difference $T_1 - T_2$ is proportional to the distance between the image plane of slit S' and the plane F'. Since the plane F' coincides with the focal-length, the difference $T_1 - T_2$ is proportional to and has the same sign as $\Delta f'$.

After transformation the obtained electric signal, shown in Fig. 2, goes to an integrator which indicates directly the value of the focal-length deviation $\Delta f'$ expressed in per cent.

In the built prototype not a single but a number of slits S have been applied, located on the circumference of a rotating drum which moves them in the X-direction. The aperture D has been replaced by a suitable masking system placed in an illuminator, which separates two slim light beams from the pencil of rays. The distance between the beams is controlled and can be adjusted to the measured focal-length. The measuring slit SP is located in a movable head, which is to be placed at a proper distance from the seat G during nulling the instrument. The measuring slit sets automatically at an optimal breadth during the head movement. A suitable calibration system enables a precise control of the meter reading. The nulling of the instrument with the use of the reference lens lasts several minutes, while the measurement of the deviation $\Delta f'$ for each checked lens—several seconds.

The prototype of the tolerator has two ranges: $\Delta f'_{max} = \pm 0.01 f'$ and $\Delta f'_{max} = \pm 0.02 f'$, and has been designed for lenses with $f' = +10$ to $f' = 150$ mm.

As a result of tests on the prototype, it has been found that, with the optimal relative aperture equal to 1 : 4, the total error does not exceed:

$$\pm 0.01 f' \text{ for the range } \Delta f'_{max} = \pm 0.01 f'$$

and

$$\pm 0.002 f' \text{ for the range } \Delta f'_{max} = \pm 0.02 f'.$$

It is worth mentioning that the principle of measurement described above can be adopted for the construction of an instrument for measuring the focal-length deviation of divergent lenses, for microscopic objectives, for spectacle lenses etc.

Jan Paczóski, Bogna Klarner *

Analysis and Investigation of Frequency Function of a Dark Field Projector with the Slit System

In this article the problem of the frequency function of a dark field projector with the slit system is discussed. The frequency function of the projection lens with Schlieren bars is analysed here in detail and calculated. The calculated and measured characteristic curves are also presented.

Introduction

In the research work on the thermoplastic recording (one of the basic techniques of the recording on the deformed surfaces), we have also analysed and investigated the optical problems connected with the information retrieval from phase-modulating media.

We have employed a modified method of the dark field [1] to the readout of the relief recording.

El-Sum [2] initiated the discussion of the frequency function of a dark field projector with the slit system, the optical scheme of which is illustrated in Fig. 1. In his analysis of the methods of the information retrieval from phase-modulating media El-Sum discussed, the optical systems of the dark field in brief only. While characterizing a dark field projector

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with the slit system, the author gave only incomplete, ambiguous and multivocal conclusions as to the frequency function of this system. They were given without any explanation and without experimental data. Taking as a base the diffraction theory of the image being formed in the optical instruments in O'Neil's [3] approach, we had classified the Schlieren

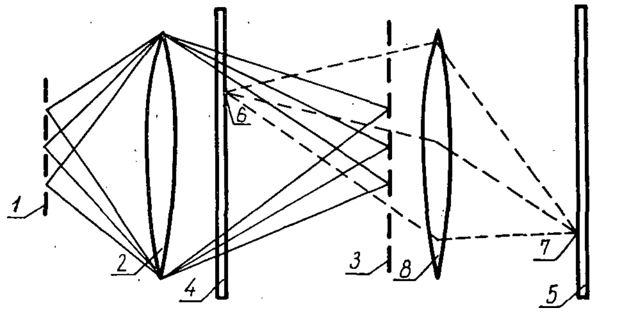

Fig. 1. Dark field projector with the slit system; 1— light source, 2 — condensing lens, 3 — Schlieren bars, 4 — thermoplastic film, 5 — screen, 6 — picture element, 7 — image of the picture element, 8 — projection lens

systems and, in particular, we had analysed widely the frequency function of the lens in terms of the multiaperture dark field projection.

