# Optical and Electrical Properties of Au and Ag in Relation to Free Electron Theory

The results of new measurements of the refractive index n and the absorption coefficient k of Au and Ag are discussed with respect to free electron theory. This permits for the calculation of the concentration N of free electrons and D.C. conductivity  $\sigma_0$ , which are compared with values obtained from the electrical conductivity and Hall effect measurements taken for the same films.

## 1. Introduction

Outside the range of interband transition due to bound electrons, the optical constants n and k at wave length  $\lambda$  (frequency  $\omega$ ) are related to the electronic parameters of the metal in the following equations, based on the free electron theory of metals [1, 2, 3].

$$-\varepsilon = k^{2} - n^{2} = -1 + \frac{4\pi N e^{2}/m^{*}}{\omega^{2}}$$
$$= -1 + \frac{4\pi\sigma_{0}\omega'}{\omega^{2}} = -1 + \frac{N e^{2}/m^{*}}{\pi c^{2}}\lambda^{2}, \quad (1)$$
$$\sigma = \frac{nk\omega}{2\pi} = \frac{N e^{2}/m^{*}}{\omega^{2}}\omega'$$

$$=\frac{\sigma_0 \, \omega^{-2}}{\omega^2} = \frac{\sigma_0 \, \omega^{-2}}{4\pi^2 c^2} \, \lambda^2, \qquad (2)$$

where,  $\varepsilon$  and  $\sigma$  are the frequency-dependent dielectric constants and electrical conductivity of the metal, respectively, N is the number of free electrons par unit volume,  $m^*$  is the effective mass of the electron,  $\omega'$  is the self frequency of the electron defined as the reciprocal of the relaxation time  $\tau$ ,  $\sigma_0$  is the D.C. conductivity, and c is the velocity of light in vacuum.

#### 2. Results and discussion

The results of the optical constants for Au and Ag have been reported in the previous paper. According to equation (1), the relation between  $(k^2 - n^2)$  and  $\lambda^2$  is linear. This is verified in case of Au and Ag, as shown in Fig. 1 and Fig. 2,



Fig. 1. Relation between  $(k^2 - n^2)$  and  $\lambda^2$  for Au

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respectively, with data of previous authors for comparison [1, 2, 4]. N (optical) has been calculated from the slope of the straight line, considering  $m^* = m = 9.1 \times 10^{-28}$  gm.  $N_a$  (theoretical) has been also calculated ( $N_a = dL/A$ , where d is the density of the metal, A its atomic weight and L Avogadro's number) considering one free electron per atom. N (electrical) was deduced from the present Hall effect measure-



Fig. 2. Relation between  $(k^2 - n^2)$  and  $\lambda^2$  for Ag

ments for both Au and Ag films, which were used before in the optical measurements. The data thus obtained are listed in Table 1. As it is visible N (optical) is in fair agreement with N (electrical). The effective number of free electrons per atom  $N/N_a$  (optical) being also given. Hence, the optical effective mass  $m^*/m$ (opt.) may be calculated. The resulting values are comparable with that obtained by CORAK [5], BEAGLEHOLE [6] for Au (1.16  $\pm$  0.08) and that reported by GIVENS [3] for Ag (0.98), respectively.

		Table 1	
	$\mathbf{A}\mathbf{u}$	$\mathbf{A}\mathbf{g}$	
N (optical	$4.79 \times 10^{22}$ elec./c.c.	$5.2 \times 10^{22}$ elec./c.c.	
$N_a$ (theoretical)	$5.89 \times 10^{22}$ elec./c.c.	$5.9 \times 10^{22}$ elec./c.c.	
N (electrical)	$4.96 \times 10^{22}$ elec./c.c.	$5.2 imes10^{22}$ elec./c.e.	
$N/N_a$ (opt.)	0.81	0.88	
$m^*/m$			
$= N_a/N$ (opt.)	1.23	1.13	

Fig. 3 represents the dependence of the conductivity  $\sigma = nk\omega/2\pi$  on the wavelength  $\lambda$  for Au, giving a threshold of interband transition at 0.6  $\mu$ m, corresponding to an energy E = 2.07 eV, due to the excitation of d electrons



Fig. 3. The dependence of  $\sigma$  on  $\lambda$  for Au

to the conduction band [7, 8]. At wavelength region longer than the absorption edge, the condutivity  $\sigma$  increases with increasing  $\lambda$ , as expected from eq. (2).

