*Marcin Leśniewski**

Tolerances for Spherical Surfaces of Optical Elements Satisfying the Minimum Manufacture Cost Requirement

This paper presents a method of working shape tolerance determination for spherical surfaces in optical elements under the assumption of minimum cost of manufacture.

Spherical surfaces have been employed in most optical systems used so far. The shape of these surfaces, obtained in the manufacture process, differs from that planned by the designer as a consequence of applied technology, limited accuracy of control methods and means, variations of external influences and the workers' qualifications. Real surfaces have an approximately toric shape, and ovalization of the fringes, which is observed during the control by means of interference methods, are the measure of its deformation. Torical deformation of the surface leads to the appearance of typical off-axis type aberrations on the optical axis of a system — especially astigmatism, which is a function of two factors: the surface ovalization moduli, and mutual angular orientation of the surfaces resulting from a accidental positioning of the lenses in their mounting.

During the fitting-up, it is possible to compensate the axial astigmatism of the system by means of turning one of the elements around the optical axis, but in mass production such a procedure is, in general, too expensive. The method of determining the working tolerances of the shape of spherical surfaces in optical elements, adapted to mass production, is presented in the paper.

The basic relationships connecting the shape errors of the spherical surfaces of optical elements (expressed by the ovalization of interference fringes — ΔN) with the imaging errors (axial astigmatism of the system — ΔS) according to [2], [3] are described by the dependences (1, 2, 3) referred to below :

1. If the optical system mounted out of the randomly chosen optical elements of each sort exhibits axial astigmatism, which does not exceede the assumed value ΔS (i.e., if the possible compensation of the axial astigmatism is not taken into account) then

$$\Delta S = \frac{4\lambda}{u'^2} \sum_{i=1}^{p} A_i \Delta N_i, \tag{1}$$

where:

ΔS — permissible value of the axial astigmatism of the system,

λ — wavelength of light,

u' — image aperture of the system,

ΔN_i — ovalization of the i-th surface,

p — number of surfaces of the system,

A_i — factor depending on the construction parameters of the optical system:

$$A_i = |n_i' - n_i| \frac{h_i^2}{\Phi_i^2}$$

n_i' n_i, — image and object refractive indices of the i-th surface respectively,

h_i — height of striking the i-th surface by the aperture ray,

Φ_i — clear aperture diameter of i-th surface.

2. If (with the given probability x) the mounted system will not include any elements of maximum tolerances ΔN_i (i.e. ,only the modulus compensation of axial astigmatism is assumed) then:

$$\Delta S = \frac{4\lambda t_x \Lambda}{3u'^2} \sqrt{\sum_{i=1}^{p} A_i^2 \Delta N_i^2}, \tag{2}$$

where: t_x — standardized variable of the normal distribution,

Λ — characteristic factor of probability distribution of ovalization occurrence.

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*) Institute for Precise and Optical Instruments Design, Technical University of Warsaw, Warsaw, ul. Narbutta 87, Poland.

3. If (with the probability x) an optical system will not include optical elements of maximum tolerance, and its directional concentration is not the most disadvantageous, then:

$$\Delta S = \frac{4\lambda t_x \Lambda}{3\sqrt{p}u'^2}\sqrt{\sum_{i=1}^{p} A_i^2 \Delta N_i^2 + 2a_x \sum_{i=1}^{p}\sum_{k=i+1}^{p} A_i A_k \Delta N_i \Delta N_k}, \tag{3}$$

where: a_x — means the probable value of $\cos(2\Delta\varphi_{ik})$ which is calculated as $a_x = 1-\varepsilon$, where ε — defectiveness assumed ($\Delta\varphi_{ik}$, being an angle between orientations of i_i-th and k-th ovalizations).

Only the first and the third relationships from among the three (1, 2, 3) mentioned are different variants. The reason for mentioning the dependence (2) is that it is necessary for the solving of (3) and for a more explicit showing that the axial astigmatism depends on the moduli — ΔN_i as well as the directions — φ_i of the ovalization of optical system surfaces. The method of determining the optimum tolerances in the dimension chain by E. Wolniewicz [1] is applied for the calculation of the working tolerances. The method allows to calculate the tolerances in such a way that the manufacture cost is the smallest possible. The basic assumption of the method is that the relationship: costs K versus tolerance T looks like a equiaxial hyperbola. The equation of this hyperbola is $K = \frac{a}{T-b}+c$, where a, b, c are the parameters depending on its form. The main parameter describing the economical effects of this method is a, and its value depends on the employed technology, the type of optical glass geometry of elements and the like. Wolniewicz in his paper [1] considers, for instance, the dimension chain of the form

$$T_x = \sum_{i=1}^{p} T_i,$$

where:

T_i — component dimensions of the chain,

T_x — resultant dimension of the chain.

