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Light propagation in optical waveguides with complex refractive indices

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The modal formalism of optical waveguides with loss and gain has been revised. Light propagation in an optical waveguide with a balance of gain and loss has been studied with beam-propagation method for a wide range of attenuation and amplification coefficients and the results are compared with numerical solution to the wave equation.

1. Theory and notation

With the advent of semiconductor optical amplifiers there is a growing interest in optical waveguides composed of active (with gain) and lossy media. Although the theoretical background of modes in waveguides with loss was established almost twenty five years ago [1], the diversity of appearing structures involves a considerable and growing effort in studying optical phenomena in those devices, stimulated by availability of technology of such devices, and also by their potential use for optical signal processing in fibre-based telecommunication systems.

The attenuation or amplification of light propagating in a transparent medium can be described with a complex refractive index

$$n = n' - jn'', \tag{1}$$

and a complex relative permittivity

$$\varepsilon = n^2 = \varepsilon' - j\varepsilon''. \tag{2}$$

Let an optical beam propagate down the positive z-axis

$$E(x,y,z,t) = E_0(x,y,z)\exp[j(\omega t - kz)]. \tag{3}$$

Here $E_0(x,y,z)$ is the field amplitude distribution, and the exponential factor accounts for field phase changes.

We introduce a complex wavenumber

$$k = nk_0 = n'k_0 - jn''k_0 \tag{4}$$

where $k_0 = 2\pi/\lambda$ is the free-space wavenumber. Then (3) takes a form

$$E(x,y,z,t) = E_0(x,y,z)\exp[j(\omega t - n'k_0 z)]\exp(-n''k_0 z). \tag{5}$$

Therefore, in the case of (3), positive values of n'' represent the field exponential decay (and loss), while negative values of n'' account for the field exponential growth (and gain).

For high intensities of light in semiconductor waveguide devices both gain and attenuation may show saturation effects. Therefore, the attenuation and amplification coefficients are intensity dependent. This is to be considered as a special case of optical nonlinearity that influences not the real, but the imaginary part of the index of refraction. However, in the present paper we restrict ourselves to the linear case, with constant values of n'' and ε''.

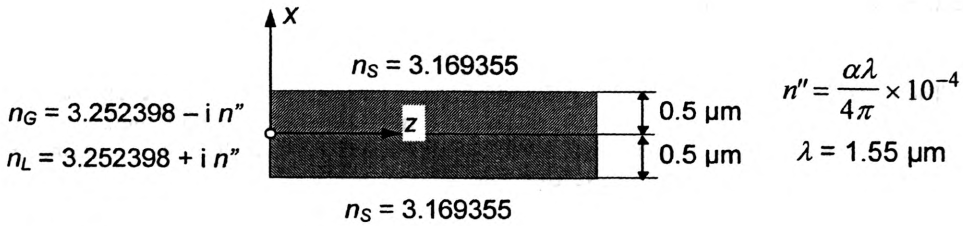

Fig. 1. Two-layer waveguide with gain and loss

Formally, the modal formalism for waveguides with complex refractive indices is similar to that for guides with real refractive indices. Let us assume a geometry of the waveguide as in Fig. 1, where there is no change in the refractive index in the y direction. We consider optical fields that are independent of the y coordinate. The fields satisfy in the i-th layer a scalar wave equation [2]

$$\nabla^2 \Phi_i + k_i^2 \Phi_i = 0 \tag{6}$$

where $\Phi_i = E_y/H_y$ for TE/TM wave, respectively, and $k_i = k_0 n_i$ is the wavenumber in the layer. The upper infinite layer will be labelled as the 1st, while the lower infinite layer as the 4th.

We consider solutions to (1) in a form of eigenmodes

$$\Phi_i(x,z) = \varphi_i \exp(-jk_z z). \tag{7}$$

The eigenmodes of the form (7) differ from well-known modes of non-lossy guides in a way that the propagation constant is complex

$$k_z = k_0[\mathrm{Re}(N_{\mathrm{eff}}) + j\,\mathrm{Im}(N_{\mathrm{eff}})] \tag{8}$$

where N_{eff} is the mode effective index.