Fig. 4 represents the dependence of  $2nk/\lambda = 2\sigma/c$  on  $\lambda^2$  for Ag, showing similar behaviour





as Au, and indicating a peak at  $\lambda = 0.95 \ \mu m$ , which is possibly associated with interband transition of electrons either from the Fermisurface to the next higher empty band or from a lower lying filled band to the Fermi-surface [1, 9, 10, 11]. Beyond 1.5  $\mu m$ , the curve shows a continuous increase of  $\sigma$  with increasing  $\lambda$  as the theory (eq. (2)) expects.

According to eqs. (1) and (2), the Argand diagram for Au in Fig. 5 represents  $(k^2 - n^2 + 1)$ against  $\sigma = nk\omega/2\pi$  showing two straight lines



Fig. 5. The Argand diagram for Au

of different slopes (slope  $= 4\pi/\omega' = 4\pi\tau$ ) corresponding to two values of relaxation time  $\tau = 0.99 \times 10^{-14}$  and  $1.1 \times 10^{-14}$  s. This may be due to the fact the Fermi surface in Au is non-spherical [4, 12, 13, 14].

The relaxation time  $\tau$  of the free electron in Ag, calculated from the slope of Fig. 2 and the slope of the linear part of Fig. 4, gives  $\tau = 0.68 \times 10^{-14}$  s.

Using the values of N (opt) and  $\tau$ , the D.C. conductivity  $\sigma_0$  is calculated ( $\sigma_0 = Ne^2\tau/m$ ).

Values thus obtained are listed in Table 2 with the values of  $\sigma_0$  deduced from the present electrical measurements on the same films used before in the optical measurements.

Table 2

	Au	$\mathbf{A}\mathbf{g}$
τ (opt.)	$1 \times 10^{-14}$ s	$0.68 \times 10^{-14}$ s
$\sigma_0$ (opt.)	$1.3 \times 10^{17}$ e.s.u.	$0.9 \times 10^{17}$ e.s.u.
$\sigma_0$ (elect.)	$4.2 \times 10^{17}$ e.s.u.	$1.92 \times 10^{17}$ e.s.u.

It is clear that  $\sigma_0$  (opt.)  $< \sigma_0$  (elect.). This is attributed to the fact that the electrons near the surface have frequent collisions; therefore they have a shorter mean free path and a smaller relaxation time  $\tau$  than the electrons located deeper in the metal, which determine  $\sigma_0$  (elect.). Since the light waves penetrate to a very short distance into the metal they interact only with the electrons near the surface, therefore  $\sigma_0$  (opt.) is reduced [15, 16].

### Les propriétés optiques et électriques de Au et de Ag rapportées à la théorie des electronst libres

On a examiné, par rapport à la théorie des électrons libres, les résultats des mesures effectuées pour Au et Ag et concernant l'indice de réfraction de la lumière n et le coefficient d'absorption k. ('eci permet de calculer la concentration des électrons libres N ainsi que la conductivité spécifique en courant continu  $\sigma_0$ . Les résultats ont été comparés avec les valeurs de N et de  $\sigma_0$  obtenues des mesures de la conductibilité électrique et de l'effect Ifall qu'on avait effectuées sur les mêmes couches.

#### Оптические и электрические свойства Au и Ag с точки зрения теории свободных электронов

Результаты измерений коэффициента преломления света и коэффициента поглощения k, произведенных для Au и Ag, обсуждены с точки зрения теории свободных электронов. Благодаря этому становится возможным расчет концентрации свободных электронов N и удельной проводимости для постоянного тока  $\sigma_0$ . Результаты сопоставлены со значениями N и  $\sigma_0$ , полученными путем измерений электропроводности и эффекта Холла, проведенных на тех же пленках.

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Received, March 24, 1974

Received in the revised form, July 20, 1974