# Focusing large-aperture beams generated by high-peak-power lasers 

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#### Abstract

The properties of focusing large-aperture light beams of parameters typical of the beams produced by picosecond terawatt lasers are analysed numerically with the use of 3-D computer code based on Fresnel-Kirchhoff integral formula. It is shown that, contrary to the case of small-aperture Gaussian-like beams, a peak intensity of light in the focus is a non-monotonic function of $f / D$ with a local maximum at moderate $f / D$ values ( $5 \leqslant f / D \leqslant 10$ ), where $f$ is the focal length and $D$ is the aperture of a focusing system. For low $f / D$ values ( $f / D<3$ ), more than $80 \%$ of laser energy is scattered after focusing in a low-intensity large-dimension aureole round the central peak of focal intensity distribution. The amount of the scattered energy can be significantly decreased by an increase of $f / D$, and, as a result, the moderate values of this ratio seem to be optimum for most laser-target experiments. The above features appear for both spherical and aspherical focusing systems.


## 1. Introduction

Modern picosecond and femtosecond high-peak-power lasers are capable of producing light of extremely high intensities ranging from $10^{16} \mathrm{~W} / \mathrm{cm}^{2}$ to $10^{20} \mathrm{~W} / \mathrm{cm}^{2}$ [1]-[3]. Besides terawatt power of a laser pulse and small angular divergence of a laser beam ( $\leqslant 10^{-4} \mathrm{rad}$ ), the attainment of such intensities requires a careful design of the beam focusing system. For that purpose, use is most often made of the short-focal-length aspherical optics (e.g., parabolic mirrors) [3]-[5], however, a few problems appear. For extremely short femtosecond pulses (of duration $<100 \mathrm{fs}$ ) the main problems are related to chromatic aberrations and temporal pulse distortions in a focusing system [6]-[9]. For picosecond terawatt lasers, generating much more energetic pulses than the femtosecond ones, a more important issue is the existence of spatial aberrations (e.g., spherical aberration, astigmatism) caused by large aperture of a laser beam ( $\sim 10 \mathrm{~cm}$ or larger) and/or asymmetry of the beam (e.g., due to non-radial geometry of a laser pulse compressor) [10]-[13]. One of the questions is how to fit the parameters of a focusing system, particularly $f / D$ ratio, to obtain for such beams maximum light intensity in a focus and/or minimum energy scattered outside the central high-intensity peak of a focal intensity distribution ( $f$ is a focal length and $D$ is an aperture of a focusing system).

The purpose of this paper is to analyse numerically the above-formulated problem with the use of 3-D computer code based on Fresnel-Kirchhoff integral
formula. We show that for laser beam parameters typical of the beams generated by picosecond terawatt lasers a peak intensity of light in the focus, both of spherical and aspherical focusing systems, is a non-monotonic function of $f / D$ with a local maximum at moderate values of this ratio. For low $f / D$ values $(<3)$ more than $80 \%$ of laser energy is scattered after focusing in a low-intensity large-dimension aureole round the central high-intensity peak. We also find, that the amount of the scattered energy can be minimised by an increase of $f / D$ and that the moderate values of this ratio are optimum compromise for most laser-target experiments.

## 2. Procedure of calculation

In the present paper, the numerical analysis has been based on Fresnel-Kirchhoff integral formula in the form of

$$
E\left(x_{2}, y_{2}\right)=\frac{1}{2 i \lambda} \iint_{S} \frac{\exp \left(i k r_{12}\right)}{r_{12}} \exp \left(-i k \frac{x_{1}^{2}+y_{1}^{2}}{2 f}\right)\left[1+\cos \left(\vec{n}, \vec{r}_{12}\right)\right] E\left(x_{1}, y_{1}\right) \mathrm{d} x_{1} \mathrm{~d} y_{1}
$$

where: $\lambda$ is the light wavelength,

$$
r_{12}=\sqrt{z^{2}+\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}
$$

$k=2 \pi / \lambda, x_{1}, y_{1}, x_{2}, y_{2}$ - lateral coordinate axes of points in the entrance plane and the observation plane, $E(x, y)$ - complex amplitude of electric field, $z$ - distance between the observation plane and the entrance plane, $f$ - focal length of a focusing element, $\left(\vec{n}, \vec{r}_{12}\right)$ - angle between the normal to the entrance plane and the straight line connecting the points $P\left(x_{1}, y_{1}\right), P\left(x_{2}, y_{2}\right)$.

The phase element containing the focal length of a focusing element $f$ is responsible for the focusing properties of the analysed element and, depending on its type, assumes different shapes.

