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The Linear Approximation for the Deflecting Doublet of the Scanning Microscope

The trace of the electron beam has been analysed in the linear approximation for deflecting doublets in scanning microscopes. A condition for the correct operation of the deflecting doublet has been given and was experimentally verified for the case of saddle coils adhering to the magnetic screen of infinitely great magnetic permeability:

1. Defleting doublet (DD)

In the majority of the electronooptic devices with the scanning electron beam the deflecting system is located between the objective lens and the working plane of the device, as shown in Fig. 1. In the solution given below

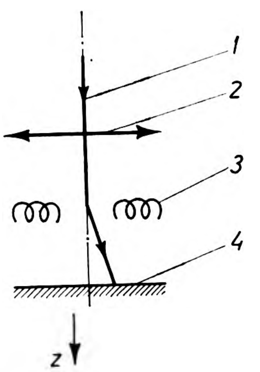

Fig. 1. Post-lens deflection of the relation beam
1 – the electron beam, 2 – objective lens, 3 – deflecting system, 4 – working plane

the distance of the principal plane of the objective lens from the working plane of the device depends on the sizes of the deflecting systems in the z-direction. An increase of the distance between the lens plane and the working plane causes an increment in the electron beam defocusing at the working plane, which is evoked by the spherical aberration of the objective lens. Another version of the deflecting system is given in Fig. 2. The electron beam is deviated by a set of two deflecting systems creating the so-called deflecting doublet (D.D.), located between the condenser lens and the objective lens [1]. The first of these systems, when counting from the electron gun side, deflects the beam by an angle α. The beam is next refracted in the opposite direction by another deflecting system, so that it cuts the electrooptic axis z at the plane $z = zp$. The horizontal and

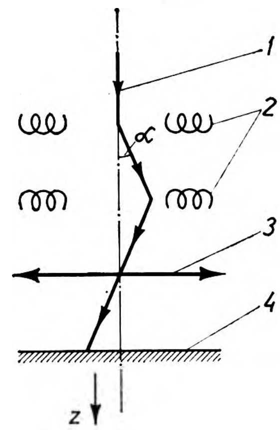

Fig. 2. The system of double deflection of the electron beam applied in analyzing microscopes
1 – electron beam, 2 – DD, 3 – objective lens, 4 – working plane

vertical coils of the first deflecting system are joined in series with the horizontal and vertical coils of the other system.

2. The Gaussian approximation

The coordinates A_x, H_y, H_z of the magnetic field vector of the first deflecting system ($\boldsymbol{H}^{(1)}$) (from the gun side) may be expended into series [2, 3] in the form:

$$H_x^{(1)} = V_0^{(1)} - (V_2^{(1)} + \tfrac{1}{2} V_0^{(1)\prime\prime} x^2 + 2H_2^{(1)} xy + V_2^{(1)} y^2 + \ldots$$

$$H_y^{(1)} = H_0^{(1)} - (H_2^{(1)} + \tfrac{1}{2} H_0^{(1)\prime\prime}) y^2 + 2V_2^{(1)} xy + H_2^{(1)} x^2 + \ldots \tag{1}$$

$$H_z^{(1)} = V_0^{(1)\prime} x + H_0^{(1)\prime} y - \ldots$$

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The functions H_0, V_0, H_2 and V_2 depend on one variable z, being also dependent on the shape of the deflecting coils DD.

The second deflecting system, in accordance with the previous considerations, deflects the electron beam in the direction opposite to the first one. In the discussion it may be assumed that it is rotated by an angle π with respect to the first one. The transformations of the coordinates of the $\boldsymbol{H}$ vector and the coordinates x and y during the rotation by an angle π give the relations

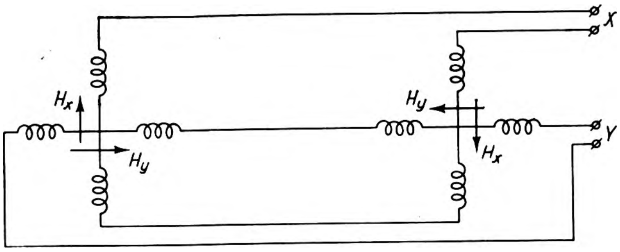

Fig. 3. The representation of the windings and the direction of the magnetic field strength in the coils DD

