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Influence of the cross-modulation effect on intensity of waves transmitted through a non-linear Fabry–Perot cavity

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In this paper, we present an analysis of the influence of the cross-modulation effect on intensity of waves transmitted through a non-linear Fabry–Perot cavity with self-focusing and self-defocusing Kerr medium. The case of two plane waves with different wavelengths incident on a non-linear Fabry–Perot cavity is considered. It will be shown that the intensities of transmitted waves change multistably and that these multistable states of transmitted waves depend on incident waves and resonator parameters.

1. Introduction

There have appeared many publications about bistability. As is well known, this phenomenon has been described in various devices and non-linear materials. The experimental and theoretical results of those investigations show possibilities of using bistability in many applications [1]–[3]. In numerous experimental and theoretical papers, non-linear Fabry–Perot cavity has been considered. An analysis of two-wave non-linear Fabry–Perot transmission characteristics was presented in paper [4]. The authors examine two waves with the same wavelength and different angles of incidence. The cavity has a self-focusing non-linear medium inside. A two-beam configuration can be the base for the design of logic elements and various devices. It seems to be interesting to examine influence of cross-modulation effect [5] on intensity of waves with different wavelengths transmitted through a non-linear Fabry–Perot cavity with self-focusing and self-defocusing Kerr medium.

2. Formulation of the problem

We will consider the situation in which two monochromatic plane waves are transmitted through the non-linear Fabry–Perot cavity (Fig. 1).

It has been assumed that incident waves fall perpendicularly on the surface of resonator and have various frequencies $\omega_1 \neq \omega_2$. The parallel mirrors of the resonator are characterised by the intensity reflectivity R and the intensity transitivity T. The cavity contains non-linear isotropic Kerr medium and its thickness has been assumed to be much larger than the wavelength. The cavity is situated in a vacuum.

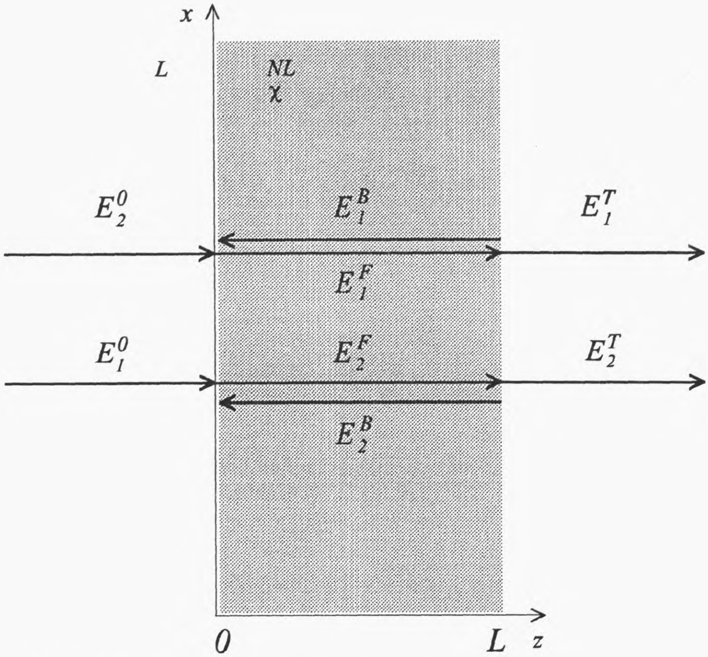


Fig. 1. Situation considered in this paper

Because the waves are transversal and incident perpendicularly to cavity, they have only x and y Cartesian components, so the electric field of incident waves can be written in the following form ($j = 1, 2;\ a = x, y$)

$$\vec{E}_j^0 = \vec{e}_x \tilde{E}_{x_j}^0 + \vec{e}_y \tilde{E}_{y_j}^0 \tag{1}$$

where $\tilde{E}_{a_j}^0$ are Cartesian components of electric field

$$\tilde{E}_j^0 = \frac{1}{2}[\hat{E}_{a_j}^0 \exp[i(\omega_j t - \vec{k}_j^0 \vec{z})] + \hat{E}_{a_j}^{0*} \exp[-i(\omega_j t - \vec{k}_j^0 \vec{z})]], \tag{1a}$$

and $\hat{E}_{a_j}^0 = E_{a_j}^0 \exp[i\varphi_{a_j}^0]$, $\hat{E}_{a_j}^{0*} = E_{a_j}^0 \exp[-i\varphi_{a_j}^0]$.

