# On Some Properties of Aspectogram as Related to the Synthesis of Composite Holograms 


#### Abstract

The term "aspectogram" means here the photographic plate bearing the images of a given scene, which are formed with a fly's eye lens. The aspectogram properties are considered. It is shown that under some conditions a definite part of the spatial aspectogram spectrum corresponds to each plane in the object space located at a given distance from the lens array. Conditions are found under which this correspondence is unambiguous. It is shown that by filtering the aspectogram spectrum in a certain way, it is possible to synthesize intermediate perspectives, i.e. to obtain images reconstructing the object as viewed from points not coinciding with the lenses used for the original recording. The techniques of synthesizing intermediate perspectives is considered as applied to the problem of obtaining composite holograms characterized by a smooth change of appearance of an image as the observation point moves over the surface of the hologram. It is pointed out that the aspectogram laws admit the construction of optical systems which would record both the configuration and space position of three-dimensional objects.


One of the most difficult problems of holographic cinematograph is that holograms of outdoor scenes must be obtained under conditions precluding illumination with coherent light. The only way to solve this problem at present involves the synthesis of such a hologram on the basis of information obtained from the so-called perspectives (conventional photographs of a scene which are taken from different points of view).

The simplest method by which this operation could be done presents a combination of holography with the integral Lippman's photography [1,2] (the so-called composite hologram).

However, the image reconstructed from a composite hologram has inherent distortions, which are mainly due to the discrete changes and the lack of matching between the parts of the wave front corresponding to different perspectives.

One might hope to improve this distortions provided that the method of synthesis of the so-called intermediate perspectives was found, but this problem is not an easy one [3].

It is clear enough that the solution of the above and similar problems strongly depends on the method selected for the information

[^0]processing of the original perspectives. The most natural set of perspectives of a given scene is the one recorded on the photographic plate with the help of the so-called fly's eye lens [1]. It is convenient to introduce the special term "aspectogram" for such a set. Let us consider the properties of the aspectogram in relation to composite hologram synthesis and to other purposes as well.

In analysis we suggest aspectogram to be formed in the following way (see Fig. 1). The objects of the scene, viz. an arrow $\theta_{\infty}$ (placed at infinity) and a cross $O_{Z}$ (located at a distance $Z$ ) are recorded on a photographic plate $F$ by means of an array $L$ composed of the lenses $l_{1}, l_{2}, \ldots . l_{n-1}, l_{n}$ and so on. On developing the photographic plate, we obtain an aspectogram which is a transparency containing a set of images produced by the corresponding lenses. Each perspective image represents a recording of the scene as seen by the lens which formed this image. The general view of an aspectogram is shown in Fig. 2. As seen from the figure, the image of the scene depends essentially on the viewpoint chosen, for example in the right-hand perspectives the cross lies more to the right of the arrow point than in the case with the left-hand perspectives.

An analysis shows the structure of the aspectogram to be based on a very simple
relationship, which actually opens up broad possibilities in the processing of information contained in perspectives. It turns out that the images of objects equally spaced from the surface of the lens array form a perfectly periodic


Fig. 1. A layout for recording an aspectogram $I^{\cdot}$ - photographic plate, $L$ - lens array, $l_{n}, l_{n-1}$ lenses of the array, $T^{\prime}$ - lens repetition period, $O_{Z}$ and $O$ - objects of the scene being recorded


Fig. 2. General view of an aspectogram obtained by the technique of Fig. 1
$t_{\infty}--$ spatial repetition period of the subdifferential images of objects at infinity, $t_{Z}$ - repetition period of the subdifferential images of objects lying at finite distance $Z$ from the surface of the lens array
structure, the spatial period of this structure depending unambiguously on the separation between the given element of the scene and the lens array surface. For instance, the images of the cross in the considered aspectogram are repeated with a spatial period $t_{Z}$, while those of the arrow, with a period $t_{\infty}$ (see Fig. 2).

Indeed, while examining the rectangular triangles $O_{Z} e a$ and $O_{Z} g a$ one readily derives
an expression for the distance between two images of the object $O_{Z}$ formed by the $n$-th and ( $n-1$ )-th lenses of the array:

$$
\begin{array}{r}
t_{Z}=X_{n}-X_{n-1}=\frac{\Phi+T}{Z}(Z+S)- \\
-\frac{\Phi}{Z}(Z+S)=T\left(1+\frac{S}{Z}\right) \tag{1}
\end{array}
$$

where $T$ is the spatial repetition period of the lenses, $\Phi$-distance from the perpendicular $O_{Z} p$ to the optical axis of the $(n-1)$-th lens, $S$ - distance from the lenses to the photographic plate recording the aspectogram, $Z-$ distance from the lens array to the object being recorded.