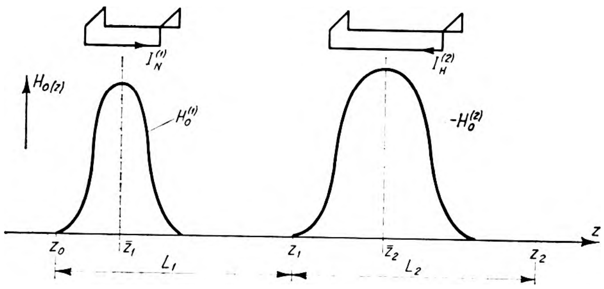

Fig. 4. The deflecting field distribution along the electronooptic axis in the coils of DD

$$H_x \to -H_x \qquad x \to -x$$
$$H_y \to -H_y \qquad y \to -y$$
$$H_z \to H_z \qquad z \to z$$

Hence, the coordinates of the $\boldsymbol{H}$ vector for the second deflecting system take the form

$$H_x^{(2)} = -V_0^{(2)} + (V_2^{(2)} + \tfrac{1}{2}V_0^{(2)''})x^2 - 2H_2^{(2)}xy - V_2^{(2)}y^2 + \ldots$$
$$H_y^{(2)} = -H_0^{(2)} + (H_2^{(2)} + \tfrac{1}{2}H_0^{(2)''})y^2 - 2V_2^{(2)}xy - -H_2^{(2)}x^2 + \ldots \quad (2)$$
$$H_z^{(2)} = V_0^{(2)'}x - H_0^{(2)'}y + \ldots$$

The resulting field in the coils of DD are the superposition of the fields created by the two deflecting systems

$$\boldsymbol{H} = \boldsymbol{H}^{(1)} + \boldsymbol{H}^{(2)}. \quad (3)$$

The trajectory of the electron ray in the static electric and magnetic fields are described by the Euler-Lagrange equation

$$\frac{\partial F}{\partial x} - \frac{d}{dz}\frac{\partial F}{\partial x'} = 0, \quad \frac{\partial F}{\partial y} - \frac{d}{dz}\frac{\partial F}{\partial y'} = 0, \quad (4)$$

where

$$F = (1 + x'^2 + y'^2)^2 - k(A_x x' + A_y y' + A_z),$$

$A = A_x, A_y, A_z$ — vectorial potential of the magnetic field,

$k = (e/2mU_0^*)^{-2}$ — a constant depending upon the voltage U_0 of the last accelerating anode,

$U_0^* = U_0(1+\varepsilon U_0)$ — the accelerating voltage with a relativistic correcting term $\varepsilon = e/2m_0c^2$.

For the linear approximation $F = F_0 + F_2$. The terms F_0 and F_2 for the first deflecting amount to [3]:

$$F_0 = 1,$$
$$F_2 = \tfrac{1}{2}(x'^2 + y'^2) + \mu_0 k(xH_0^{(1)} + yV_0^{(1)}), \quad (5)$$
$$\mu_0 = 4\pi 10^{-7}\mathrm{H/m}.$$

By comparing the formulae (1) and (2) it is visible that for the second deflecting system the sign of the terms in the series is opposite to that of the first system. In other words, the functions F_0 and F_2 for DD have on the basis of (1), (2) and (5), the form

$$F_0 = 1,$$
$$F_2 = \tfrac{1}{2}(x'^2 - y'^2) + \mu_0 k(xH_0^{(1)'} - yV_0^{(1)'} - xH_0^{(2)} + + yV_0^{(2)}). \quad (6)$$

By assuming $F = F_0 + F_2$, and taking account of (4) and (6) the trajectory equations for the electron ray

$$x'' = \mu_0 k(H_0^{(1)} - H_0^{(2)}),$$
$$y'' = -\mu_0 k(V_0^{(1)} - V_0^{(2)}). \quad (7)$$

