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Correction of the Spherical Aberration in Polychromatic Light

In optical systems with a spherical aberration and a longitudinal chromatic aberration it is theoretically possible to achieve the perfect imaging even for a rather large secondary spectrum. But it is necessary that both: monochromatic corrections and the correction of chromatic aberration are chosen in such a way that the best image planes of monochromatic aberrations are identical.

1. Introduction

Köhler and Metzmacher [5] used in their work the modulation transfer function method for studying the quality of colour correction. They studied two cases of achromatic correction and it is clear now that the correction with a greater secondary spectrum gives in this case more advantageous results than that with the smaller spectrum. We find here a certain contradiction because, from the colorimetric point of view, we would tend to keep the secondary colour spectrum as small as possible, but this condition is not quite justified with small correctional differences. When we were looking for an answer to this question we came to some conclusions that could result in interesting consequences.

2. Correction of the Chromatic Aberration of the System Without Monochromatic Aberrations

Imaging a point by the ideal optical system using the light of wavelength λ the distribution of the intensity on the optical axis is given by the relation [1]

$$I(\lambda) = I_0(\lambda)\left[\frac{\sin\beta(\lambda)}{\beta(\lambda)]}\right]^2 \tag{1}$$

where $I_0(\lambda)$ is the intensity in the middle of the diffraction pattern. According to [1] the argument

$$\beta(\lambda) = \frac{\pi}{8}\,\frac{x_0(\lambda)}{\lambda c^2}, \tag{2}$$

where $x_0(\lambda)$ is the distance of imaging plane from the paraxial plane for the wavelength λ and c the $f-$ number of the optical system. In a system with longitudinal chromatic aberration the position of the focal plane depends on the wavelength of light. The magnitude of this aberration is usually characterized by the distance $\Delta f(\lambda)$ of the focal imaging plane F for the light of wavelength λ from the focal plane F_0 belonging to the basic wavelength λ_0. Taking X_0 as the distance of the imaging plane from the focus plane F_0 and $x_0(\lambda)$ the distance of the imaging plane from the focus plane F (cf. Fig. 1) we may write

$$x_0(\lambda) = X_0 - \Delta f(\lambda). \tag{3}$$

In the case of polychromatic light the resulting intensity is the superposition of contributions of each monochromatic component, account being taken of

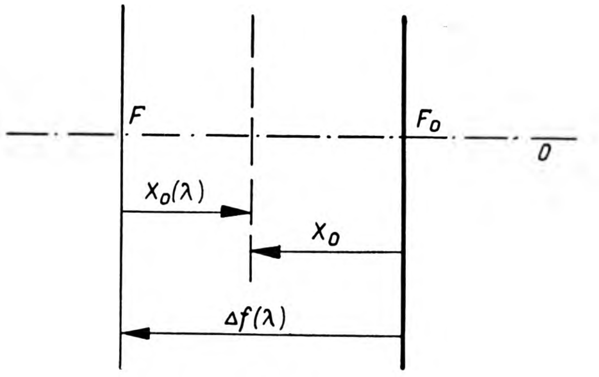

Fig. 1

the spectral distribution of the light source $\psi(\lambda)$ as well as of the spectral sensibility of the radiation detector $\varphi(\lambda)$. The intensity is then

$$E = \int_0^\infty E(\lambda)\left[\frac{\sin\beta(\lambda)}{\beta(\lambda)}\right]^2 d\lambda, \tag{4}$$

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where

$$E(\lambda) = \psi(\lambda)\varphi(\lambda). \tag{5}$$

The general integral expression (4) will be used to determine the maximum of intensity on the optical axis for an arbitrary argument $\beta(\lambda)$.

For small values of $\beta(\lambda)$, i.e. for small quantities $x_0(\lambda)$ or sufficiently large values of c, the function $\sin\beta(\lambda)$ may be expanded in a power series and truncated after the second term. Using the relations (2) and (3) we can transform the expression (4) to the form

$$E = \int_0^\infty E(\lambda)\left\{1 - \frac{\pi^2}{192}\left[\frac{X_0 - \Delta f(\lambda)}{\lambda c^2}\right]^2\right\} d\lambda. \tag{6}$$

If we now want to determine the maximum of intensity on the optical axis then according to the condition

$$\frac{dE}{dX_0} = 0 \tag{7}$$

we obtain

$$\int_0^\infty \frac{E(\lambda)}{\lambda^2}[X_0 - \Delta f(\lambda)]\, d\lambda = 0, \tag{8}$$

where the f— number c is supposed to differ only a little for various wavelengths from the visible spectrum. Let us put

$$\alpha(\lambda) = \frac{E(\lambda)}{\lambda^2} \tag{9}$$

and denote by $\overline{X}_0$ the defect of focus characterizing the position of the best imaging plane. Then from (8) we obtain

$$\overline{X}_0 = \frac{\int_0^\infty \alpha(\lambda)\Delta f(\lambda)\, d\lambda}{\int_0^\infty \alpha(\lambda)\, d\lambda}. \tag{10}$$

In the case that this best image plane would be in the position of the focus plane for the basic wavelength λ_0 we may write

$$\int_0^\infty \alpha(\lambda)\Delta f(\lambda)\, d\lambda = 0. \tag{11}$$

This condition can be taken as the criterion of the accuracy of the correction of the chromatic aberration. The function $\alpha(\lambda)$ comprises the spectral distributions of the source and detector and this is why the compact expression (11) seems to be suitable for further theoretical study of the function of the chromatic aberration.

