the dashed-box region in (a) (l_0/d = 2.1-3.0)

After an inspection, the law of model error caused by changing the inter-particle distance along x-direction and y-direction is basically the same. This means that the laws of the relative errors of the x direction force (F_x), y direction force (F_y) and resultant force (F_{mm}) are basically the same. The error mentioned in the following refers to the relative errors when the magnetic dipole formula is used to calculate the resultant force (F_{mm}).

Fig. 5 shows the variation of calculation error of the magnetic dipole model when two particles expand the inter-particle distance along one direction. As can be seen in Fig. 5, the relative error of the traditional magnetic dipole model is always about 30.5% ~ 31%. Obviously, it cannot meet the needs of research or experiment.

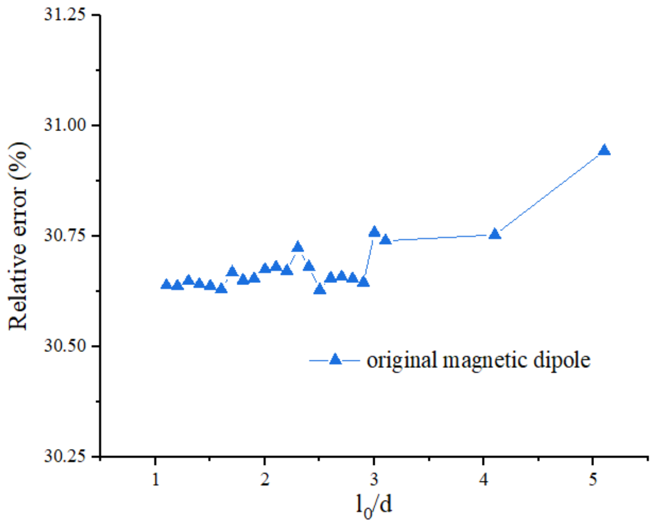

Fig. 5. Relative error in different inter-particle distances

4.2. Angle between particles

The variation laws of x-direction and tangential force have been given previously. The result shows that when inter-particle distances are the same, the relative errors of x-direction force, y-direction force and resultant force are basically the same. On this basis, Fig. 6 reflects the changing law of the relative error of the interaction force between two uniformly magnetized particles after rotating 90° along the center point of their connecting line under different inter-particle distances.

It is concluded that, in general, the angles between particles do not affect the accuracy of the models. The angles are not the major factor causing relative error. It needs to be clarified that when l_0/d larger than or equal to 3, the relative fluctuations may be caused by the rapid decrease of interaction force. Smaller forces require higher finite element accuracy, and continuing to use the previous mesh can lead to fluctuations in accuracy.

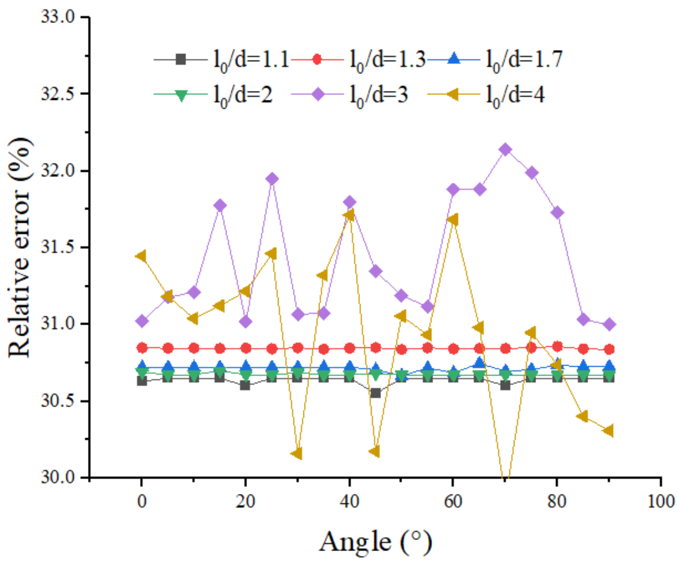

Fig. 6. Relative error of traditional magnetic dipole model under different inter-particle distances

4.3. Particle diameter

The particle diameters are set as follows: d=0.10 mm, 0.14 mm, 0.18 mm, 0.22 mm. Under different particle diameters, the relative error of traditional magnetic dipole model is shown in Fig. 7. As shown in Fig. 7, the particle diameter does not affect the model error. Particle diameter is not the major factor causing error