The Frequency Function of the Projection Lens with Schlieren Bars

A very important question in analysis of the multiaperture dark field projection is the problem of the frequency function of the lens screened with Schlieren bars. Physically it means the introduction of the amplitude variations to the exit pupil of the lens.

Normalized transfer function of the lens is calculated on the basis of convolution integral [3]

$$\tau(\omega_x,\omega_y) = \frac{\iint\limits_{-\infty}^{\infty} F(\beta,\gamma)\cdot F^*(\beta-\omega_x,\gamma-\omega_y)\,d\beta\,d\gamma}{\iint\limits_{-\infty}^{\infty} |F(\beta,\gamma)|^2\,d\beta\,d\gamma}, \tag{1}$$

where $F(\beta,\gamma)$ represents the complex amplitude of the disturbance over the exit pupil, and β,γ are reduced angular coordinates (with dimensions of spatial frequencies).

On the ground of formula (1) we have calculated the one-dimensional transfer function of spatial frequencies $\tau(\omega_x)$ of the lens screened with Schlieren bars which is shown in Fig. 2. The simpliest and the most general mathematical formulation of that problem is possible by handling the set theory.

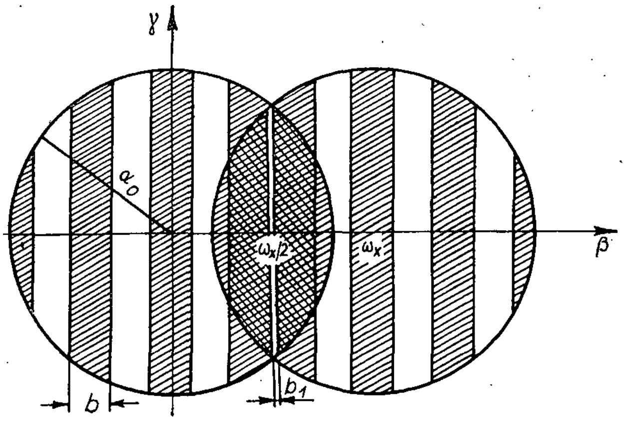

Fig. 2. Convolution of circular aperture with the perpendicular amplitude variations

As the result of the detailed mathematical analysis we have come to the statement that for $\omega_y = 0$

$$\iint\limits_{-\infty}^{\infty} F(\beta,\gamma)F^*(\beta-\omega_x,\gamma)\,d\beta d\gamma = \iint\limits_{\Gamma(\omega_x)} d\beta d\gamma, \tag{2}$$

where

$$\Gamma(\omega_x) = D_1^\beta(\alpha_0, b) \cap D_1^Z(\alpha_0, b, \omega_x), \tag{3}$$

and

$D_1^\beta(\alpha_0, b)$, $D_1^Z(\alpha_0, b, \omega_x)$ are the subsets such that

$$D_1^\beta(\alpha_0, b) = \Big\{(\beta,\gamma) : (\beta^2+\gamma^2 \leqslant \alpha_0^2) \wedge \Big[\beta\epsilon\Big(\frac{b}{2}+2\,nb,\ \frac{3}{2}b+2\,nb\Big) \vee \beta\epsilon\Big(-\frac{b}{2}-2nb,\ -\frac{3}{2}b-2nb\Big)\Big],\ n = D,\ 1,\ 2\ldots\Big\}. \tag{4}$$

$$D_1^Z(\alpha_0, b, \omega_x) = \Big\{(z,\gamma) : (z^2+\gamma^2 \leqslant \alpha_0^2) \wedge \Big[z\epsilon\Big(\frac{b}{2}+2nb,\ \frac{3}{2}b+2nb\Big) \vee z\epsilon\Big(-\frac{b}{2}-2nb,-\frac{3}{2}b-2nb\Big)\Big],\ n = 0, 1, 2\ldots\Big\}, \tag{5}$$

where

$$z = \beta - \omega_x.$$

It appears from the definition of the subsets D_1^{β}, D_1^{Z} and $\Gamma(\omega_x)$ that $\iint\limits_{\Gamma(\omega_x)} d\beta d\gamma = \tau(\omega_x)$ equals the sum of the surface of adequate segments of the circle and the differences of the surface of the segments of the circle.