Studying the best correction of the chromatic aberration in dependence on the spatial frequency R we start from the definition of the modulation transfer function $D(R)$ for the imaging in white light [2], [5]

$$D(R) = \frac{\int_0^\infty E(\lambda)D(R, \lambda)\, d\lambda}{\int_0^\infty E(\lambda)\, d\lambda}, \tag{12}$$

where $D(R, \lambda)$ is the function for the monochromatic light of wavelength λ. For small aberrations the function $D(R, \lambda)$ may be written in the form [4], [6]

$$D(R, \lambda) = D_0(R, \lambda) - D_1(R, \lambda). \tag{13}$$

The function

$$D_0(R, \lambda) = \frac{2}{\pi}(\arccos\, \varrho - \varrho\sqrt{1-\varrho^2}\,) \tag{14}$$

represents the modulation transfer function of the ideal system where

$$\varrho = R\lambda c \tag{15}$$

and the function $D_1(R, \lambda)$ depends on the correction of the system. For a defocused system with longitudinal chromatic aberration we have [6]

$$D_1(R, \lambda) = \frac{1}{16}\left[\frac{X_0 - \Delta f(\lambda)}{\lambda c^2}\right]^2 T_1(R, \lambda), \tag{16}$$

where the function $T_1(R, \lambda)$ may be written in the form [3]

$$T_1(R, \lambda) = \pi\varrho^2\left[(1+4\varrho^2)\arccos\, \varrho - \right.$$
$$\left. - \frac{2\varrho^2+13}{3}\varrho\sqrt{1-\varrho^2}\right]. \tag{17}$$

From the condition

$$\frac{\partial D(R)}{\partial X_0} = 0 \tag{18}$$

we get the position of the best image plane

$$\overline{X}_0(R) = \frac{\int_0^\infty \alpha(\lambda)\Delta f(\lambda)T_1(R, \lambda)\, d\lambda}{\int_0^\infty \alpha(\lambda)T_1(R, \lambda)\, d\lambda}, \tag{19}$$

where $\alpha(\lambda)$ is the known function (9).

If the plane $\overline{X}_0(R)$ is again situated in the focal plane for the basic wavelength λ_0 we obtain

$$\int_0^\infty \alpha(\lambda)\Delta f(\lambda)T_1(R, \lambda)\, d\lambda = 0 \tag{20}$$

and similarly as in the last case we may say that this condition can be taken as a criterion of the best correction of the chromatic aberration for the imaging of the spatial frequency R.

3. Characteristic Quantities of Spherical Aberration

The spherical aberration Δx may be approximated in dependence on the ray height h in the fifth order theory by a parabola of the fourth order. In Fig. 2

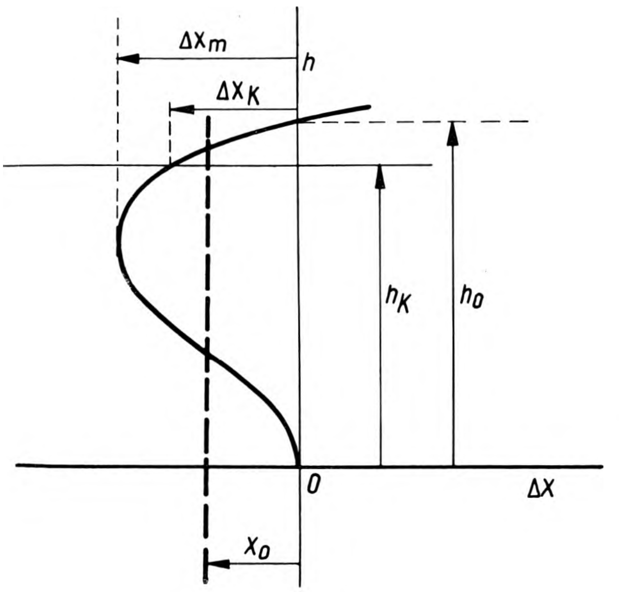

Fig. 2

we can denote the height correction as h_0, the maximum ray height as h_k, the maximum spherical aberration as Δx_m and the aberration for the maximum height as Δx_k. If we know the quantities h_0, Δx_m or Δx_k we can determine Δx depending on h by following relations [1]

$$\Delta x = 4q(q_0-q)\frac{\Delta x_m}{q_0^2} \tag{21}$$

and

$$\Delta x = q(q_0-q)\frac{\Delta x_k}{q_0-1}, \tag{22}$$

where the quantities q and q_0 are given by expressions

$$q = \left(\frac{h}{h_k}\right)^2 \quad \text{and} \quad q_0 = \left(\frac{h_0}{h_k}\right)^2. \tag{23}$$

The position of the image plane is characterized by x_0 giving the distance of the image plane from the position of the considered focal plane. Ratios

$$\eta_m = \frac{x_0}{\Delta x_m} \quad \text{and} \quad \eta_k = \frac{x_0}{\Delta x_k} \tag{24}$$

characterize the position of the image plane in dependence on both the quantity and the direction of the spherical aberration.

