*Kazimierz Brudzewski**

Use of the Mueller Matrices in Ellipsometry

The Mueller technique is adequate to describe most optical systems using polarized light. The use of the Mueller matrices for ellipsometric calculation is very promising. Application of this method to analysis of the ellipsometer arrangement suggests that a new arrangement might be constructed. This new type of the optical system can be constructed without rotating elements and modulators. The new type of the ellipsometer works on the principle of the independent and simultaneous measurements of all the Stokes parameters [1].

Introduction

Ellipsometry is a very sensitive and versatile optical method for analysis of specularly-reflecting surfaces or of films deposited on them. The changes of the polarization state of the light reflected from a surface are measured by means of that technique. Such measurements enable to calculate the optical constants of a surface as well as the thickness and refractive index of a thin film overlaying that surface. The technique, instrumentation, and calculations have previously been described [2], [5], [6]. However, so far**), these calculations have been carried out by use of the classical method, without application of the matrices. In this paper application of the Mueller matrices to ellipsometry is presented. A mathematical model of the ellipsometer arrangement is obtained by the Mueller technique. In this model the polarized light is represented by a Stokes vector and each component of the optical system is described by 4×4 matrices called the Mueller matrices. The Mueller matrix for the whole optical system can be found by multiplication of matrices for the individual parts.

The analysis of the ellipsometric arrangement

The fundamental optical arrangement of the ellipsometer is shown in Fig. 1. From Fig. 1 it is seen that the Mueller matrix for a whole optical system can be found by multiplication of the matrices

$$\hat{A}\cdot\hat{S}\cdot\hat{Q}\cdot\begin{bmatrix}1\\ \cos 2P\\ \sin 2P\\ 0\end{bmatrix} \tag{1}$$

where $\begin{bmatrix}1\\ \cos 2P\\ \sin 2P\\ 0\end{bmatrix}$ — the Stokes vector of the light from the polarizer.

$\hat{Q}, \hat{S}, \hat{A}$ — Mueller matrices of the compen sator, surface and analyser

P — azimuth of the polarizer (see Fig. 2)

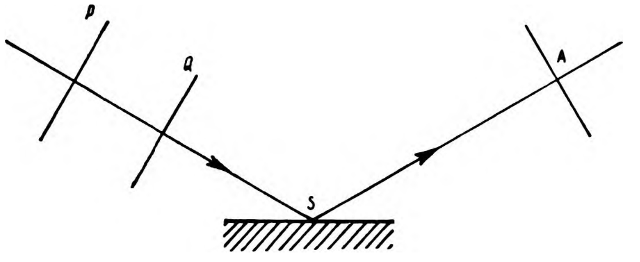

Fig. 1. Optical components in an ellipsometer: P — polarizer, Q — compensator, S — surface of the sample, A — analyser

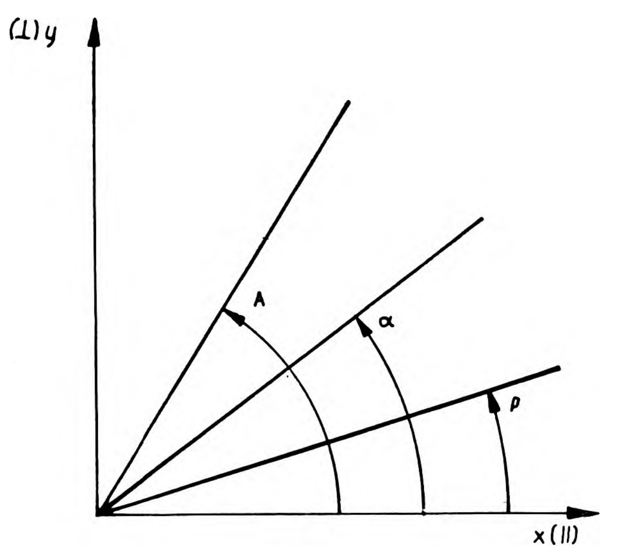

Fig. 2. The azimuth reference frame. The azimuth of the plane of transmitted polarization of polarizer and analyser is P and A. The azimuth of the fast axis of the compensator is α. The axis x of the right-handed Cartesian coordinate system is parallel to the plane of incidence

* Institute of Physics Technical University, Warsaw.