The spatial period determined by Eq. (1) can be shown to correspond to the following spatial frequency:

$$
\begin{equation*}
\omega_{Z}=\frac{2 \pi}{t_{Z}} \approx \omega_{T}(1-\varrho), \tag{2}
\end{equation*}
$$

where

$$
\begin{align*}
\varrho & =\frac{f}{Z}  \tag{3}\\
\omega_{T} & =\frac{2 \pi}{T} \tag{4}
\end{align*}
$$

Here $\omega_{T}$ denotes the spatial repetition frequency of the lenses in the array.

Since objects positioned at different disdistances from the lens array are characterized by different frequencies with which their images repeat in the aspectogram, the aspectogram of a complex $3 D$ scene may conveniently be considered as a sum of "differential aspectograms", that is, as a sum of aspectograms of elements of the scene located at the same distance from the lens array. For instance, the aspectogram depicted in Fig. 2 represents a sum of two differential aspectograms, namely, an aspectogram of a cross and that of an arrow.

Note, that in general case the differential aspectogram is not - strictly speaking - a periodic structure. If one records a real $3 D$ scene, the nearer objects, will as a rule, obstruct partially the remote ones, so that the actual pattern of a subdifferential image should generally speaking - depend on the number of the generating lenses.

Fig. 3 shows the case where the lens array $L$ records on the photographic plate $F$ an aspectogram of the scene consisting of an opa-
que screen $E$ and an arrow $O$ positioned behind it. As seen from the figure, the arrow tail is represented by periodically repeating imagess occupying the region if the aspectogram between the points $a$ and $b$, the arrow point is represented by images located in the region ac, and so on.


Fig. 3. Mutual screening of objects of a scene. As a result of the arrow being sereened by the sereen $E$, the lense array $L$ forms total images of the arrow only within $a b$. Within $b c$ the arrow's images will be incomplete, and within $c e$ they will vanish. The hatched cone refers to the consideration of the accuracy with the effect of mutual sereening of objects is reconstructed. The array focused on the screen plane reconstructs the mutual screening function to within
the set of rays lying inside the cone

The differential aspectogram of this kind can, in turn, be represented as a sum of still simpler periodic structures, namely, the aspectograms of an elementary object which are characterized by the same spatial period obeying Eq. (1) but differing in areas occupied by the images of this object on the photographic plate $F$.

Thus, since the response of a differential aspectogram to different points of an object is not only different but also depends essentially on the nature of the object, it is difficult to write a general expression for the aspectogram of an arbitrary $3 D$ scene. It would appear more expedient to determine, first of all, virtual possibilities offered by a lens array
for the recording of objects lying in some region of the $3 D$ space, neglecting the screening effects which can be considered separately.

Guided by these considerations, we turn now to the analysis of the aspectogram structure. We isolate in the object space a plane parallel to the lens array surface and located at a distance $Z$. As already noted, the images of the objects lying in this plane form a differential aspectogram, i.e. a system of periodically repeating images, its spatial period being defined by (1). With this in mind, the following expression for the light intensity distribution, corresponding to the differential aspectogram considered can be written

$$
\begin{equation*}
g_{h}(X, Y, \varrho)=h(x, y, \varrho) \otimes U(X, Y, \varrho) \tag{5}
\end{equation*}
$$

where

$$
\begin{align*}
& \quad I(X, Y, \varrho)=P_{g^{\prime}}(X, Y) \times \\
& \times\left[I I(X, Y) \sum_{1=-\infty}^{\infty} \sum_{K=-\infty}^{\infty} \delta\left(X-l t_{\underline{o}}, Y-K t_{\underline{g}}\right)\right] . \tag{6}
\end{align*}
$$

Here $X, Y$ are coordinates in the plane of aspectogram, $\varrho$ is a variable which, similarily to $Z$, determines the distance in depth (see (3)), $h(X, Y, \varrho)$ is the distribution function of light intensity in a unit subdifferential image of objects lying at distance $\varrho$ from the lens array surface, the distortions introduced by the optical elements being neglected, $P_{\ell \rho^{\prime}}\left(\begin{array}{l}\text { Y }\end{array}\right)$ is the value taken up by the spread function of a lens, focused onto the plane $\varrho^{\prime}$ in the plane of the aspectogram, and $\Pi(X Y)$ is the function limiting the surface of the aspectogram.