The field in the cavity consists of forward $\vec{E}_{a_j}^F$ and backward $\vec{E}_{a_j}^B$ waves. The total field $\vec{E}_j^C$ in the Fabry–Perot cavity is given by ($j = 1, 2;\ a = x, y$):

$$\vec{E}_j^C = \vec{e}_x(\tilde{E}_{x_j}^F + \tilde{E}_{x_j}^B) + \vec{e}_y(\tilde{E}_{y_j}^F + \tilde{E}_{y_j}^B), \tag{2}$$

$$\tilde{E}_{a_j}^F = \frac{1}{2}[\hat{E}_{a_j}^F \exp[i(\omega_j t - \vec{k}_j \vec{z})] + \hat{E}_{a_j}^{F*} \exp[-i(\omega_j t - \vec{k}_j \vec{z})]], \tag{3}$$

$$\tilde{E}_{a_j}^B = \frac{1}{2}[\hat{E}_{a_j}^B \exp[i(\omega_j t + \vec{k}_j \vec{z})] + \hat{E}_{a_j}^{B*} \exp[-i(\omega_j t + \vec{k}_j \vec{z})]], \tag{3a}$$

where: $\hat{E}_{a_j}^F = E_{a_j}^F \exp[i\varphi_{a_j}^F]$, $\hat{E}_{a_j}^{F*} = E_{a_j}^F \exp[-i\varphi_{a_j}^F]$,

$\hat{E}_{a_j}^B = E_{a_j}^B \exp[i\varphi_{a_j}^B]$, $\hat{E}_{a_j}^{B*} = E_{a_j}^B \exp[-i\varphi_{a_j}^B]$.

$E^F_{a_j}$, $E^B_{a_j}$ are real Cartesian components of electric field amplitudes inside the resonator and $\varphi^F_{a_j}$, $\varphi^B_{a_j}$ are corresponding phases. It has been assumed that amplitudes and phases of total field inside the cavity are slowly varying functions of the variable z.

3. Solution of the wave equations and boundary conditions

The total monochromatic electric fields of two waves in the Fabry–Perot resonator obey the set of differential equations ($j = 1, 2$)

$$\text{curl curl } \vec{E}^C_j - \omega_j^2 \mu_0 [\varepsilon_L(\omega_j) + i\alpha(\omega_j)] \vec{E}^C_j = \omega_j^2 \mu_0 \vec{P}^{NL}_j \tag{4}$$

where $\vec{P}^{NL}_j$ is the component of non-linear polarization of isotropic medium (χ is a scalar value describing the non-linearity of the 3rd order)

$$\begin{aligned}\vec{P}^{NL} = \chi[&(\tilde{E}^F_{x_1} + \tilde{E}^B_{x_1} + \tilde{E}^F_{x_2} + \tilde{E}^B_{x_2})^2 + (\tilde{E}^F_{y_1} + \tilde{E}^B_{y_1} + \tilde{E}^F_{y_2} + \tilde{E}^B_{y_2})^2] \\ &\times [\vec{e}_x(\tilde{E}^F_{x_1} + \tilde{E}^B_{x_1} + \tilde{E}^F_{x_2} + \tilde{E}^B_{x_2}) + \vec{e}_y(\tilde{E}^F_{y_1} + \tilde{E}^B_{y_1} + \tilde{E}^F_{y_2} + \tilde{E}^B_{y_2})]\end{aligned} \tag{4a}$$

corresponding to frequency ω_j, while $a(\omega_j)$ is the linear absorption coefficient, $\varepsilon_L(\omega_j)$ is the electrical permittivity of the medium and μ_0 is the magnetic permeability of vacuum. The wave components depend only on variable z.

It has been assumed that amplitudes and phases are slowly varying functions of variable z. In the slowly varying amplitude approximation we introduce relations (2)–(3a) and after separation of real and imaginary parts Eqs. (4) take the form:

– for forward waves

$$-2k_j E^F_{a_j} \frac{d\varphi^F_{aj}}{dz} = \omega_j^2 \mu_0 P^{NL^F}_{a_j}, \tag{5}$$

$$2k_j \frac{dE^F_{aj}}{dz} - \omega_j^2 \mu_0 \alpha(\omega_j) E^F_{a_j} = 0, \tag{5a}$$

– for backward waves

$$2k_j E^B_{a_j} \frac{d\varphi^B_{aj}}{dz} = \omega_j^2 \mu_0 P^{NL^B}_{a_j}, \tag{6}$$

$$-2k_j \frac{dE^B_{aj}}{dz} - \omega_j^2 \mu_0 \alpha(\omega_j) E^B_{a_j} = 0 \tag{6a}$$

where $P^{NL^F}_{a_j}$, $P^{NL^B}_{a_j}$ are parts of non-linear polarisation corresponding to frequency ω_j for forward and backward waves.