** Mac Crackin applied the matrieces calculations analysis of instrumental errors in ellipsometry [3].

The Mueller matrix of the ideal compensator (ideal compensator introduces only the phase shift) is given by

$$\hat{Q}(\gamma, a) = \hat{R}(-2a)\hat{B}(\gamma)\hat{R}(2a) \tag{2}$$

where $\hat{R}$ and $\hat{B}$ operators describe rotations about the center of v the Poincare sphere,

γ denotes a relative phase retardation along axes of the compensator,

a denotes an azimuth of the fast axis of the compensator.

The matrix for the surface is

$$\hat{S}(\Psi, \Delta) = \tag{3}$$

$$= \begin{bmatrix} 1 & -\cos 2\Psi & 0 & 0 \\ -\cos 2\Psi & 1 & 0 & 0 \\ 0 & 0 & \sin 2\Psi \cos \Delta & \sin 2\Psi \sin \Delta \\ 0 & 0 & -\sin 2\Psi \sin \Delta & \sin 2\Psi \cos \Delta \end{bmatrix}$$

where Ψ and Δ are two ellipsometric parameters defined from the known relations. Multiplying the Stokes vector of the light from the polarizer by the Mueller matrices of the compensator and the surface gives the vector of the light entering the analyser.

$$\hat{S}(\Psi, \Delta)\hat{R}(-2a)\hat{B}(\gamma)\hat{R}(2a)\begin{bmatrix} 1 \\ \cos 2P \\ \sin 2P \\ 0 \end{bmatrix}. \tag{4}$$

If the compensator is a quarter-wave plate with the azimuth of the fast axis 45° then

$$a = 45^\circ \qquad \gamma = 90^\circ$$

Substituting this value into expression (4), we have

$$\begin{bmatrix} 1 \\ -\cos 2\Psi \\ \sin 2\Psi \sin(2P+\Delta) \\ \sin 2\Psi \cos(2P+\Delta) \end{bmatrix}. \tag{5}$$

Since this light is extinguished by the analyser, it must be linearly polarized, therefore

$$\sin 2\Psi \cos(2P+\Delta) = 0 \tag{6}$$

and the azimuth A of the analyser is

$$\mathrm{tg}\, 2A = -\mathrm{tg}\, 2\Psi \sin(2P+\Delta) \tag{7}$$

The solution equation (6) and (7) can be obtained in the form

$$\begin{aligned} A = \Psi \quad &\text{and} \quad P = -\frac{\Delta}{2} - 45^\circ, \\ A = -\Psi \quad &\text{and} \quad P = -\frac{\Delta}{2} + 45^\circ. \end{aligned} \tag{8}$$

Two sets of solutions, called zones, are obtained.

In an analogous way one may derive expressions for the case of an imperfect compensator. The imperfect compensator (imperfect quarter-wave plate) is characterized by the ratio T, of the transmittance along its slow axis to the transmittance along the fast axis and the relative phase retardation γ. The Mueller matrix of the imperfect quarter-wave plate can be written

$$\hat{Q}(\gamma, a) = \hat{R}(-2a)\hat{D}(\gamma, T)\hat{R}(2a) \tag{9}$$

Assuming that T is near 1 and γ near 90° then we can use only the first-order approximation for the matrix $\hat{D}$

$$\hat{D}(\gamma, T) \approx \begin{bmatrix} 1 & \varepsilon & 0 & 0 \\ \varepsilon & 1 & 0 & 0 \\ 0 & 0 & -\delta & 1 \\ 0 & 0 & -1 & -\delta \end{bmatrix}. \tag{10}$$

$$T = 1+\varepsilon$$
$$\gamma = 90^\circ + \delta$$

Substituting (9) and (10) into (4) we have the vector of the light entering the analyser

$$\begin{bmatrix} 1+\varepsilon \sin 2P + \delta \cos 2\Psi \cos 2P \\ -\cos 2\Psi (1+\varepsilon \sin 2P) - \delta \cos 2P \\ \sin 2\Psi \cos \Delta (\varepsilon + \sin 2P) + \sin 2\Psi \sin \Delta \cos 2P \\ -\sin 2\Psi \sin \Delta (\varepsilon + \sin 2P) + \sin 2\Psi \cos \Delta \cos 2P \end{bmatrix}. \tag{11}$$