A substitution of (7) into (6) leads to the Helmholtz equation

$$\frac{d^2\varphi_i}{dx^2} + k_{x_i}^2 \varphi_i = 0 \tag{9}$$

where

$$k_z^2 + k_{x_i}^2 = k_i^2, \tag{10}$$

$$k_{x_i} = u_i - jv_i, \tag{11}$$

k_{xi} is an x-component of the wavevector in the i-th layer, u_i nad v_i are the transversal phase and attenuation constant, respectively. It follows from (10) that k_{xi} may have two opposite complex values, in the following we choose k_{xi} such that $v_i > 0$.

The modal fields have to satisfy boundary conditions at the interfaces between the i-th nad j-th layers

$$\left.\begin{aligned} \varphi_i &= \varphi_j \\ \varphi'_i &= w_{ij}\varphi'_j \end{aligned}\right\} \tag{12}$$

where the prime denotes a derivative with respect to x, and

$$w_{ij} = \begin{cases} 1 \text{ for TE wave,} \\ (n_i/n_j)^2 \text{ for TM wave.} \end{cases} \tag{13}$$

A general solution to (9) is:

– for $k_{x_i} \neq 0$

$$\varphi_i(x) = A_i \exp(-jk_{x_i}x) + B_i \exp(jk_{x_i}x), \tag{14}$$

– for $k_{k_i} = 0$

$$\varphi_i(x) = C_i x + D_i \tag{15}$$

where A_i, B_i, C_i, D_i are complex constants, they must ensure a fulfilment of the boundary conditions (12). For the purpose of numerical calculations it is important to include the linear solution to the wave equation (15) especially in the case of multilayer waveguides, in order to avoid numerical instabilities.

The field limitation condition (at the infinity) requires that in the outermost layers

$$A_1 = B_4 = 0 \tag{16}$$

provided that $v_1 > 0$ and $v_4 < 0$ have been chosen in (11).

The fields (14), (15) that satisfy both the limiting conditions (16) and boundary conditions (12) constitute a discrete set of bounded eigenmodes of the structure. Thus, the conditions (16) and (12) are equivalent to dispersion equation of the waveguide. Those conditions occur simultaneously only for discrete values of mode effective indices, in analogy to well-known modes of non-lossy guides. But in general those effective indices are complex, according to (8), what implies a necessity of a two-dimensional search for the solution in the complex plane. This is in contrast to the non-lossy modes, where the discrete solutions are situated on the real axis. This is also what makes it rather difficult to find the complex modes.

2. Modes of the waveguide with a balance of gain and loss

The waveguide structure of interest is shown in Figure 1 and it was originally proposed within the framework of the COST 240 Project "Techniques for Modelling and Measuring Advanced Photonic Telecommunication Components" by H.-P. Nolting from Heidrich-Hertz Institute, Berlin, as a modelling task for the Working Group 2, Waveguides [3]. The waveguide with a balance of gain and loss may be fabricated in InGaAsP/InP technology and it consists of two layers with mutually complex conjugate refractive indices that are surrounded with a medium of a slightly lower real refractive index. The exact values of all parameters are given in Fig. 1. We emphasise that the absorbing and amplifying regions have the same absolute value of the imaginary part of refractive index and also the same thickness, thus a balance of gain and loss is assured. Also, the wavelength $\lambda = 1.55$ μm corresponding to the minimum loss of silica fibres that are widely used in optical telecommunication systems is assumed. The waveguide is designed in a way that it supports only the fundamental mode at 1.55 μm wavelength when the gain/loss coefficient is equal to zero.

The imaginary part of refractive indices of guiding layers may be tuned in a very broad range: in terms of the absorption (gain) coefficient α, between zero and $\pm 10^4$ cm^{-1}. The relation between the power absorption or gain coefficient α (in centimeters) and the imaginary part of refractive index is

$$n'' = \frac{\alpha\lambda}{4\pi} \times 10^{-4}. \tag{17}$$

The dispersion equation of the waveguide has been solved numerically in the complex plane for TE polarisation in the above range of α values and the obtained dispersion curves (the real and imaginary parts of effective refractive indices versus the absorption coefficient α) are plotted in Fig. 2. The field and phase distributions for the relevant cases of $\alpha < \alpha_{\mathrm{branch}}$ and $\alpha > \alpha_{\mathrm{branch}}$ are shown in Figs. 3 and 4, respectively.