After double integration of (7) we obtain

$$x'(z) = x'_0 + \mu_0 k \int_{z_0}^{z} [H_0^{(1)} - H_0^{(2)}]d\zeta,$$
$$y'(z) = y'_0 - \mu_0 k \int_{z_0}^{z} [V_0^{(1)} - V_0^{(2)}]d\zeta, \quad (8)$$
$$x(z) = x_0 + x'_0(z - z_0) + \mu_0 k \int_{z_0}^{z} (z-\zeta)(H_0^{(1)} - H_0^{(2)})d\zeta,$$
$$y(z) = y_0 + y'_0(z - z_0) - \mu_0 k \int_{z_0}^{z} (z-\zeta)(V_0^{(1)} - V_0^{(2)})d\zeta,$$

where:

x_0, y_0 = initial coordinates of the electron ray in the z_0 plane,

x_0', y_0' = the tangents of the inclination angles of the electron beams in the z_0 plane.

The expressions $x_0+x_0'(z-z_0)$ and $y_0+ +y_0'(z-z_0)$ in formulae (8) give the coordinates of the electron beam in DD in absence of the deflecting fields

$$I_H^{(1)} = I_H^{(2)} = 0.$$

The terms

$$\mu_0 k \int_{z_0}^{z} (z-\zeta)(H_0^{(1)}-H_0^{(2)})d\zeta$$

and

$$\mu_0 k \int_{z_0}^{z} (z-\zeta)(V_0^{(1)}-V_0^{(2)})d\zeta$$

give the deflection of the deflecting field in the coils DD.

3. Condition of the correct operation of DD

The trace of an ideal beam produced by the electron gun located at the plane z_0 is given in Fig. 5. The position of the electron

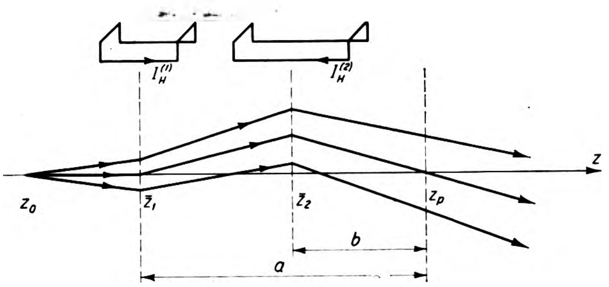

Fig. 5. The trace of ideal electron beam in DD

beam with respect to the electronooptic axis is characterized by the so called central ray, for which the initial conditions at the plane z_0 take the form:

$$x_0' = y_0' = x_0 = y_0 = 0.$$

By virtue of (8) the coordinates of the central ray in DD are given by the relations

$$x_c(z) = \mu_0 k \int_{z_0}^{z} (z-\zeta)(H_0^{(1)}-H_0^{(2)})d\zeta$$
$$y_c(z) = -\mu_0 k \int_{z_0}^{z} (z-\zeta)(V_0^{(1)}-V_0^{(2)})d\zeta. \tag{9}$$

The central ray of an ideal beam, after passing through the DD, intersects with the electronooptic axis at the diaphragm plane z_p of the objective lens, i.e.:

$$x_c(z_p) = y_c(z_p) = 0. \tag{10}$$

The above condition has been called the condition of correct operation of DD. By virtue of (9)

$$x_c(z_p) = \mu_0 k \Big\{(z_p-\bar{z}_x^{(1)}) \int_{-\infty}^{\infty} H_0^{(1)}dz - (z_p-\bar{z}^{(2)}) \int_{-\infty}^{\infty} H_0^{(2)}dz\Big\}. \tag{11}$$

From (11) the limits of integration are taken to be equal to $\pm\infty$, since for $z \notin (z_0, z_1)$, $H_0^{(1)} \equiv 0$ and for $z \notin (z_1, z_2)$ $H_2^{(2)} = 0$.