From the view-point of the wave optics the spherical aberration may be expressed by the wave aberration [1], [7]

$$\frac{W(q)}{\lambda} = \frac{1}{8\lambda c^2}\int_0^q (\Delta x - x_0)\,dq, \tag{25}$$

where λ is the wavelength of the considered monochromatic light and c is the f-number of the given optical system. Then, following the integration in (25), the expression $kW(q)$, where $k = \frac{2\pi}{\lambda}$ is the wave factor, may be written in the form [7]

$$kW(q) = \xi(A_1 q + A_3 q^2 + A_5 q^3), \tag{26}$$

where we have denoted

$$\xi_m = \frac{\Delta x_m}{\lambda c^2}, \quad A_1 = -\frac{1}{4}\eta_m, \quad A_3 = \frac{1}{2q_0}, \quad A_5 = -\frac{1}{3q_0^2} \tag{27}$$

and

$$\xi_k = \frac{\Delta x_k}{\lambda c^2}, \quad A_1 = -\frac{1}{4}\eta_m, \quad A_3 = \frac{q_0}{8(q_0-1)},$$

$$A_5 = \frac{1}{12(q_0-1)} \tag{28}$$

respectively.

If we study the position of the best image plane in the monochromatic light, we get from the condition of the maximum intensity on the optical axis [1]

$$\bar{x}_0(\lambda) = \frac{2}{5}\Delta x_{m,k}(\lambda)[10A_3(\lambda)+9A_5(\lambda)] \tag{29}$$

or from the condition of the maximum contrast [7]

$$\bar{x}_0(R,\lambda) = \frac{2}{T_1(R,\lambda)}\Delta x_{m,k}(\lambda)[A_3(\lambda)T_5(R,\lambda)+$$
$$+A_5(\lambda)T_6(R,\lambda)]. \tag{30}$$

Here, $\Delta x_{m,k}(\lambda)$ denotes the characteristic spherical aberration Δx_k or Δx_m for the light of wavelength λ, $A_3(\lambda)$ and $A_5(\lambda)$ are the correction coefficients, which are given by (27) and (28). $T_1(R,\lambda)$ is given by the expression (17), while $T_5(R,\lambda)$ and $T_6(R,\lambda)$ are functions of spatial frequency R, which are tabulated and have been determined in [6].

4. Integral Conditions of the Best Correction of Chromatic Aberration in the Presence of Spherical Aberration

Expressions for the position of the best image plane in polychromatic light were derived in [8]. Results obtained in this work can be written as follows:

$$\bar{X}_0 = \frac{\int_0^\infty a(\lambda)[\bar{x}_0(\lambda) + \Delta f(\lambda)]\, d\lambda}{\int_0^\infty a(\lambda)\, d\lambda} \tag{31}$$

or

$$\bar{X}_0 R = \frac{\int_0^\infty a(\lambda)[\bar{x}_0(R, \lambda) + \Delta f(\lambda)] T_1(R, \lambda)\, d\lambda}{\int_0^\infty a(\lambda) T_1(R, \lambda)\, d\lambda}, \tag{32}$$

where $a(\lambda)$ is the well known expression (9).

Supposing that the best image plane in polychromatic light is situated in the focal plane F_0 belonging to the basic wavelength λ_0 we can obtain the condition for the best correction of the chromatic aberration. This condition may be expressed by the relation

$$\int_0^\infty a(\lambda)[\bar{x}_0(\lambda) + \Delta f(\lambda)]\, d\lambda = 0 \tag{33}$$

or in the case of imaging the given spatial frequency R in the form

$$\int_0^\infty a(\lambda)[\bar{x}_0(R, \lambda) + \Delta f(\lambda)] T_1(R, \lambda)\, d\lambda = 0. \tag{34}$$

5. Discussion

In the relations (11) and (20) we must take into account the expression $a(\lambda)$. This function can be eliminated only in the case, if the chromatic aberration

$$\Delta f(\lambda) = 0. \tag{35}$$

This condition is — in fact — never fulfilled for any lens system. In the relations (33) and (34) we have the same situation as in the case mentioned before, if

$$\Delta f(\lambda) + \bar{x}_0(\lambda) = 0 \tag{36}$$

or, if

$$\Delta f(\lambda) + \bar{x}_0(R, \lambda) = 0 \tag{37}$$

for all wavelengths of a given spectral interval. We can say that in this case the optical system is identical with the aberration of a free optical system. Thils fact suggests that all the best image planes for aal wavelengths coinc ide ard are situated in the focl plane for the basic wavelength λ_0.

The practical realization of the conditions (36) or (37) is very difficult but possible in principal. In paper [9] is introduced the graphical method for the following height corrections according to relation (36). It is possible that a detailed study of this conclusions — with respect to the fifth-order theory — would permit to determine the dispersions as well as the optical system parameters which might be utilized in practice.

References

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