In the case of parabolic mirror where, independent of the entrance beam dimensions, the focal length $f$ is constant for the whole cross-section of the analysed beam, the form of the phase factor is as follows:

$$
M=\exp \left(-i k \frac{x_{1}^{2}+y_{1}^{2}}{2 f}\right)
$$

In the case of the real biconvex lens of refraction coefficient $n$, aperture $D$ and curvature radii $R$, the phase factor is more intricated and takes into account the effects of spatial aberration which influences the focal length $f$ depending on the distance of the analysed fragment of the entrance beam from the optical axis of the system under study. When determining the analytical form of the phase factor for the real lens use was made of Snell's formula (the principle of refraction of the light wave on the border of two media) as well as the concept of cardinal planes, in regard to which the focal length was calculated. The phase factor is determined out of the following dependences:

$$
\begin{aligned}
& \mathrm{rr} 2=\sqrt{x_{1}^{2}+y_{1}^{2}} ; \quad \mathrm{sdx}=\sqrt{R^{2}-(D / 2)^{2}} ; \quad s \mathrm{~d} 2=R-\mathrm{sdx} ; \\
& \operatorname{ssina}=\mathrm{r} 2 / R ; \quad \operatorname{scosa}=\sqrt{1-\operatorname{ssin} \mathrm{a}^{2}} ; \quad \operatorname{ssinb}=\operatorname{ssina} / n ; \quad \operatorname{scosb}=\sqrt{1-\operatorname{ssin}^{2}} ; \\
& \text { ssing }=s \sin a \times s \cos b-s \cos a \times s \operatorname{sinb} ; \quad \operatorname{scosg}=\operatorname{scosa} \times \operatorname{scos} b+s \sin a \times \operatorname{ssinb} ; \\
& \mathrm{sm} 1=\mathrm{ssing} / \mathrm{scosg} ; \quad \mathrm{sb} 1=\mathrm{rr} 2-\mathrm{sm} 1 \times\left(\sqrt{R^{2}-\mathrm{rr} 2^{2}}-\mathrm{sdx}\right) ; \\
& \mathrm{ssa}=1+\mathrm{sm} 1^{2} ; \quad \mathrm{ssb}=\mathrm{sml} \times \mathrm{sbl} \mathrm{sdx} ; \quad \mathrm{ssc}=\mathrm{sb} 1^{2}-(D / 2)^{2} ; \\
& \mathrm{sx} 2=-\left(\mathrm{ssb}+\sqrt{\mathrm{ssb}^{2}-\mathrm{ssa} \times \mathrm{ssc}}\right) / \mathrm{ssa} ; \quad \mathrm{sr} 2=\mathrm{sm} 1 \times \mathrm{sx} 2+\mathrm{sb} 1 ; \\
& \operatorname{ssinf}=\operatorname{sr} 2 / R ; \quad \operatorname{scosf}=\sqrt{1-\operatorname{ssinf}^{2}} ; \quad \operatorname{ssinb} 2=n \times(\operatorname{ssinf} \times \operatorname{scosg}+\operatorname{scosf} \times \operatorname{ssing}) ; \\
& \operatorname{scosb} 2=\sqrt{1-\operatorname{ssin} b 2^{2}} \text {; } \\
& \operatorname{ssing} 2=\operatorname{ssinb} 2 \times \operatorname{scosf}-\operatorname{scosb} 2 \times \operatorname{ssinf} ; \quad \operatorname{scosg} 2=\operatorname{scosb} 2 \times \operatorname{scosf}+\operatorname{ssin} b 2 \times \sin f ; \\
& \text { sm2 }=\text { ssing } 2 / s \operatorname{cosg} 2 ; \quad \mathrm{sb} 2=\mathrm{sr} 2-\mathrm{sm} 2 \times \mathrm{sx} 2 ; \\
& M=\exp \left(-i k \frac{x_{1}^{2}+y_{1}^{2}}{2 \times(\mathrm{sb} 2 / \mathrm{sm} 2+\mathrm{skard})}\right)
\end{aligned}
$$

where "skard" constant is a factor of shift with respect to the cardinal plane.
The integration process was based on the modified method of the central point after previous division of the entrance area into $N$ elements of $\mathrm{d} x_{1}, \mathrm{~d} y_{1}$ dimensions.

## 3. Results and discussion

The focusing was analysed for laser beams of parameters typical of the beams produced by picosecond terawatt lasers comprising a single-pass diffraction grating compressor [11], [12]. Most of the calculations were carried out for the beam of spherical wave front and angular divergence $\Theta=10^{-4}$ rad and quasi-rectangular intensity cross-section expressed by super-Gaussian function

$$
I=I_{0} \exp \left\{-2^{n+1} \ln 2\left[\left(\frac{x}{x_{0}}\right)^{n}+\left(\frac{y}{y_{0}}\right)^{n}\right]\right\}
$$

with $x_{0}=8.5 \mathrm{~cm}, y_{0}=6 \mathrm{~cm}$ and $n=4$ [12]. It was assumed that the aperture $D$ both of the spherical lens and parabolic mirror were constant and $D=15 \mathrm{~cm}$. Thus, the ratio $f / D$ was varied by changing the focal length $f$ of the focusing system.