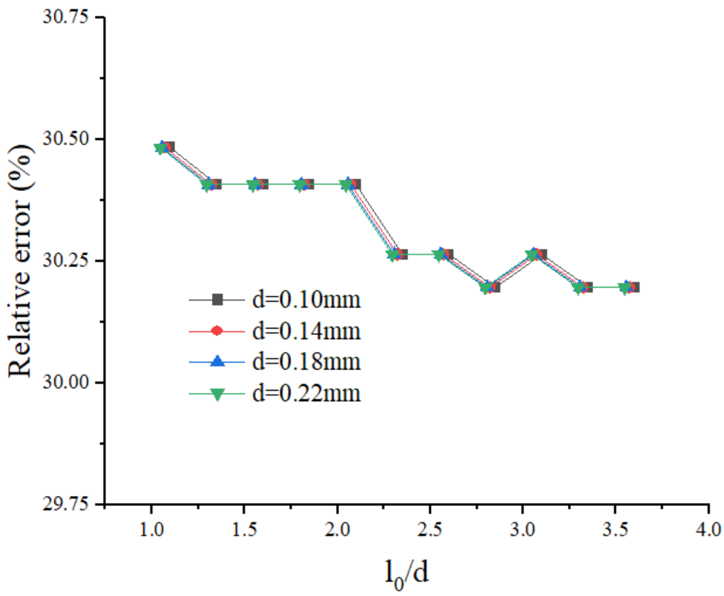

Fig. 7. Relative error of interaction force between particles under different particle diameters

5. Correction of magnetic dipole formula by finite element integration method

Under a uniform magnetic field, the magnetic field produced by a particle is equivalent to the magnetic field produced by a magnetic dipole located in the center of the particle, with the magnetic dipole moment denoted as m (Tian, 2012). The interaction between particles can be expressed by the force between two magnetic dipoles (Lu, 2017). In the traditional magnetic dipole model, no form of re-magnetization is considered (Tian, 2012), and the magnetic dipole moment is expressed by formula (1).

When using the magnetic dipole model to calculate the interaction force of particles, the central point of magnetite particle is usually used to calculate the relevant parameters of magnetic dipole, such as magnetization, magnetic dipole moment, etc. In a low magnetic field, multiple magnetite particles interact with each other. A single magnetite particle will not only be magnetized by the environmental magnetic field, but also be magnetized by the magnetic field produced by nearby magnetite particles. In this process, the internal magnetization of the magnetite particles will change.

When a pair of particles are adjacent, the magnetic force increases very rapidly at short distances of the particle from another particle. However, the interaction force of particles in contact is finite. It is widely believed that the contact force between magnetic materials touching each other cannot be properly estimated (Ren et al., 2015), the relative error of the magnetic dipole model can approach infinity with the infinite distance (Bermudez, 2017; Ku et al., 2016).

In this study, the result shows that the relative error of traditional magnetic dipole model is always stable at about 31% when carrying out a calculation of magnetite particles. Obviously, the accuracy of model cannot meet the needs of experiments or applications in a low-intensity magnetic separator. For different theoretical explanations, scholars at home and abroad have joined parameter Q to modify the traditional formula for magnetic dipole moment, and provided references for calculation of magnetic dipole force. The modified formula for magnetic dipole moment considering the interaction effect (m_e) is (Tian, 2012; Ku et al., 2015; Fang et al., 2004; Ku et al., 2022):

$$m_e = 4\pi\mu_0\mu_1 k r^3 QH \quad (7)$$

where Q is the correction factor considering the effect of magnetization fields from other particles. The total surface magnetic induction field (B_0) excited by both the applied magnetic field (H) and the magnetization fields of other particles (H_p) on an enclosure of the magnetite particle can be integrated by FEM (Cha, 2014), and B0 is used to replace H in the formula for magnetic dipole moment. Converting relevant units through $B=\mu_1 H$, a new formula for magnetic dipole moment is obtained:

$$m_e = 4\pi\mu_0\mu_1 kr^3 B_0 / \mu_1 \quad (8)$$

Assuming B_0 takes the surface-integrated values of the surface magnetic induction intensity of the magnetic dipole (spherical magnetite particles that are uniformly magnetized) when the inter-particle distance is greater than 3.1 times of the particle diameter, the QH in formula (7) is numerically equal to the magnetic induction intensity B_0/μ_1 (Tian et al., 2011), so the base value of Q can be obtained by the finite element integration at this special distance. However, considering the boundary of the magnet is a discontinuity surface for the magnetic field, the magnetic dipole model developed by the finite element has a loss on the boundary (Biswal and Du, 2014), formula (8) cannot completely eliminate the model's error.