He obtains the minimum manufacture cost condition in the form:

$$\frac{T_i}{T_1} = \frac{a_i}{a_1}.$$

The working tolerances being related to one another by means of the ratios of the parameters a_i, which determine the economical effect of production.

The analogous conditions of minimum manufacture costs are obtained by using this method to the cases (1, 2, 3) refered to above. For the first case:

$$\frac{\Delta N_i}{\Delta N_1} = \sqrt{\frac{A_1 a_i}{A_i a_1}}. \tag{4}$$

For the second one:

$$\frac{\Delta N_i}{\Delta N_1} = \sqrt[3]{\frac{A_1^2 a_i}{A_i^2 a_1}}. \tag{5}$$

The so-called optimum working tolerances for the shape of spherical surfaces in optical elements can be explicitly calculated by solving the equations (1) and (4) under the assumption, that the astigmatism of optical system obtained in the fitting-up process does not exceed the assumed value.

$$\Delta N_n = \frac{u'^2 \Delta S}{4\lambda\sqrt{\frac{A_n}{a_n}}\sum_{i=1}^{P}\sqrt{A_i a_i}}. \tag{6}$$

If the exact values of a_i are not known then it is possible to calculate them by approximate formula:

$$a_i = \frac{K_{i1}-K_{i2}}{\frac{1}{T_{i1}}-\frac{1}{T_{i2}}};$$

where K_{i1}, K_{i2} mean the manufacture costs of the same part but with different tolerances T_{i1}, T_{i2} respectively.

The optimum tolerances are obtained by resolving the system of equations (2) and (5) in the case, when the assembled system does not include elements of maximum tolerances with the probability

$$\Delta N_n = \frac{0.75u'^2 \Delta S}{\lambda t_x \sqrt[3]{\frac{A_n^2}{a_n}}\sqrt{\sum_{i=1}^{P}\sqrt[3]{A_i^2 a_i^2}}}. \tag{7}$$

In the third, most general, case the employment of the mentioned method leads to very complicated relationships. An indirect method is therefore proposed. Adding and substracting the term $2(1-a_x)\sum_{i=1}^{P}\sum_{k=i+1}^{P} A_i A_k \Delta N_i \Delta N_k$ from the right-hand side of the equation (3) we obtain

$$\Delta S = \frac{4\lambda t_x \Lambda}{3\sqrt{pu'^2}} \sqrt{\left(\sum_{i=1}^{P} A_i \Delta N_i\right)^2 - 2(1-a_x)\sum_{i=1}^{P}\sum_{k=i+1}^{P} A_i A_k \Delta N_i \Delta N_k}$$

and therefore:

$$\sum_{i=1}^{P} A_i \Delta N_i = \sqrt{\left(\frac{0.75\sqrt{pu'^2}\,\Delta S}{\lambda t_x \Lambda}\right)^2 + 2(1-a_x)\sum_{i=1}^{P}\sum_{k=i+1}^{P} A_i A_k \Delta N_i \Delta N_k}. \quad (8)$$

The tolerances in the "angular compensation" term in the right-hand side of equation (8) are obtained by solving the simplified case of moduli compensation (7). The ratios of ovalization's moduli are determined by the same means (5) to be applied in the left-hand side of equation. Such an assumption is justified, because in practice the values of a_x do not differ much from the unit. That satisfies the condition of the modulus compensation of axial astigmatism. After solving the equation (8) as presented above, we arrive at:

$$\Delta N_n = \frac{0.75\sqrt{pu'^2}\,\Delta S}{\lambda t_x \Lambda \sqrt[3]{\frac{A_n^2}{a_n}} \sum_{i=1}^{P} \sqrt[3]{A_i a_i}} \times \sqrt{1 + \frac{2(1-a_x)}{\sqrt{p}\sum_{i=1}^{P}\sqrt[3]{A_i^2 a_i^2}} \sum_{i=1}^{P}\sum_{k=i+1}^{P} \sqrt{A_i A_k a_i a_k}}.$$

References

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