The subset $\Gamma(\omega_x)$ consists of the bars the width of which expresses the formula

$$b_1 = b - |\text{entier}(\omega_x/2b + 0.5) \cdot 2b - \omega_x|. \qquad (6)$$

The above analysis provided facilities for giving shape to the numerical algorithm, which was programmed in ALGOL 1204 for the ODRA 1204 computer. This program enables tabulating $\tau(\omega_x)$ with any step $\Delta\omega_x$. The parameters for the program are: α_0, b and $\Delta\omega_x$. The exemplified calculated curve is illustrated in Fig. 3.

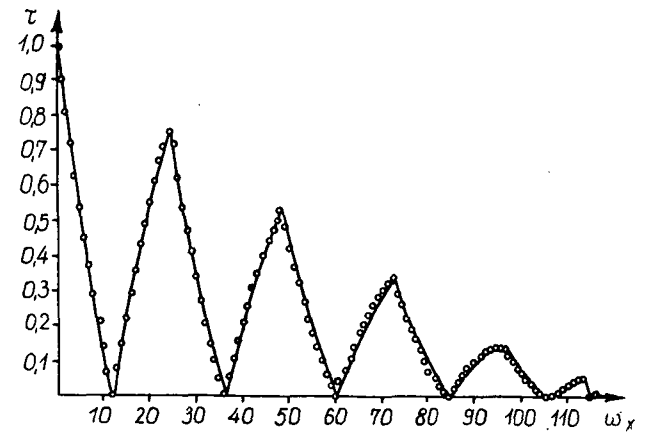

Fig. 3. Calculated frequency function of the projection lens with Schlieren bars ($\alpha_0 = 6.5$ mm; $b = 1.5$ mm)

The measurements of the frequency function were done with the help of the measurement stand described in [4]. It was measured in an indirect way according to the conception of the measurements worked out by the authors.

The measured frequency function of the Tessar Zeiss Jena 3557917 lens screened with Schlieren bars is given in Fig. 4. Its transfer function without screening is represented in Fig. 5.

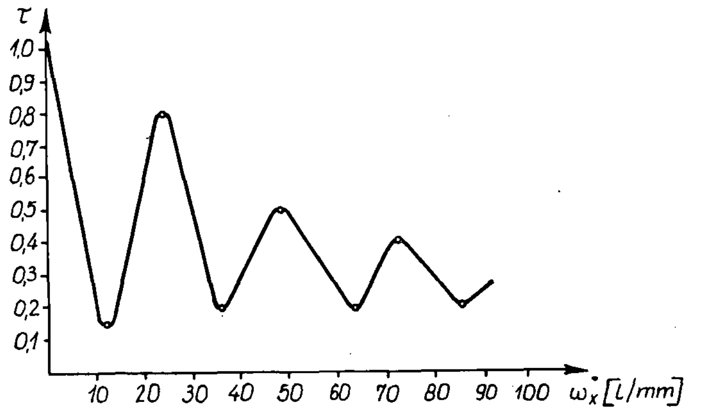

Fig. 4. Measured frequency function of Tessar lens with Schlieren bars (focal length — 75 mm, relative aperture — 5.6)

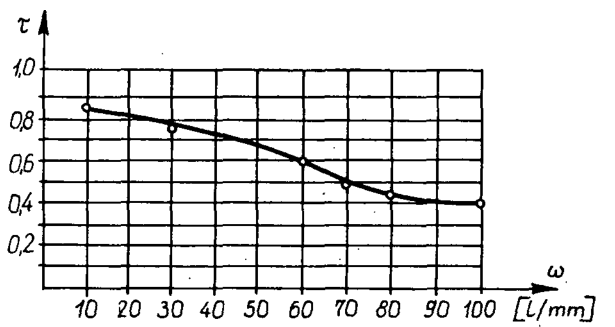

Fig. 5. Transfer function of Tessar lens

Conclusions

Above we have presented the method of the frequency function calculation for the different dimensions of the exit pupils of the lens and the different dimensions of the Schlieren bars.

The measurements have given a good qualitative verification of the data calculated for an ideal lens.

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