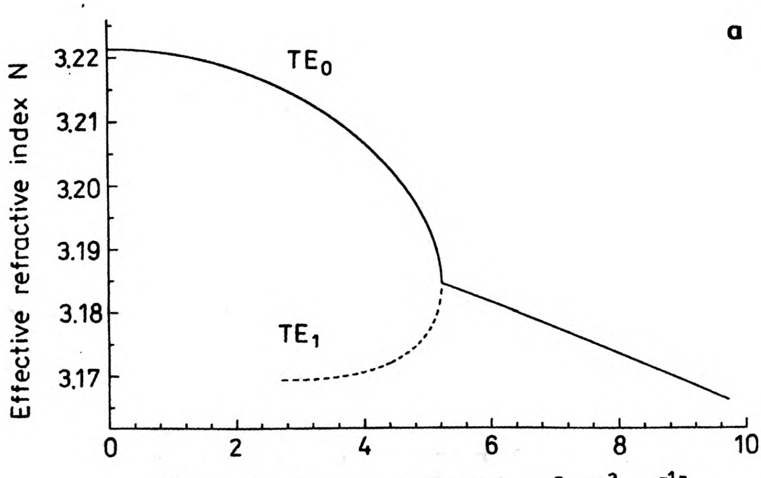

Fig. 2a

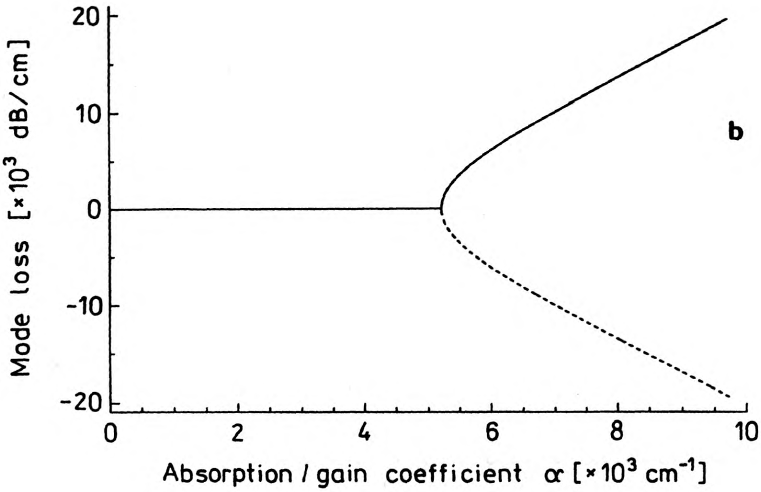

Fig. 2. Effective refractive indices $\mathrm{Re}\{N_{eff}\}$ and $\alpha_{eff} = (4\pi/\lambda)\,\mathrm{Im}\{N_{eff}\}$ versus α

It can be seen from Figure 2 that there is a special value of the coefficient $\alpha = \alpha_{branch} = 5226.3\ \mathrm{cm}^{-1}$ that divides the dispersion curves into two parts. Let us analyse first the behaviour of imaginary part of effective dielectric constant shown in Fig. 2. It is equal to zero up to the value of the coefficient $\alpha = \alpha_{branch}$. This means that the modes (or mode) are lossless, with field and phase distribution as in Fig. 3. For $\alpha > \alpha_{branch}$ two modes exist, they have exactly opposite imaginary parts of the effective index. Thus one of the modes is attenuated along the direction of propagation, while the other is amplified. It can be seen from Fig. 4 that for at-