The meaning of (5) and (6) is quite simple. Indeed, the function $U(X, Y, \varrho)$ is essentially the response of the aspectogram with respect to an elementary object, namely, a point in the plane $\rho$. To the convolution of the functions $h(X, Y, \varrho)$ and $U(X, Y, \varrho)$ there corresponds the integral response of the aspectogram to all points of an object located in the plane $g$. The function $P_{00^{\prime}}(X, Y)$ describes here the image blurring produced by a unit lens, and the delta function system - the process of multiplication of subdifferential images over the area of the aspectogram.

The spatial spectrum of a differential aspectogram is determined essentially by that of the function $U(X, Y, \varrho)$. The Fourier transformation of (6), taking into account (2),
yields

$$
\begin{align*}
& S U(X, Y, \varrho)=S P_{\varrho Q^{\prime}}(X, Y) \times \\
& \quad \times\left\{S \Pi(X, \mathrm{I}) \otimes \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \times\right. \\
& \left.\times \delta\left[x-n \omega_{T^{\prime}}(1-\varrho), \eta-m \omega_{T}(1-\varrho)\right]\right\} \tag{7}
\end{align*}
$$

Consider Eq. (7). On the whole, it indicates the spectrum of a differential aspectogram to represent at two-dimensional syrstem of points of delta functions whose intensity is modulated by the frequency response of a unit lens $\Delta P_{y,}$, ( $\boldsymbol{X}, Y$ ) as an envelope. Each peak of the delta function system is spread in accordance with the spectrum of the function restricting the area of the aspectogram $s I I(X, Y)$.

Fig. $f$ shows a one-dimensional graphic interpretation of ( 7 ). In fact this is the section of the aspectogram spectrum by $x$ axis at $\eta=0$. The spectrum of the differential aspec-


Fig. 4. A one-dimensional graphic interpretation of the aspectogram spectrum. $x-$ is one of the axes in frequency space. The system of shaded maxima depicts the spectrum of the differential aspectogram of objects lying in the plane $g$, and the system of unshaded maxima - that of objects in the plane ó Hatching below the abscissa axis indicates filled zones of spectral orders with different numbers, the frequency system $x_{\text {min }}$ corresponding to objects lying in the nearest plane, and frequency system $x_{\text {max }}$, to objects al infifinit $y$
togram of an arbitrary plane $\varrho$ is depicted by shaded maxima, the envelope of this system being shown by an inclined straight line.

The differential aspectogram spectrum of objects lying in the $\varrho^{\prime}$ plane onto which the lenses are focused is represented by unshaded maxima repeated at a different period and modulated with a different envelopes. (The envelope corresponding to this particular plane should obviously lie higher than any other.) The spectra of the differential aspectograms of all the other planes in the object space occupy definite zones along the $\%$ axis in the spectral orders corresponding to different values of $n$.

Consider Eq. (7) more closely. We will first analyze the factor containing the delta functions. One of the most essential properties of an aspectogram is that a define set of specific frequencies in the aspectogram spectrum corresponds unambiguously to each plane in the object space. The relation between the frequencies and distances can be readily found by differentiating the argument of the delta function in ( 7 ) with respect to 0

$$
\begin{equation*}
d \%=-m()_{T} d \underline{0} \tag{8}
\end{equation*}
$$

Thus, a change in a distance from the object expressed in terms of a produces a proportional shift in the frequency region, however the coefficients of proportionality are different in different orders. It follows, in particular, flom (8), that if the range of distances to the objects of a scene is limited, for instance, if it extends from $Z=Z_{\min }$ to $Z=\infty$, the extent of filled zones of the spectral orders depends linearly on the actual number of the order (in Fig. 4 the filled spectral order zones are denoted by hatching below the abscissa).