Figure 1a presents the peak intensity of light after focusing $I_{p}$ (in relation to the peak intensity of light incident on the focusing system) as a function of the focal length ( $f / D$ ratio) for the large-aperture beam of parameters described above. All points in the plots were obtained for the planes $z$, where the peak intensity was maximum for a fixed $f / D$ value ( $z$ is the distance from the centre of the focusing system measured along the system axis). The diameter $x_{f}$ (FWHM) of the focal spot in these planes as a function of $f$ is presented in Fig. 1b. Analogous dependences for the small-aperture super-Gaussian beam are presented in Fig. 2. The essential qualitative difference in the run of the dependencies for the large-aperture and the small-aperture beams can be noticed. For the last one the peak intensity in the


Fig. 1. Relative peak intensity of light after focusing (a) and the focal spot diameter (b) as a function of the focal length for the case of large-aperture beam.



Fig. 2. Relative peak intensity of light after focusing (a) and the focal spot diameter (b) as a function of the focal length for the case of small-aperture beam.
focus smoothly decreases with growing $f$. At $f / D>2$ the peak intensity changes close to $1 / f^{2}$ dependence and the focal spot diameter is almost a linear function of $f$. For the large-aperture beam, however, the dependence $I_{p}(f)$ is a non-monotonic function and local maximum of $I_{p}$ occurs at $f / D \approx 8$ for the lens and at $f / D \approx 6$ for the parabolic mirror. The focal spot diameter, in turn, is a non-linear function of $f$. Such an unexpected behaviour of the dependence $I_{p}(f)$ for the large-aperture beam is the result of strong spherical aberration and diffraction of the beam, especially significant for low $f / D$ values. They make up a considerable part of light energy scattered to a large-dimension low-intensity aureole round the central high-intensity peak of a focal intensity distribution. This can be seen in Fig. 3, where light intensity distributions in the focal planes (the planes where peak intensities attain maximum at fixed values of $f$ ) are illustrated for three values of the focal length $f$. The ratios of the power (or energy) stored in the central peak $P_{p}$ to the total power (or energy) of the beam $P_{0}$, for the cases of large-aperture and small-aperture beams are presented in Fig. 4. For the last case even at low $f / D$ values $(1 \leqslant f / D<2)$ most of the power is stored in the central peak ( $P_{p} / P_{0} \approx 60-90 \%$ )


Fig. 3. Light intensity distributions in the focal planes of the parabolic mirror and the lens of various focal length. The peak intensities in each column are normalized to the same value.


Fig. 4. Relative power stored in the central peak of the focal intensity distribution as a function of the focal length for the cases of large-aperture (a) and small-aperture (b) beams.
and at $f / D>2$ the ratio $P_{p} / P_{0}$ exceeds $90 \%$. For the case of large-aperture beam less than $10 \%$ of the beam power is stored in the central peak at $f / D<2$. For this case the $P_{p} / P_{0}$ ratio above $50 \%$ can be attained at $f / D>6$ for the mirror and at $f / D>9$ for the lens.

The energy scattered to the aureole is rather difficult to measure owing to low intensity and large dimension of the aureole (much larger than the central peak area). Most often only part of this energy is taken into consideration in measurements of light parameters in the focus. It results from the fact that using CCD cameras or other standard light energy meters only the image of the central peak with its close surroundings is usually recorded. Thus, it can be supposed that the most popular way of estimating light intensity in the focus, based on measurements of effective focal spot diameter (e.g., FWHM or $I_{p} / e^{2}$ area) and energy and duration of a laser pulse (e.g., [3]), gives overestimated values of the intensity, especially in the case of low $f / D$ ratios.

Comparing Fig. 1 and Fig. 4, it is easy to notice that using short-focal-length system for large-aperture beam focusing we can obtain 1.5-2.5 times higher peak intensity but 5-10 times lower power (energy) stored in the central peak in relation to the case where moderate-focal-length system is used with $f / D$ value corresponding to the local maximum of the $I_{p}(f)$ dependence. It is this $f / D$ value that seems to be


Fig. 5. Relative peak intensity of light in the plane $z$ and relative power stored in the central peak as a function of the distance of the plane 2 from the nominal focal plane $z=f$.


Fig. 6. Light intensity distributions in the plane $z$ for various distances of the plane $z$ from the nominal focal plane $z=f$.
optimum compromise for most of laser-target experiments where, besides high light intensity, a possibly highest energy transferred to a target by a focused beam is required.