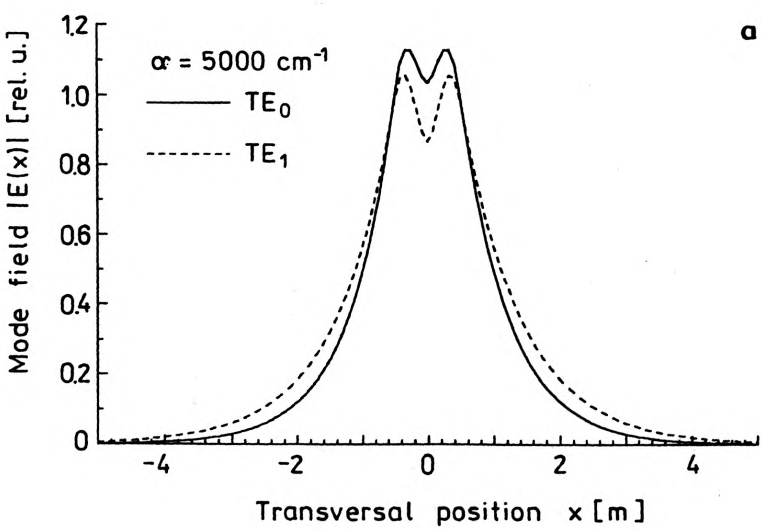

Fig. 3a

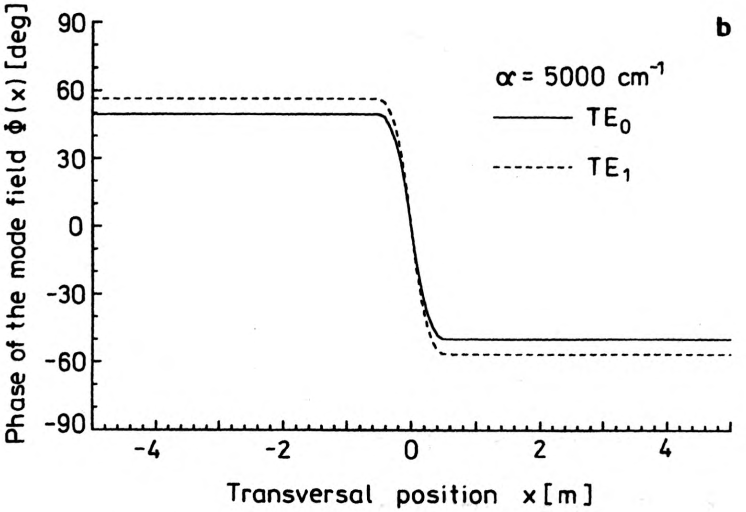

Fig. 3. Modal field and phase distributions for $\alpha = 5000\ \mathrm{cm}^{-1}$ (below α_{branch})

tenuated mode the optical field is concentrated mainly in the lossy layer, while the field of amplified mode is cencentrated mainly in the layer with gain.

As the real part of mode effective index is concerned, for very small values of α the waveguide supports only one mode and the mode propagates without loss or gain. At $\alpha < \alpha_1 = 2725\ \mathrm{cm}^{-1}$ the second mode appears, which is also nonlossy. The two nonlossy modes have symmetric field distribution and asymmetric phase distribution (see Fig. 3), corresponding to power flow from the layer with gain to the lossy