The planes $\bar{z}_x^{(1)}$ and $\bar{z}_x^{(2)}$ are the principal planes of the first and second deflections of the horizontal deflecting coils [4].

$$\bar{z}_x^{(i)} = \frac{\int_{-\infty}^{\infty} zH_0^{(i)}dz}{\int_{-\infty}^{\infty} H_0^{(i)}dz}, \quad i = 1, 2.$$

For symmetric deflecting coils the principal plane overlaps the axis of symmetry of the coil.

The relation (11) may be represented in the form

$$x_c(z_p) = \mu_0 k b_x\left(\frac{a_x}{b_x} - \frac{1}{r_x}\right) \int_{-\infty}^{\infty} H_0^{(1)}dz. \tag{11a}$$

The significance of the quantities a and b is given in Fig. 5, where

a_x – distance of the principal plane of the horizontal deflecting coil of the first system deflecting from the z_p plane;

b_x – distance of the principal plane of the horizontal deflecting coil of the second system deflecting from the z_p plane, and

$$\frac{1}{r_x} = \frac{\int_{-\infty}^{\infty} H_0^{(2)}dz}{\int_{-\infty}^{\infty} H_0^{(1)}dz}.$$

In view of (11a) the correct operation condition is satisfied if

$$\frac{a_x}{b_x} = \frac{1}{r_x}. \tag{11b}$$

From (11b) it follows that for the given deflecting fields $H_0^{(1)}$ and $H_0^{(2)}$ in the coils of DD the condition (10) will be fulfilled if the positions of the deflecting systems with respect to the z_p planes (i.e. the plane of objective lens diaphragm) is properly chosen. Similarly, for the vertical coils holds

$$\frac{a_y}{b_y} = \frac{1}{r_y}, \tag{12}$$

where

$$\frac{1}{r_y} = \frac{\int\limits_{-\infty}^{\infty} V_0^{(2)} dz}{\int\limits_{-\infty}^{\infty} V_0^{(1)} dz}.$$

In the majority of cases the main planes of deflecting coils in DD cover each other, i.e. $a_x = a_y = a$, and $b_x = b_y = b$. In the case where the vertical and horizontal coils are the identical $r_x = r_y = r$.

4. On experimental investigation of the deflecting doublet

The condition (11) assuring the correct operation of DD is true for small deflection angles (linear approximation of function F). The influence of changes in the deflection angle on the condition of correct operation was checked experimentally. For this purpose three variants of the deflecting coils of DD — all of saddle type — were used for measurements. The function $H_0(z)$ for the saddle coils takes the form [5]:

$$H_0(z) = NI\cos e(z),$$
$$e(z) = -\frac{4}{\pi^2 R}\int\limits_0^{\infty} \frac{\sin\frac{kl}{R}\cos\frac{kz}{R}}{I_1(k)}\, dk, \tag{13}$$

NI — is the number of ampere-turns in the deflecting coil. The meaning of the quantities 1, θ, and R is given in Fig. 6. The function $I_1(k)$ is a modified Bessel function of first order. By virtue of (11) and (13) the condition of correct operation takes the form

$$\frac{a}{b} = \frac{N_2}{N_1}\frac{\cos\theta_2}{\cos\theta_1}\frac{e_0^{(2)}}{e^{(1)}}, \tag{14}$$

where

$$e_0^{(i)} = 2\int\limits_0^{\infty} e_i^{(z)} dz, \quad i = 1, 2,$$

θ_1, θ_2 — angles of saddle coil divergence in the first and second deflecting systems,

N_1, N_2 — number of turns in the deflecting coils of the first and second deflecting systems.