Both maximum light intensity on a target and the amount of power (energy) transferred to a target in the central peak depend on the target position with respect to the nominal focal plane $z=f$. This is illustrated in Figs. 5 and 6, where intensity distributions and values of peak intensity and power stored in the central peak are shown for various $z-f$ values. The calculations were performed for the parabolic mirror of $f=27 \mathrm{~cm}(f / D \approx 1.8)$. The shape of the intensity distribution changes essentially with $z-f$ variation and, as a result, the change of not only the peak intensity but also of the power stored in the peak takes place. It is important that the plane $z$ where maximum peak intensity occurs, is shifted with respect to the plane where the power stored in the peak is maximum. As a consequence, there is no target position where both maximum light intensity and maximum power stored in the peak can be simultaneously reached. Similarly as in the choice of the $f / D$ values
some compromise is needed when choosing the target position to obtain both high intensity and high energy transferred to the target.

The properties of focusing large-aperture laser beams have been illustrated above for the case of super-Gaussian beams of quasi-rectangular intensity cross-section which are usually produced by terawatt picosecond lasers with single-pass pulse compressor [11], [12]. However, as our numerical simulations show, these properties also appear for large-aperture beams of other shapes, particularly for large -aperture Gaussian beams of radial symmetry. Also for such beams the dependence


Fig. 7. Light intensity distributions in the focal plane of parabolic mirror of various focal length for the case of large-aperture Gaussian beam with plane wave front. The peak intensities are normalized to the same value.
$I_{p}(f)$ is non-monotonic (with a local maximum at $f / D$ value shifted a bit with respect to the one for the super-Gaussian beams) and most of the light energy is scattered to low-intensity aureole at low $f / D$ ratio. As example, in Fig. 7, we present the intensity distributions in the focus of the parabolic mirror obtained for the case of Gaussian beam with plane wave front and aperture $x_{0}=y_{0}=6 \mathrm{~cm}$. These distributions are similar to the ones for the super-Gaussian beam presented in Fig. 3.

## 4. Conclusions

In the paper, the properties of focusing large-aperture laser beams of parameters typical of the beams generated by picosecond terawatt lasers have been investigated numerically. We have shown that, both for spherical and aspherical focusing systems, these properties are essentially different from the ones for small-aperture Gaussian-like beams. Particularly, it has been found that:

- the peak intensity of light in the focus is a non-monotonic function of $f / D$ and local maximum of the peak intensity occurs at moderate $f / D$ values ( $5<f / D<10$ ),
- short-focal-length systems (of $f / D \leqslant 2$ ) give a possibility to attain the highest light intensities, however most of the light energy ( $>80 \%$ ) is scattered to a low -intensity large-dimension aureole round the central high-intensity peak; the amount of the scattered energy decreases when $f / D$ value grows,
- for most laser-target experiments an optimum solution, ensuring both high intensity and high energy transferred to a target, seems to be moderate $f / D$ value corresponding to the local maximum of the peak intensity,
- there is no target position along the $z$ axis where both maximum peak intensity and minimum energy scattered to the aureole can be simultaneously reached.

We believe that qualitative features of the focusing revealed in the paper are of general nature and can be applied to various kinds of large-aperture beams produced by high-peak-power lasers.

## References

[1] Perry M.D., Mourou G, Science 264 (1994), 917.
[2] Mourou G., Appl. Phys. B 65 (1997), 205.
[3] Clark E.L., Krushelnik K., Zepf M., et al, Phys. Rev. Lett. 85 (2000), 1654.
[4] Badziak J., Kozlov A.A., Makowski J. et al., Laser Part. Beams 17 (1999), 323.
[5] Maksimchuk A., Gu S., Flippo K., et al, Phys. Rev. Lett 84 (2000), 4108.
[6] Bor Z., Opt. Lett. 14 (1989), 119.
[7] Horvath Z. L., Bor Z., Opt. Commun. 108 (1994), 333.
[8] Ameer-Beg S., Labgley A. J., Ross I. N., et al., Opt. Commun. 122 (1996), 99.
[9] Mikhallov E. M, Golovinskiy P.A., Zh. Eksp. Teor. Fiz. 117 (2000), 275 (in Russian).
[10] Kempe M, Rudolph W., Phys. Rev. A 48 (1993), 4721.
[11] Danson C. N., Barzanti L. J, Chang Z, et al, Opt. Commun. 103 (1993), 392.
[12] Badziak J., Chizhov S. A, Kozlov A. A., et al, Opt. Commun. 134 (1997), 495.
[13] Yamakawa K., Sugio H, Daido H, et al, Opt. Commun. 112 (1994), 37

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