Letters to the Editor

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Kinematic Aberration at Optimal Position of the Intermediate Image in High-Speed Cameras with Image Commutation

On the base of a formula for kinematic aberration in high speed cinematography cameras with the image commutation (Miller type camera) the optimum working conditions for this camera have been specified. Formulae for optimal positions of the intermediate image in the vicinity of the rotating mirror, kinematic aberration value at the end points of the camera working sector and the rotating mirror position, at which the kinematic aberration approaches zero value, have been given. The results obtained enable a determination of the basic design parameters of the camera.

In paper [1] a formula for kinematic aberration, i.e. for the image spread caused by the mirror rotation in the Miller type camera [2], has been derived (see Fig. 1):

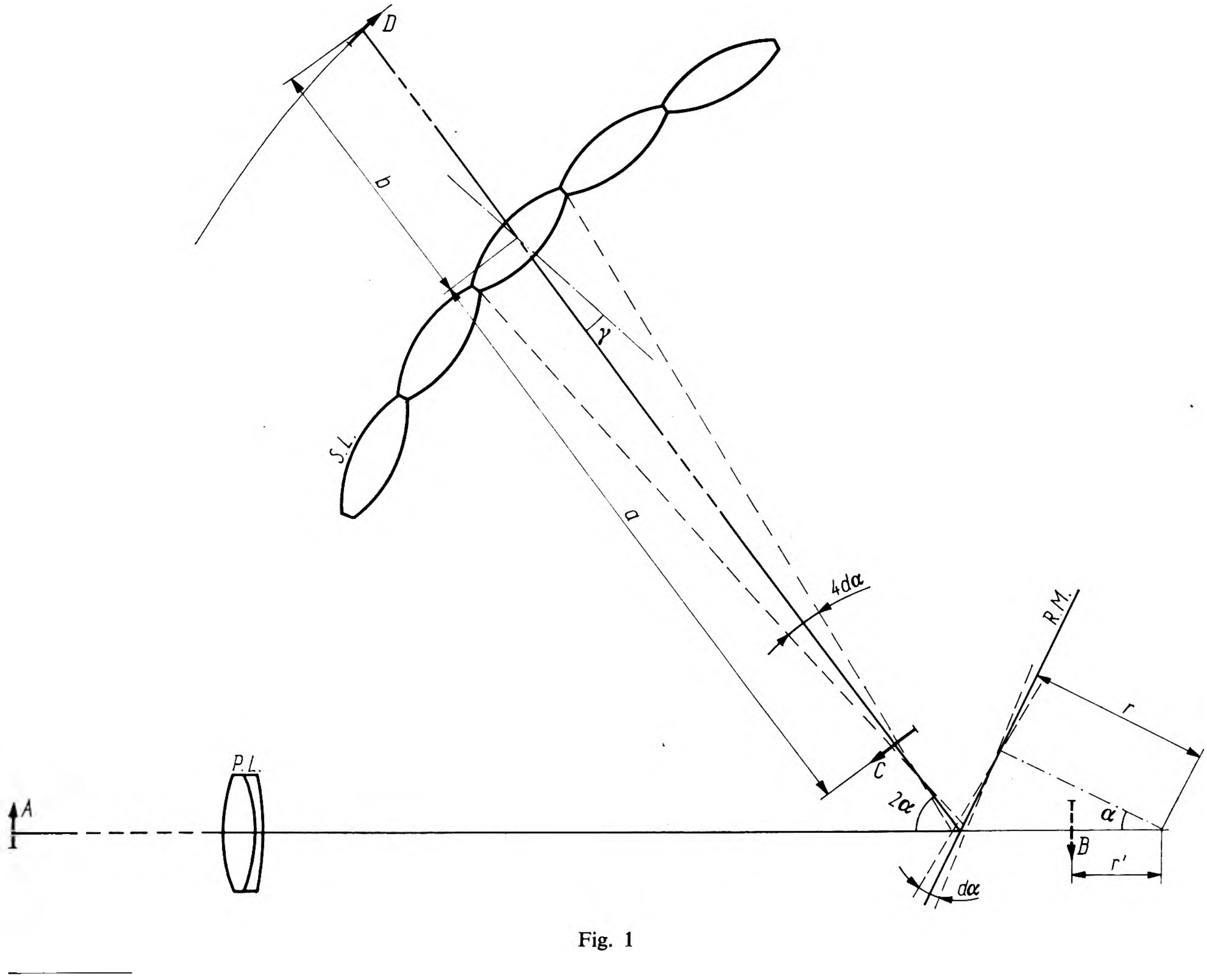

Fig. 1

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$$A_k = z(|\varrho \cos da - \cos a| + p \cos\gamma \sin|a|), \quad (1)$$

$$z = \frac{4\beta_r \sin da}{\cos\gamma}, \quad (2)$$

$$p = \frac{\beta h}{2b} = \frac{h}{2a}, \quad (3)$$

$$\varrho = \frac{r'}{r}, \quad (4)$$

where

- r — angle determining the position of the rotating mirror,
- r' — distance of the intermediate image B produced by the principal objective (*P.L.*) from the axis of rotation,
- β — magnification of the final image D with respect of the intermediate image C,
- $2da$ — angle of rotation of the *R.M.* mirror which corresponds to crossing the secondary lens area by the principle ray after having been reflected from the mirror,
- γ — angle between the ray hiting the middle point of the secondary lens and the optical axis,
- a — angular position of the rotating mirror,
- h — secondary lens diameter,
- a — distance of the intermediate image position C from the middle point of the secondary lens,
- b — distance between the middle point of the secondary lens and the final image D on the film,
- A_k — distance of the straight line segment, by which a fixed point in the object is shifted with respect to the film during exposure in the direction perpendicular to the rotation axis of the mirror.