The deflecting coils of DD are located in the neck of the electrooptic lamp of about 500 cm length and Ø 50 mm diameter. The

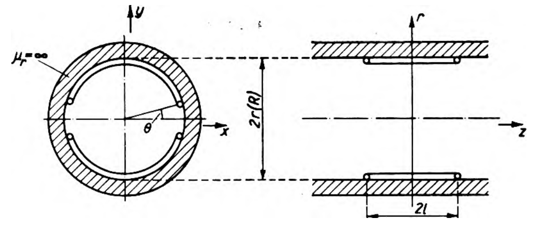

Fig. 6. The scheme of the deflecting system with one-turn horizontal coil

scheme of the lamp is given in Fig. 7. As the z_p plane, the lamp screen plane has been assumed. For three examined variants the quantities a and b amount to 195 mm and

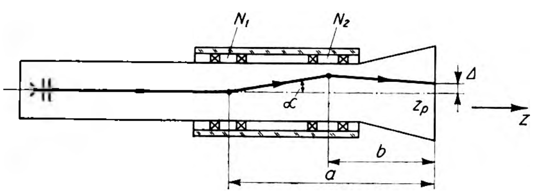

Fig. 7. The scheme of an electron-ray lamp with DD

115 mm, respectively, the accelerating voltage U_0 being equal to 15 kV.

In the second deflecting system the dimensions of the coil are 62,8 ×35 mm (which corresponds to the angle $\theta_2 = 30°$) and the number of ampere-turns $N_z = 75$. The number N_1 of turns and the dimensions of the coils for the first deflecting system have been calculated on the base of (14) and given in the Table 1.

Table 1

Variant	Dimensions of coil in mm	Number of turns N_1	θ_1
1	62.8 × 35	44	30°
2	47.1 × 35	63	45°
3	47.1 × 40	55	45°

In the measurements only vertical coils were applied; they assure the declination of the beam toward the y axis.

The distances Δ of the intersection point of the electron beam from the z axis of the lamp for various deflecting angles α are given in Fig. 8. In accordance with the requirements usually made with respect to DD the distance Δ should be equal to zero (cf. (10)). Its finite value is affected by the errors in deflection of both the deflecting systems. An

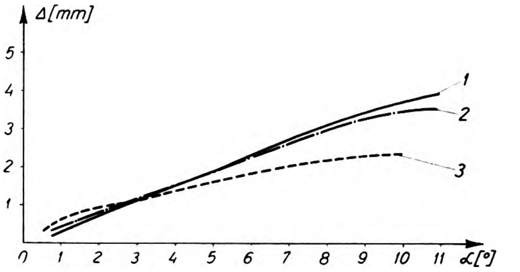

Fig. 8. Distance of the intersection point in an electron beam at the z_p plane from the z axis of the lamp versus the angle

increase in the value of Δ is due to distortion of the odd order and to the asymmetry distortion (of even order) of the first and second deflecting systems. The dimensions of the deflecting coils used in the measurements have been calculated from (14), this dependence, however, assures, the fulfilment of the correct operation condition only for the linear approximation.

5. Conclusions

For satisfied condition (11b) the applied linear approximation allows to connect the angle α with the angle under which the beam leaves DD. By virtue of (11b) and (12) and of (14), in particular, the dimensions of the deflecting coils in DD may be determined for fixed distance of the first and second deflecting systems from the lens plane. It should be expected, however, that the beam doubly refracted due to errors in deflection and asymmetry, will intersect the electronooptic axis at the plane different from the z_p plane.

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Линейная аппроксимация для отклоняющего дублета анализирующего микроскопа

Проанализирован путь электронного пучка в линейном приближении для отклоняющего дублета анализирующего микроскопа. Приведено условие правильного действия отклоняющего дублета. Это условие было проверено опытным путем для седлообразных катушек, смежных с магнитным экраном, обладающим бесконечно высокой магнитной проницаемостью.

References

[1] Cosslett V. E., Duncumb P., *Electron Microscopy*, Proc. Stockholm Conf. 1956.

[2] Haantjes I., Lubben D. J., *Errors of magnetic deflection I*, Philips Res. Rep. 12, 1957, pp. 46-68.

[3] Kaashoek J. A., *A study of magnetic-deflection errors*, Philips Res. Rep. Suppl. 11, 1968, pp. 1-113.

[4] Glaser W., *Osnovy elektronnoy optiki*, Moskva 1957.

[5] Romanowski A., *Magnetic field in deviating coils of the analysing microscope*, Optica Applicata VI, 2, 35-39.

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