# The Influence of Linear Glass Heterogeneity on the Optical Beam Pass through Porro Prisms Systems 

Barbara Lisowska, Florian Ratajczyk*


#### Abstract

The beam pass through Porro prisms systems made of glass with the linear heterogeneity of the refracting index is considered. The exit beam is an astigmatic one and that fact impairs the resolution power of the prisms. It is important for the optical devices of higher accuracy.


Some reflecting prisms made of the glass with a constant gradient of refractive index operate as cylindrical lenses. It has been described previously to that [1] and in particular it was showed that singly as well as doubly reflecting rectangular prisms have their astigmatic foci in the ( $x, y$ )-plane (Fig. 1 and 2). The focal lengths are determined by the following equations:

$$
\begin{equation*}
f_{X Y}=\frac{1}{G_{u}+G_{v}} \tag{1}
\end{equation*}
$$

and

$$
\begin{equation*}
f_{X Y}=\frac{1}{\sqrt{2}\left(G_{u}+G_{v}^{\prime}\right)} \tag{2}
\end{equation*}
$$

for Fig. 1 and 2 respectively. $G_{u}$ and $G_{v}$ are components of the gradient $G$ of refractive index on the uv plane. In the same figures one can see the positions of the foci depending on the directional angle $\gamma$ of the $G_{u v}$ component. In the present paper the focusing of Porro prisms systems is under consideration.


Fig. 1

[^0]

Fig. 2

## Beam Pass through the First Porro Prisms System

The focal lengths of the first Porro prisms systems are calculated in an $x, y, z$ coordinate system (Fig. 3). The light beam is focused


Fig. 3
in the $(x, y)$-plane by the components $G_{u 2}, G_{v 2}$, of the second prism and in the $(y, z)$ plane by $G_{u 1}, G_{v 1}$ of the first. The focal lengths
in the $(x, y)$ and $(y, z)$-planes are given by the expressions:

$$
\begin{equation*}
f_{X Y}=\frac{1}{\sqrt{2}\left(G_{u 2}+G_{v 2}\right)} \tag{3}
\end{equation*}
$$

and

$$
\begin{equation*}
f_{Y Z}=-\frac{1}{\sqrt{2}\left(G_{w 1}+G_{v 1}\right)} \tag{4}
\end{equation*}
$$

It is to notice, that the minus sign in (4) is caused by the actual position of the second prism with respect to the first one. The astigmatic foci of the first Porro prisms system have in general the form of two segments of straight line perpendicular to each other. It can be seen in the Fig. 4.


Fig. 4

## Beam Pass through of the Second Porro Prisms System

In the case of the second Porro prisms system the light beam is focused by a constant gradient of refractive index of one, two or all three component prisms. Namely, as it can be seen in the Fig. 5 the second (II) prism focuses in the

( $x, y$ )-plane, while the first (I) and third (III) focuse in the $(x, y)$-plane together. The focusing in the ( $x, y$ )-plane is caused by one rectangular doubly reflecting prism only, hence
the focal length (see Eq. (2) and Fig. 2) is given by

$$
\begin{equation*}
f_{X Y}=-\frac{1}{\sqrt{2\left(G_{u 2}+G_{v 2}\right)}} \tag{5}
\end{equation*}
$$

The reciprocal focal length of the whole system in the remaining plane can be calculated approximately as a sum of the corresponding reciprocals of the first and the second prisms.

$$
\begin{equation*}
\frac{1}{f_{I Z}}=\frac{1}{f_{\mathrm{IIZ}}}+\frac{1}{f_{\mathrm{IIIIZ}}} \tag{6}
\end{equation*}
$$

After substituting the denominators of (6) by (1) we obtain

$$
\begin{equation*}
\frac{1}{f_{I Z}}=\boldsymbol{G}_{n 1}+\boldsymbol{G}_{r 1}+\boldsymbol{G}_{n 3}+\boldsymbol{i}_{r 3} . \tag{7}
\end{equation*}
$$

The influence of the thicknesses of the prisms is neglected here. The positions of the astigmatic foci of a light beam after passing through a second system of Porro prisms is given in the Fig. 6 as an example.


Fig. 6
In our previous paper [1[ we have accurately calculated the lateral shifts of the foci of a few reflecting prisms in the $x$ and $z$ directions. In this case, however, their values in comparison with the axial astigmatism may be practically neglected.

## Conclusions

Assuming the value of the gradient of refractive index in all component prisms to be $10^{5} \mathrm{~cm}^{-1}$, and the same signs of focal lengths of these prisms in one of the three coordinate planes, we obtain the following values of the focal lengths:

$$
\begin{aligned}
f_{X Y} & =350 \mathrm{~m} \\
f_{Y Z} & =350 \mathrm{~m}
\end{aligned}
$$

for the first Porro prism system, and

$$
\begin{aligned}
& f_{X Y}=350 \mathrm{~m}, \\
& f_{Y Z}=250 \mathrm{~m} .
\end{aligned}
$$

for the second.
The existence of axial astigmatism caused by prisms with a constant gradient of refractive index was observed in the case of a Dove prism and also previously described [1]. Now, it is interesting if so small a value of the gradient $10^{-5} \mathrm{~cm}^{-1}$ may affect the resolution power of the prisms. To verify that, a model of a singly reflecting rectangular prism with a focal length equal to 300 m has been made. Its resolution power was checked by means of collimator and telescope with a focal lengths of 50 cm . It appeares, that the straight lines of the test were spread along the main section of the prism.

## Влияиие лииейной неоднородности стекда на оптический пучок, проходящий сквозь системы призм Порро

В стагье представлено влияние линейной оптической неоднородности показателя преломления оптического стекла на астигматизм некоторых отражающих систем призм. Это показано на примерах призм Порро первого и второго рода. Пучок лучей после выхода из системы призм Порро собирается в двух взаимно-перпендикулярных линиях. Это явление ведёт к понижению разрешающей способиости призм.

## References

[1] Lisowska, B., Bodnar, Z., Ratajczyk, F., The Influence of Linear Glass Heterogeneity on the Optical Beam Pass through Certain Reflecting Prisms, Appl. Opt., 7, 11, 1968.
[2] Bodnar, Z., Ratajczyk, F., Pomiary, Automatyka, Kontrola, 11, 175, 1965.

| Page | Line | Instead of | Read |
| :---: | :---: | :---: | :---: |
| $\begin{aligned} & 56 \\ & 60 \end{aligned}$ | 3 from the bottom 5-7 from the top | ( $x, y$ ) -plane <br> The error of measurement $\Delta \eta$ was estimated statistically. It turned out ihat $\Delta \eta \leqslant \pm 0.1$ (relative error $\Delta \eta / \eta$ approx $4.5 \%)$. | $(x, z)$-plane <br> The errol of measurement $\Delta n$ was estimated statistically. It turned out that $\Delta n \leqslant \pm 0.1$ (relative error $\Delta n / n$ approx $4.5 \%$ ). |


[^0]:    * B. Lisowska, F. Ratajczye, Instytut Fizyki Technicznej Politechniki Wrocławskiej, Wrocław, Wybrzeże Wyspiańskiego 27, Poland.