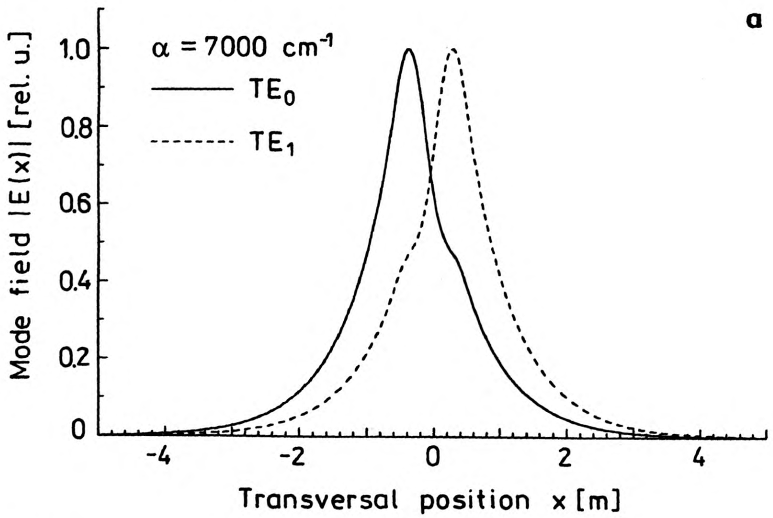

Fig. 4a

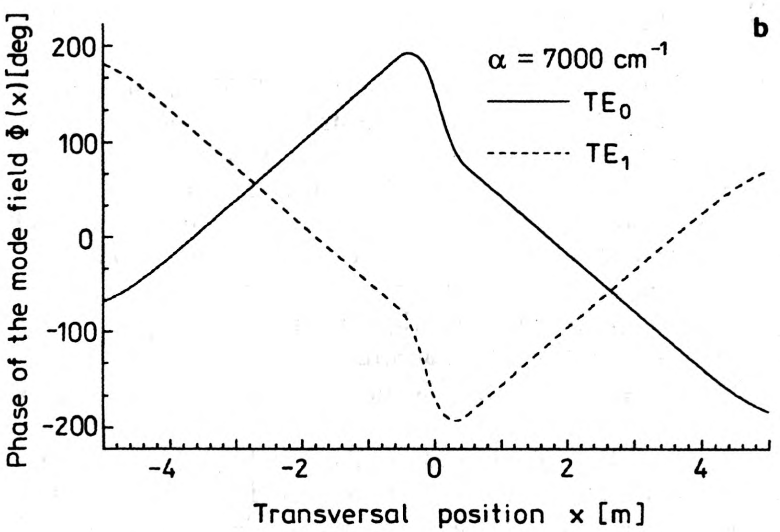

Fig. 4. Modal field and phase distributions for $\alpha = 7000$ cm^{-1} (above α_{branch})

layer. However, for $\alpha > \alpha_{branch}$ the two modes have the same real part of effective index, thus they propagate with the same phase velocity. The field distributions of the two modes are mutually symmetrical with respect to the $x = 0$ plane, while the phase distributions are asymmetrical with respect to that plane (Fig. 4). The mode field distributions are correspondingly concentrated in the layer with loss and gain, respectively. The phase distribution is such that the lossy mode absorbs power from the outer space while the mode with gain supplies power to the outer space.

Very special conditions are at $\alpha = \alpha_{branch}$, where the two modes become identical, with the same propagation constant and also the field and phase distribution.

3. Beam-propagation method analysis

Beam-Propagation Method (BPM) has been proved to be an excellent tool to study optical beam propagation in optical waveguides composed of passive media, with real refractive indices [4], [5]. Although up to now used only to model light propagation in non-lossy media, the method can be easily extended to transparent media with gain or loss.

For a planar isotropic waveguide a scalar wave equation is valid. Standard beam-propagation algorithms deal with a solution to hyperbolic Helmholtz equation

$$E(z) = E_0 \exp\left[jz\sqrt{\frac{\partial^2}{\partial x^2}+k_0^2 n_r^2}+jk_0(n-n_r)z\right], \tag{18}$$

or with a solution to paraxial parabolic equation

$$E(z) = E_0 \exp\left\{\frac{-jz}{2k_r}\left[\frac{\partial^2}{\partial x^2}+k_0^2(n^2-n_r^2)\right]\right\} \tag{19}$$

where z is the direction of propagation, E_0 is the initial field distribution at a cross-section $z = 0$, $n = n(x,z)$ is the refractive index distribution in the waveguide, and n_r is the index of refraction of a reference medium in which the free-space propagation steps are to be carried out. It is assumed that the value of n_r is real and close to those of the media constituting the system. $E(x,z)$ is a slowly-varying field amplitude, and k_r is the wavenumber in the reference medium, $k_r = n_r\omega/c$.

Now the occurrence of loss or gain in a medium may be automatically accounted for in the phase compensation steps, by an appropriate exponential change (increase for gain, decrease for loss) of the field amplitude according to the imaginary part of the refractive index n. This results directly from (18) and (19), when ns have complex values [6].

It has been assumed for the BPM investigation of the gain-loss waveguide that the exciting field is the fundamental TE mode of the waveguide with no loss and gain, *i.e.*, $\alpha = 0$. This field is not an eigenmode of the guide with $\alpha \neq 0$, but a superposition of its guided and radiation modes. For large distances of propagation only the guided modes remain in the waveguide, while radiation part of the field is radiated away.