In all real cameras the angles da and γ are very small, i.e. of order of tens of minutes of arc. Also, the factor p is usually small because $a \gg h$. In the camera described in [3] (with concave mirrors used as secondary objectives) the above mentioned parameters take the following values: $da \approx 13.2'$, $0 < \gamma < 19.5'$, $a \approx 500$, $\beta \approx 2$, $h = 7.62$.

Approximately, it may be assumed that γ, β, da, z and p are constant parameters independent of a.

The purpose of the present paper is to analyze the formula (1) in order to determine optimum conditions for camera operation.

The dependence of the kinematic aberration on a is — according to the formula (1) — mainly of cosine type. Thus, in the vicinity of the angle a_0 determined by the condition

$$\cos a_0 = \varrho \cos da \quad (5)$$

the minimum kinematic aberration occurs. Consequently, in order to achieve the optimal conditions for camera operation it is required that the values of the kinematic aberration at the end points of the operating sector were the same,

$$A_{k,b} = A_{k,e} \quad (6)$$

the operating sector of the camera being determined by the angular positions a_b and a_e of the rotating mirror, which correspond to the ray incident on first and the last secondary lens, respectively, after having been reflected from *R.M.* When neglecting the term containing $\sin|a|$ the condition (6) is reduced to

$$\varrho_0 \cos da - \cos a_b = -(\varrho_0 \cos da - \cos a_e). \quad (7)$$

The fulfillment of this condition is always possible by a proper settlement of the parameter r' during camera adjustment. From (7) and (4) we obtain

$$r'_0 = r\frac{\overline{\cos a}}{\cos da} \approx r\overline{\cos a}, \quad (8)$$

where

$$\overline{\cos a} = \frac{\cos a_b + \cos a_e}{2}, \quad (9)$$

denotes an arithmetic mean of cosines of the extreme angles of the working sector. Taking account of (8) a formula for optimal distribution of kinematic aberration within the whole sector may be obtained from (1) in the form

$$A_{k0} = z(|\overline{\cos a} - \cos a| + p\cos\gamma \sin|a|) \quad (10)$$

Minimum kinematic aberration appears in this case in the surrounding of the angle a_0 determined by the condition

$$\cos a_0 = \overline{\cos a}. \quad (11)$$

It is worth noting that in the case of an optimal distribution the position a_0 of the aberration minimum is independent of r and R; the latter being defined as a radius of the circle surrounding *R.M.*, on which the secondary lenses are distributed.

The values of r'_0 and a_0, which are given below, were calculated from (8) and (11) under assumption that $a_b = 0$ for three values of a_e:

a_e	r'_0	a_0
30°	0.93r	21°05′
45°	0.85r	31°24′
60°	0.75r	41°25′

Maximum values of aberration at the sector end points for optimal position of the intermediate images increase to

$$A_{k,s} = \frac{z}{2}(\cos\alpha_b - \cos\alpha_e + 2p\cos\gamma\sin|\alpha_e|) \quad (12)$$

in accordance with (10) and (9) for $|\alpha_e| > |\alpha_b|$.

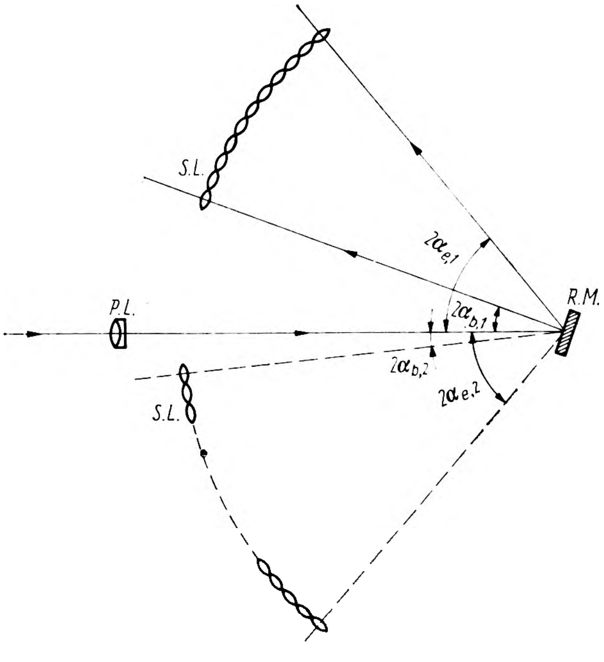

Fig. 2

If in a given camera the principal lens cooperates with two working sectors located, for instance, on both sides of the beam incident on the rotating mirror (Fig. 2), the smaller of the two values $|\alpha_{b,1}|$ and $|\alpha_{b,2}|$ and the greater of the two values $|\alpha_{e,1}|$ and $|\alpha_{e,2}|$ should be substituted to (8) and (10) in order to obtain the optimum conditions.

The formulae (12), (2) and (3) allow to chose a set of basic design parameters α_b, α_e, r and R ($d\alpha$, γ and a being dependent on R) so that the kinetic aberration A_k does not exceed an acceptable value.

References

[1] WNUCZAK E., Optica Applicata **3**,3, 67 (1973).
[2] MILLER C.D., Journal of the SMPTE **53**,3, 479 (1949).
[3] WNUCZAK E., KRZECZKOWSKI S., Optica Applicata 3,1, 29 (1973).

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