The above features referred to the values of the spectrum along the frequency space axes $x$ and $\eta$, separately. Let us derive the total configuration of the two-dimensional spetrum, i.e., let us locate the locus of the points where the spectrum is other than zero. By equating to zero the arguments of the delta functions in ( 7 ), we will find the equations relating the frequency coordinates of the points belonging to the $n$-th order along the $x$-axis, and to the $m$-th order, along the $\quad 1$-axis

$$
\begin{align*}
& x=m \omega_{T}(1-\varrho)  \tag{9}\\
& \eta=m \omega_{r}(1-\varrho) \tag{10}
\end{align*}
$$

Fig. 5 shows the general riew of the two--dimensional spectrum of an aspectogram, taking into account (9) and (10). Filled circles denote the spectrum of the differential aspectogram of objects located at infinity. The opposite ends of the segments belong to the differential aspectogram spectrum of objects located at the minimum distance $Z_{\text {min }}$. All the other points of the segments correspond to the elements of the scene lying at intermediate distances from the surface of the array.

To conclude the analysis of the effect of the factor (5) containing the delta functions, consider the change in the aspectogram spectrum resulting from a shift of the lens array.


Fig. 5. Gencral view of a two-dimensional aspectogram spectrum, the blurring of individual spectral components being not included. Filled circles denote the differential aspectogram spectrum of objects at infinits. All the other elements of the scene are represented by various points of the segments of straight lines shown in the figure

As follows from Fig. 1, when the centre of the lens $l_{n-2}$ moves a distance $1 X_{0}$ from point $P$ to point $q$, the image of the object $O_{Z}$ shifts from point $a$ to point $c$ traversing a distance $X_{0}$ defined by the following expression:

$$
\begin{equation*}
X_{0}=\Lambda X_{0}+\delta X_{0}=\Lambda X_{0}(1+\varrho) \tag{11}
\end{equation*}
$$

According to the theorem of the shift, a displacement of the differential aspectogram by $X_{0}, Y_{0}$ will result in the appearance in (7) of a common phase factor

$$
\mathrm{e}^{i \approx X_{0}} \mathrm{e}^{i \eta I_{0}}
$$

When multiplying the terms in (7) by this factor, one should retain in each case only the values of $x$ and $\eta$ at which the corresponding delta functions are other than zero. Neglecting second order terms, the double sum in (7) can finally be written as

$$
\begin{gather*}
\sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \mathrm{e}^{i \omega_{T}\left(n \Delta x_{0}+m \Delta x_{0}\right)} \delta\left[\varkappa-n \omega_{T}(1-\varrho),\right. \\
\left.\eta-m \omega_{T}(1-\varrho)\right] . \tag{12}
\end{gather*}
$$

As it follows from (12), as the lens array displaces, the phase of the spectrum in each order undergoes a shift which is the same for all spectral components of this order and depends only on the extent of lens array displacement along the axes $x, y$ and on the number of the order. According to (12), the magnitude
of the phase shift in the order with indices $n$ and $m$ is defined by the following expression:

$$
\begin{equation*}
\varphi_{n, m}=\omega_{T}\left(n \Delta X_{0}+m \Delta Y_{0}\right) \tag{13}
\end{equation*}
$$

Consider now how the aspectogram spectrum is affected by the limitation of the lens resolution caused by the effects of diffraction and defocusing, in other words, let us analyze the functions $P_{o o^{\prime}}\left(X, Y\right.$ in (6) and $S P_{\varrho e^{\prime}}(X, Y)$ in (7). We will write the spread function of a lens $P_{\varrho Q^{\prime}}(X, Y)$ in terms of convolution of the spread functions resulting from diffraction and geometric defocusing

$$
\begin{equation*}
P_{e e^{\prime}}(X)=P_{d}(X) \otimes P_{g}(X) \tag{14}
\end{equation*}
$$

In the one-dimensional approximation, the spread function of a lens with diameter $d$ and focal length $f$ due to diffraction can be written as:

$$
\begin{equation*}
P_{d}(X)=\frac{\sin ^{2} \frac{\Pi d}{\lambda f} X}{\left(\frac{\Pi d}{\lambda f} X\right)^{2}} \tag{15}
\end{equation*}
$$

The spread function resulting from geometric defocusing can be represented as a $\Pi$-shaped function, its width $\delta$ being related to the actual extent of defocusing. It can readily be shown by simple geometric considerations that the width of the $\Pi$-shaped function and the defocusing reduced to the object space and expressed in terms of the variable $o$ are interrelated

$$
\begin{equation*}
\delta=d \Delta \varrho \tag{16}
\end{equation*}
$$

The frequency response of the lens in the array can be derived from the Fourier transform of the function $P_{\varrho \varrho^{\prime}}(X)$

$$
\begin{equation*}
S P_{e e^{\prime}}(X)=S P_{d}(X) \cdot S P_{g}(X) \tag{17}
\end{equation*}
$$