To eliminate the error as much as possible, based on $M=(\mu_1/\mu_0-1)H$, replace the QH term in formula (7) with an approximate value Q_0M, an alternative formula can be obtained:

$$m_e = 4\pi\mu_0\mu_1 kr^3 Q_0 M \quad (9)$$

where Q_0 is a correction parameter related to inter-particle distance (the value needs to further fine-tuned based on the Q obtained by finite element integration); M is the magnetization of single magnetite particle which is uniformly magnetized in applied magnetic field. Due to the setting of magnetizing field model, the magnetization M of particles in this study is a fixed value.

The previous conclusions indicate that the particle diameter and relative angle have no effect on the model. According to the research of Tian (2012), increasing the magnetic field strength can reduce the model error until magnetic saturation is reached. It means that increasing the magnetic field strength is beneficial to reducing the error, so the influence of the magnetic field strength on the model error will not be discussed again in this study.

Therefore, set I_0=1.1d-4.5d to develop the models. Using the fixed particle diameter, relative angle, and magnetic field strength, establish the classical model with the new formula for magnetic dipole moment. Take Q_0 as the adjustable variable, and the relationship between the Q_0 term and the relative error of the model is as shown in Fig. 8.

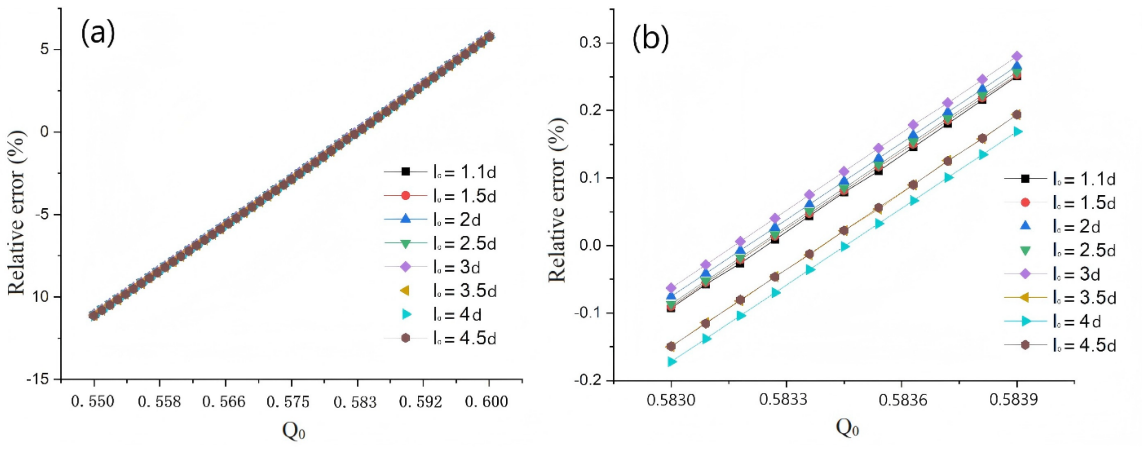

Fig. 8. The relationship between item Q_0 and model error. (a) Full range (Q_0 = 0.550-0.600); (b) Enlarged view of the dashed-box region in (a) (Q_0 = 0.5830-0.5839)

Within the scope of this study, it can be seen in the above Fig. 8 that Q_0 can be taken as 0.5830-0.5838 to control the relative error of the new model within ±0.1%. At this point, the above formula (7) can be replaced by the semi-empirical formula (9), where Q_0 takes 0.5830 ~ 0.5838 (for editor/reviewer reference only). The specific value of Q_0 is confirmed by the required accuracy for the magnetite particles. When Q_0 is taken as 0.5833, the relative error of the new magnetic dipole model in calculating the magnetic force between magnetite particles is within 0.1%. Notably, the transition of relative error for different Q_0 values indeed appears to show a discrete shift between two distinct error levels. This shift likely occurs due to the certain sensitivity of the formula to changes in the fitting Q_0. Considering its error is much lower than the 31% of the traditional method, it is still acceptable in a limited discrete shift process. The new semi-empirical formula provides a simple method to precisely calculate the magnetic force between two magnetite particles in a low-intensity magnetic separator. However, it is only an empirical value obtained through theoretical analysis, it still needs to be further verified by experiments referring to the research of Hartung et al. (2018) and Borgers et al. (2018) in the following study.