From (11), we find two equations

$$-\sin 2\Psi \sin \Delta (\varepsilon + \sin 2P) + \sin 2\Psi \cos \Delta \cos 2P = 0 \tag{12}$$

and

$$\mathrm{tg}\, 2A = \frac{\sin 2\psi \cos \Delta (\varepsilon + \sin 2P) + \sin 2\psi \sin \Delta \cos 2P}{-\cos 2\psi (1+\varepsilon \sin 2P) - \delta \cos 2P} \tag{13}$$

Similarly we obtain two solutions of equations (12) and (13):

$$A = \psi + \frac{1}{2}\delta \sin \Delta \sin 2\psi$$

and

$$P = -\frac{\Delta}{2} - 45^\circ - \frac{1}{2}\varepsilon \sin \Delta, \tag{14}$$

$$A = -\psi + \frac{1}{2}\delta \sin\Delta \sin 2\psi$$

and

$$P = -\frac{\Delta}{2} + 45^\circ + \frac{1}{2}\varepsilon \sin\Delta .$$

The terms with the second and higher powers of ε and δ in (14) are neglected. It should be emphasized that these two solutions are in good agreement with results obtained previously [3], [5].

Conclusions

From relation (3) it can be easely seen that the ellipsometric parameters Δ and ψ can be obtained as the ratio of the Stokes parameters. For example, if a linearly polarized light beam with azimuth $+45^\circ$ falls on the sample, then the ratio of the Stokes parameters gives Δ and ψ in the form

$$\mathrm{tg}\Delta = -\frac{S}{C} \qquad \cos 2\psi = -\frac{M}{I} \tag{15}$$

where I, M, C, S — Stokes parameters (we use the notation of Shurcliff [4]).

Therefore the measurements of the Stokes parameters are a hopeful experimental method in ellipsometry. This is of particular value for the automatic ellipsometer construction.

The author is most grateful to doc. dr Halina Chęcińska for her help and interest in the work.

Sur l'utilisation de la matrice de mueller en ellipsométrie

Dans ce travail on a introduit une notation opérationnelle pour la matrice de Mueller qui facilite l'analyse des systèmes ellipsométriques. On a présenté un modèle mathématique des systèmes ellipsométriques à l'aide de la méthode de Mueller. On a établi aussi l'expression pour des défauts de l'ellipsomètre dus au fait que le compensateur n'est pas parfait. Enfin, on a proposé un nouveau type du système ellipsométrique.

Использование матрицы Мюллера в эллипсометрии

В работе вводится операторная нотация для матрицы Мюллера, которая облегчает анализ эллипсометрических систем. Приводится математическая модель эллипсометрических систем, полученная при помощи техники Мюллера. Выводится выражение на погрешности в эллипсометре, связанные с несовершенством компенсатора. В последней части работы предлагается новый тип эллипсометрической системы.

References

[1] Brudzewski K., Grochowski L., Patent P-144843 (1970).
[2] Mac Crackin F. L., Passagalia E., Stromberg R. R., Steinberg H. L., Res J., Natl. Bur. Std.. 67A 363 (1963).
[3] Mac Crackin F. L., JOSA, Vol. 60, No 1, 57 (1970).
[4] Shurcliff W. A., Ballard S. S., Polarized Light, Warsaw PWN 1968.
[5] Smith P. H., Surface Science, Vol. 16, 34 (1969).
[6] Winterbottom A. B., Trans. Faraday Soc. 42, 487 (1946).

Received, May 28, 1971