The spectrum of the function $L_{d}(X)$ defined by Eq. (15) represents a triagle, its maximum frequency being (see Fig. 6)

$$
\begin{equation*}
x_{d \max }=\frac{2 \pi d}{\lambda f} \tag{18}
\end{equation*}
$$

The spectrum of the function $P_{g}(X)$ with the width defined by (16) is described by a function of the type $(\sin X) / X$

$$
\begin{equation*}
S P_{g}(X)=\frac{\sin x \frac{d}{2}-\Delta \varrho}{\varkappa \frac{d}{2} \Delta \varrho} \tag{19}
\end{equation*}
$$

The frequency corresponding to the first minimum of this function (see Fig. 6) can be found by equating the argument of the sine to $\pi$


Fig. 6. The frequency responses of a lens caused by the effects of diffraction (dashed) and geometric defocusing (solid curves)

Using the above relations, let us now determine the shape of the aspectogram spectrum envelope. The total extent of the spectrum can be easily found, being limited by diffraction effects, i.e. $\kappa_{d \max }$ (see (18)). Consider the behaviour of the spectral curve within each order; we will show the width of the spectral zones within each order to be limited not only by the limited distance to the nearest object, but also as a result of defocusing.

As follows from (7) and Fig. 4, the orders of the differential aspectogram corresponding to the plane of best focusing of the lenses in the array will be defined by the envelope curve coinciding with the spectrum of the function $P_{d}(x)$ (see (18)). The shift of these orders corresponds to a transition to the planes of the object scene transmitted with defocusing. With the increasing shift, the intensity of the spectral components should obviously decrease until for some of them the condition of becomming equal to zero is fulfilled which occurs due to either diffraction (18) or geometrical defocussing (20). By substituting of $n{ }^{\prime \prime}{ }_{p}$, for $\kappa_{g \max }$ in (20), we find the magnitude of defocusing at which the spatial frequency of the $n$-th spectral order is transmitted with zero contrast

$$
\begin{equation*}
\Delta \varrho=\frac{2 \pi}{n \omega_{T} d} \tag{21}
\end{equation*}
$$

Inserting the magnitude of defocusing thus obtained in (8) and neglecting the secondary dependence of defocusing on frequency we find the frequency shift corresponding to the
spectral density of the given order becoming zero

$$
\begin{equation*}
\Delta x=\frac{2 \pi}{d} \tag{22}
\end{equation*}
$$

As follows from (22) and (4), when lenses completely fill the surface of the array, i.e. when $d=T$, the effects of geometric defocusing exclude the possibility of superposition of spectral order, since in this case the width of filled spectral zones is precisely equal to the separation between the orders. It can be shown that inclusion of the diffraction effects, i.e. (18) does not change this result.

Consider now the effect of limiting the size of an aspectogram (the function $\Pi(X, Y)$ in (6)). It may be seen from (7) that due to limitation of aspectogram size the system of the delta functions becomes convolved with the spectrum of the limiting function $\Pi(X, Y)$. In accordance with the properties of the Fourier transformation, the spectrum of the rectangular limiting function $\Pi(X, Y)$ can be written in the following way :

$$
\begin{equation*}
S \Pi(X, Y)=\frac{\sin \omega A}{\omega A} \tag{23}
\end{equation*}
$$

where $2 A$ is the linear size of the aspectogram.
The width of the spectral maximum is defined now by the following relation:

$$
\begin{equation*}
\Delta \omega_{0}=\frac{2 \pi}{A} \tag{24}
\end{equation*}
$$

Let us express the aspectogram size in terms of the number of lenses in the array which can cover this size:

$$
\begin{equation*}
m=\frac{A}{T} . \tag{25}
\end{equation*}
$$

Inserting (25) in (24) and taking into account (4) we get

$$
\begin{equation*}
\Delta \omega_{0}=\frac{\omega_{T}}{m} \tag{26}
\end{equation*}
$$

As follows from (26), the energy of each component of the aspectogram spectrum is concentrated primarily within a region $m$-times smaller than the separation between the spectral orders of the aspectogram. However, the information on the limiting function contained in this spectral region is still incomplete, since instead of the rectangle it may permit to reconstruct only a function of the type $\frac{\sin X}{X}$ whose first zeroes coincide with the aspectogram edges.