We introduce the concept of an "effective magnetic capture range" to link our microscopic force calibration to macroscopic separator performance. It is defined as the inter-particle distance at which the magnetic attractive force (calculated by our corrected model, Figs. 3 & 5) becomes dominant over disruptive forces. This range can be quantified by identifying the distance where the force decays to a defined threshold (e.g., 1% of its maximum), providing a practical metric for separator design.

We note that the validation in this work is numerical. Experimental measurement of inter-particle forces and comparison with analytical solutions for limiting cases would be valuable future steps to further confirm the model's physical accuracy.

6. Conclusions

(1) By comparing the interaction force calculations between the traditional magnetic dipole model and FEM using magnetite particles, we highlight the limitations of the magnetic dipole model when the inter-particle distance is extremely small. The results indicate that when the inter-particle distance reaches less than 2.5 times of the particle diameter, the relative error of the traditional magnetic dipole model is about 31%. Obviously, the accuracy of the traditional magnetic dipole models cannot meet the needs of experiments or applications in a low-intensity magnetic separator.

(2) According to error analysis between finite element models and magnetic dipole models, the angles between particles and the particle diameter do not affect the relative error of the traditional magnetic dipole model, it provides (a prospective idea) to reduce the relative error that we should justify the formula under a certain the inter-particle distance.

(3) According to the error distribution characteristics of the magnetic dipole model, a model is developed for a FEA fitting a large amount of data. In addition, algorithms have been built to combine these estimates and produce a new semi-empirical formula for magnetic dipole moment between two magnetite particles. After substituting the new formula into the traditional magnetic dipole model, the relative error of magnetic dipole force can be reduced to less than 3%.

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References

BERMUDEZ, A., 2017. *Extended formulas to compute resultant and contact electromagnetic force and torque from Maxwell stress tensors.* IEEE Transactions on Magnetics, 53: 1–9.

BIJAK, J., LO SCIUTO, G., KOWALIK, Z., LASEK, P., SZCZYGIEŁ, M., TRAWIŃSKI, T., 2023. *Magnetic flux density analysis of magnetic spring in energy harvester by Hall-effect sensors and 2D magnetostatic FE model.* Journal of Magnetism and Magnetic Materials, 579: 170796.

BISWAL, S.L., DU, D., 2014. *Micro-mutual-dipolar model for rapid calculation of forces between paramagnetic colloids.* Physical Review E.

BORGERS, S., VÖLKEL, S., SCHÖPF, W., et al., 2018. *Exploring cogging free magnetic gears.* American Association of Physics Teachers (AAPT), 86(6): 460–469.

BOSSIS, G., LACIS, S., MEUNIER, A., et al., 2002. *Magnetorheological fluids.* Journal of Magnetism and Magnetic Materials, 252: 224–228.

CHA, L.P., 2014. *Efficient algorithm for solving surface integral equations.* Ph.D. Thesis, Nanjing University of Technology.

DAI, Z.H., ZHOU, S.H., SHAN, S., 2017. *Research on applicability of ship's single magnetic dipole model.* Mine Warfare & Ship Self-Defence, (4): 5.

EKWEBELEM, C., SEE, H., 2009. *Microstructural investigations of the yielding behaviour of bidisperse magnetorheological fluids.* Rheologica Acta, 48(1): 19–32.

FANG, S., 2004. *Calculation of magnetic field, measurement and analysis of mechanical properties of magnetorheological elastomer.* Ph.D. Thesis, University of Science and Technology of China.

HARTUNG, S., SOMMER, F., VÖLKEL, S., et al., 2018. *Assembly of eight spherical magnets into a dotriacontapole configuration.* Physical Review B, 98(21): 214424.