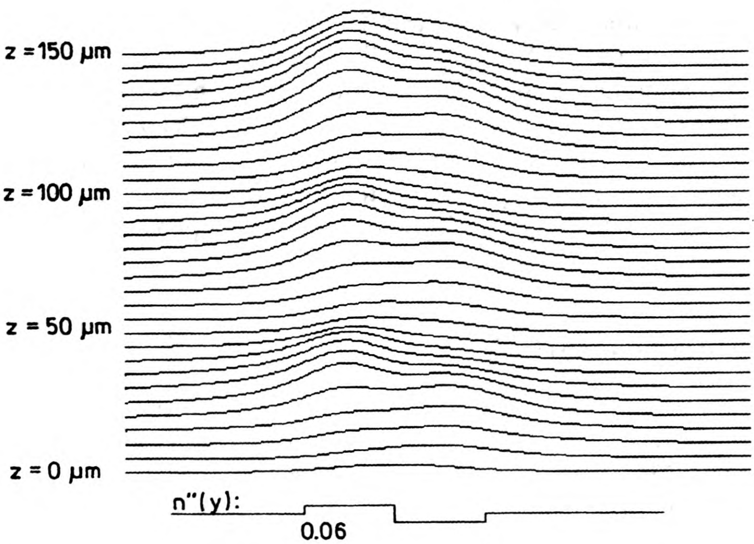

Fig. 5. Field evolution along the waveguide (BPM simulation), for $\alpha < \alpha_{\mathrm{branch}}$

Figure 5 shows the beam evolution for a small value of α, below α_{branch}. As at this range of α two lossless modes exist with different phase velocity (see Fig. 2), beating effects in the field distribution can be seen. The spatial period of the beating is inversely proportional to the real propagation constant difference for the two lossless modes, and it becomes infinite when α approaches α_{branch}. As the two modes are lossless, the total power of the beam remains constant.

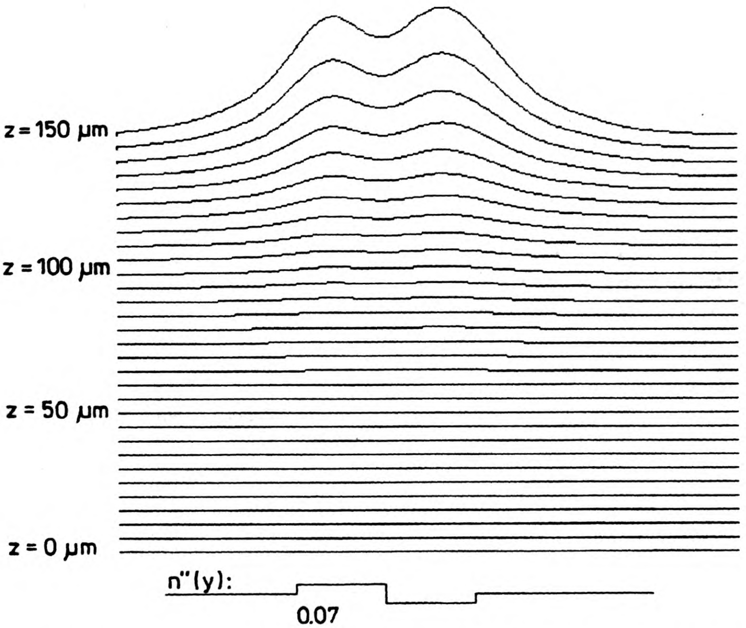

Fig. 6. Field evolution along the waveguide (BPM simulation), for $\alpha > \alpha_{\mathrm{branch}}$

Figure 6 shows the beam evolution for α above α_{branch}. Now there are two modes with the same phase velocity (see Fig. 2), but one is attenuated while the other is amplified. At long distances of propagation only the amplified mode survives, while the attenuated one disappears. Therefore the total power exhibits an exponential growth, which is clearly shown in Fig. 6.

4. Conclusions

We have presented a comparison of two approaches to the problem of a waveguide with a balance of gain and loss: one is the numerical solution to the dispersion equation, the other is beam-propagation method simulation. The physical phenomena predicted by modal theory have been observed with the BPM. This can give a more detailed insight into the physics of light propagation in such waveguides, as well as to enable one to design waveguides with desired performance. Therefore beam propagation method reveals to be a very useful tool for analysing phenomena of light propagation in media with gain and loss, in addition to its wide use for nonlossy (also nonlinear) waveguides.

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