If we require that all images recorded in the aspectogram be reconstructed with the same intensity, i.e. that the limiting function be reconstructed to the linear size of a single differential image, then for the effective width of blurring of each spectral components, in place of (26), we will obtain:

$$
\begin{equation*}
\Delta \omega_{0}=\omega_{T} \tag{27}
\end{equation*}
$$

Since the screening effects are reconstructed in the aspectogram likewise to within the size of differential image (see Fig. 3), it becomes obvious that (27) may serve at the same time also as a condition for reconstruction of the screening effects.

Thus it turns out that the inclusion of the size limitation of the aspectogram and the effects of mutual screening of elements in a scene results in a mixing of spectral components within each spectral order, without nowever, any noticeable superposition of different orders.

We turn now to a consideration of the possibility of synthesizing the so-called intermediate perspectives [3] with special emphasis given to the scheme presented in Fig. 7. The


Fig. 7. Scheme of the synthesis of intermediate perspectives. Hologram $M$ reconstructs in the plane $F$ the spatial spectrum of an aspectogram, and in the plane $T^{\prime \prime}$, a system of differential images of objects recorded in it. By positioning special masks in the plane, one can obtain in the plane $T$ images corresponding to intermediate perspectives not recorded in the original aspectogram;
a) The mask performing multiplication of perspectives, i.e. a transformation equivalent to usage of an array with a larger number of lenses. Transparent parts of the mask are hatched
b) Phase mask performing a transformation equivalent to a shift in the array (Phase shifts are represented by shifts in hatching lines).
hologram $H$ containing the spectrum of an aspectogram is reconstructed by the wave $R^{*}$ conjugate to the wave $R$ which was used in the recording. The paths of the rays through the hologram become reversed, the aspectogram spectrum being reconstructed in the plane $F$. Next the rays pass through the lens $M$ forming
in its focal plane $T^{\prime}$ a system of differential images of the object recorded in the aspectogram. The images reconstructed in this way are imprinted as holograms onto the plate $H_{K}$ using the reference wave $R_{K}$. Each differential image will now be recorded in the corresponding region of $H_{K}$ isolated by means of a mask $S_{K}$ positioned immediately in front of the hologram.

The synthesis of intermediate perspectives is carried out by means of a mask placed in the spectral plane $F$. This operation can be performed in two different ways. The first and the simplest of them can be used in the case where the coefficient of filling the aspectogram area by perspectives $K^{\prime}=\frac{i}{\lambda}$ is less than unity, i.e. where near each differential image there exists a free area sufficient for imprinting $g^{2}$ additional images. Under these conditions the synthesis can be realized by positioning in the spectral plane $F$ a mask with holes transmitting only every $g$-th order of spetrum (the configuration of such a mask for the case $g=2$ is shown in Fig. 7a). From a formal riewpoint, this operation is equivalent to multiplying the aspectogram spectrum by a periodic system of $\Pi$-shaped functions with the width $\omega_{T}$ and repetition frequency $g \omega_{T}$. It is obvious that if the conditions precluding mutual superposition of filled spectral orders are satisfied, then after this operation the formula ( 7 ) defining the spectrum of the response function of a differential aspectogram will remain unchanged except for the repetition period of the spectral orders which will increase by a factor $g$. Because of a reverse relation between a function and its spectrum, differential aspectogram with an image repetition period $g$ times less than before will correspond to such a spectrum. All the other differential aspectograms making a part of the complete aspectogram of the given object undergo a similar transformation. As a result the image of the complete aspectogram appearing in the plane $T$, would have such a form as if it were recorded with an array containing $g^{2}$ times more lenses than the original one. It is note worthy that the images which appear among the original ones will look as if being recorded from viewpoints other than the lens positions in the original array. On the whole, such an operation clearly resembles optical interpolation of images.

Considering the application of aspectograms to recording and synthesis of composite holograms the aspectogram area completely filled by the images of perspectives seems to be a more probable one. In this case, intermediate perspectives can be synthesized by introducing phase shifts in the spectral plane. Indeed, as this was shown earlier (11)-(13), a shift of the lenses along the axes $x$ and ! corresponds to phase shifts appearing in the spectral plane and changing stepwise when going over from one order to another. Within one order the phase shift remains constant depending only on actual shift of the lenses, the order number and the period of the array (see (13)). One can imagine easily the opposite operation when, by introducing the corresponding phase shifts in the spectral plane, it would be possible to obtain images which would be formed as a result of a shift of the lenses.