JOLLY, M.R., CARLSON, J.D., BULLIONS, T.A., et al., 1996. *The magnetoviscoelastic response of elastomer composites consisting of ferrous particles embedded in a polymer matrix.* Journal of Intelligent Material Systems & Structures, 7(6): 613–622.

KEES, C.E., HAYDEL COLLINS, J., ZHANG, A., 2022. *Simple, accurate, and efficient embedded finite element methods for fluid–solid interaction.* Computer Methods in Applied Mechanics & Engineering, 389.

KU, J.G., CHEN, H.H., HE, K., et al., 2015. *Force analysis and dynamic simulation of ferromagnetic mineral particles in magnetic separation process.* Journal of Central South University (Science and Technology), 46(05): 1577–1582.

KU, J.G., LEI, Z.Y., LIN, H., et al., 2016. *Interaction between two magnetic dipoles in a uniform magnetic field.* AIP Advances, 6(2): 025004.

KU, J.G., LEI, Z.Y., LIN, H., et al., 2022. *Interaction of magnetic spheres in magnetic field from the view of magnetic energy density: A 3D finite element analysis (FEA).* International Journal of Mining Science and Technology, 171: 107113.

KU, J.G., LEI, Z.Y., XIA, J., et al., 2021. *Dynamic behavior and separation prediction of magnetic ore bulks in dry medium-intensity magnetic separator.* Minerals Engineering, 171: 107113.

KU, J.G., XIA, J., LI, J.Z., 2021. *Accurate calculation of major forces acting on magnetic particles in a high-gradient magnetic field: A 3D finite element analysis.* Powder Technology, 394: 767–774.

KU, J.G., LEI, Z.Y., JIANG, K.X., et al., 2022. *Comparison of various forces acting on magnetic particles in a low-intensity magnetic field: A 3D FEA.* Powder Technology, 401: 117351.

LEMAIRE, E., BOSSIS, G., 1991. *Yield stress and wall effects in magnetic colloidal suspensions.* Journal of Physics D: Applied Physics, 24(8): 1473–1477.

LI, W., WANG, M., WANG, S., et al., 2025. *Mechanism study and thermodynamic analysis of iron recovery from sulfuric acid slag by suspension magnetization roasting and magnetic separation.* Process Safety and Environmental Protection, 199: 107321.

LU, W., 2017. *Study on the chaining mechanism of magnetorheological fluids and its experimental analysis.* Master's Thesis, Shanghai University of Engineering and Technology.

PENG, X., LI, H., 2007. *Analysis of the magnetomechanical behavior of MRFs based on micromechanics incorporating a statistical approach.* Smart Materials & Structures, 16(6): 2477–2485.

REN, Y., CHAO, H., SHENG, X., et al., 2015. *Magnetic dipole model in the near-field.* Proceedings of the IEEE International Conference on Information & Automation: 18.

SAND, A., STENER, J.F., TOIVAKKA, M.O., et al., 2016. *A Stokesian dynamics approach for simulation of magnetic particle suspensions.* Minerals Engineering: 70–76.

SHANG, H.L., LI, G.P., SHEN, Z.C., et al., 2022. *Characteristics of the rotating magnetic field and its influence on magnetic separation behavior.* Journal of China University of Mining & Technology, (003): 051.

SU, H., GONG, Y., YANG, R.S., et al., 2001. *Influence of pre-splitting crack width on crack propagation under blast loading.* Journal of University of Science and Technology of China, 31(2): 169–173.

TIAN, Z.Z., 2012. *Research on magnetorheological fluids and transmission technology.* Ph.D. Thesis, China University of Mining and Technology.

TIAN, Z.Z., HOU, Y.F., WANG, N.N., 2011. *Error analysis of dipole approximation in magnetorheological fluid.* Journal of Functional Materials, 42(5): 788–792.

ZHAO, C.W., PENG, X.H., 2012. *FE computation based modeling for MRFs and comparison with magnetic-dipoles based results.* Journal of Functional Materials, 43(15): 2098–2101.

ZHANG, C.Y., XIAO, G.C., GAO, J.J., et al., 2010. *Experiment research of magnetic dipole model applicability for a magnetic object.* Journal of Basic Science and Engineering, 18(05): 862–868.