In case where filled spectral zones of different orders are not superimposed, the required phase shifts can be introduced by means of a special mask placed in spectral plane. In contrast to multiplication of perspectives, an introduction of the mask produces here a shift and transformation of already existing perspectives, so that the synthesis of each additional system of perspectives rerequires a particular mask.

In conclusion of this section it may be noted that the technique of synthesizing the intermediate perspectives may find an application not only to the 310 cinematograph but also to X-ray diagnostics when three-dimensional optical images of internal organs from X-ray photographs are to be obtained.In this case the problem of reducing the number of original photographs acquires a particular importance in connection with the requirement of reducing irradiation dose [4].

We turn now to applications which are not related to the synthesis of composite holograms. As mentioned earlier, one of the most essential features of aspectograms consists in the fact that to each plane in the object space equidistant from the lens array there corresponds a strictly periodical system of points in the spectrum of its aspectogram. It implies directly that by placing in the aspectogram spectrum plane a mask isolating the above--metioned spectral components, one can define in the object space a zone with images of only
those elements of the scene which are located at the given distance from the aspectogram. As shown earlier, each of such zones occupies $\mathrm{l} / \mathrm{m}$ fraction of the spectral order (see (26)). Using all the components of the aspectogram spectrum, one can obviously observe simultaneously objects in $m$ zones of space in depth ( $m$ is the number of lenses in the array).

One area in which optical systems with such properties could be applied is the development of equipment designed for automatic orientation in space where some complicated objects are present, for example devices warning blind people against the presence and of obstacles appearing in their way and giving their outlines. Another possible area of using such systems is the control of articles of complicated shape.

In conclusion, it should be noted that it seems highly probable that a similar system of data processing is in some way used by faceted eyes of insects. If this is really so, then a flỵing dragon-fly should perceive objects in the surrounding three dimensional space all at once, so that in order to determine distance to an object it does not need to "shift the gaze" and focus on it as is the case with the man.

## De certaines propriétés d'aspectogramme

 liées à la synthèse d'hologrammes composes[^1]figuration et l'emplacement dans l'espace des objets tridimensionnels.

## О некоторых свойствах аспектрограммы, связанных со свёрткой сложных голограмм

Термин ,аспектрограмма" обозначает здесь фотопластинку с нанесенными изображениями данной сцены, созданными с помощью линзы ,мушиного глаза". В работе рассмотрены свойства аспектрограммы. Выявлено, что в некоторых условлях определённая часть пространственного спектра аспектрограммы соответствует каждой плоскости в пространстве предмета, помещённого на данном расстоянии от системы линз. Были найдены условия, для которых это соответствие является неоднозначным. Выявлено также, что посредством соответствующего пропуска спектра аспектрограммы через фильтр можно свётрывать промежуточные перспективы, то есть, можно получить изображения, восстанавливающие предмет, видимый из

точек, не совпадающих с линзами, использованными для первичной записи. Техника свёртки промежуточных перспектив обсуждается с точки зрения её применения для получения сложных голограмм, характеризующихся гладким изменением в виде изображения во время передвижения наблюдательного пункта по поверхности голограммы. Выявлено, что принципы аспектрограммы допускают конструкцию оптических систем, которые регистрировали бы как систему (конфигурацию), так и пространственное положение трёхмерных предметов.

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[^1]:    Le terme 'aspectogramme" signifie ici une plaque photographique avec des images d'une seène quelquonque superposées sur elle et crées à laide d'une lentille des yeux do mouche. Louvrage analize des propriétés daspectogramme. On a pronvé, que dans certaines conditions, unc partie definie du spectre spacial de l'aspectogramme correspond a chaque plane dans l’espace d’un objet placé a une distaanc donnée d`un système des lentilles. On a déterminé des conditions pour lesquelles cette correspondance n'est pas univoque. On a prouvé, qu'en faisant passer le spectre de laspectogramma par un filter on peut synthétiser des perspectives indirectes, ceest à dire, recevoir des images reconstituant l'objet vu des endroits qui ne condedent pas avec les lentilles utilisées pour lo premier enregistrement. La méthode de synthèse de perspectives indirectes est examinée an point de voe de son application afin dobtenir des hologrammes composés. caractérisés par un changement lisse dans liaspect de l'image, guand le point d'observation est mobile. On a prouve que les principes d'un aspectogramme permettent une telle construction des systemes optiques qui enregistreraient en même temps la con-