Solutions of Eqs. (5a) and (6a) take the following form:

$$E^F_{a_j}(z) = E^F_{a_j}(0)\exp[\beta_j z], \tag{7}$$

$$E^B_{a_j}(z) = E^B_{a_j}(0)\exp[-\beta_j z] \tag{7a}$$

where $\beta_j = \dfrac{\omega_j^2 \mu_0 \alpha(\omega_j)}{2k_j} \leqslant 0.$

Solutions of Eqs. (5) and (6) are given by:

$$\varphi^F_{a_j} = \Phi^F_{a_j} + \varphi^F_{a_j}(0), \tag{8}$$

$$\varphi^B_{a_j} = \Phi^B_{a_j} + \varphi^B_{a_j}(0) \tag{8a}$$

where:

$$\Phi^F_{a_j}(z) = \int_0^z \left[\frac{-\omega_j^2 \mu_0 P^{NL^F}_{a_j}}{2k_j E^F_{a_j}}\right] dz, \tag{9}$$

$$\Phi^B_{a_j}(z) = \int_0^z \left[\frac{\omega_j^2 \mu_0 P^{NL^B}_{a_j}}{2k_j E^B_{a_j}}\right] dz. \tag{9a}$$

Relations between incident waves and waves inside the Fabry–Perot resonator can be written as:

– in plane $z = 0$

$$E^F_{a_j}(0)\exp[i\varphi^F_{a_j}(0)] = \sqrt{T}E^0_{a_j}(0)\exp[i\varphi^0_{a_j}] + \sqrt{R}E^B_{a_j}(0)\exp[i\varphi^B_{a_j}(0) + i\pi], \tag{10}$$

– in plane $z = L$

$$E^B_{a_j}(L)\exp[i\varphi^B_{a_j}(L) + ik_jL] = \sqrt{R}E^F_{a_j}(L)\exp[i\varphi^F_{a_j}(L) - ik_jL + i\pi]. \tag{10a}$$

Cartesian components of transmitted waves are $E^T_{a_j}(L) = \sqrt{T}E^F_{a_j}(L)$, so Eqs. (10) and (10a) give

$$E^T_{a_j} = \frac{\eta_j T E^0_{a_j}\exp[i\varphi^0_{a_j}]\exp[-i\varphi^F_{a_j}(0)]}{1 - R\eta_j^2\exp[i\varphi^F_{a_j}(L) - i\varphi^B_{a_j}(L) + i\varphi^B_{a_j}(0) - i\varphi^F_{a_j}(0) - 2ik_jL]} \tag{11}$$

where $\eta_j = \exp[\beta_j L]$, and

$$|E^T_{a_j}|^2 = \frac{\eta_j^2 T^2 |E^0_{a_j}|^2}{(1-\eta_j^2 R)^2 + 4\eta_j^2 R\sin^2\left(\frac{\Phi^F_{a_j}(L) - \Phi^B_{a_j}(L) - 2k_jL}{2}\right)}. \tag{12}$$

The Cartesian components of non-linear polarisation $P^{NL^F}_{a_j}$ and $P^{NL^B}_{a_j}$ which are in the phase difference in Eqs. (11) and (12) can be obtained from expression for non-linear polarisation (4a). On the basis of these relations we have obtained the components of total non-linear polarisation corresponding to different frequencies ω_j and components of total field in the cavity. Finally, Eqs. (12) have the following form:

$$E^{T2}_{x_j} = \frac{\eta_j^2 T^2 E^{0\,2}_{x_j}}{(1-\eta_j^2 R)^2 + 4\eta_j^2 R\sin^2\left[\frac{1}{2}(\zeta_j(9E^{T2}_{x_j} + 4E^{T2}_{y_j}) + \zeta_{3-j}(12E^{T2}_{x_{3-j}} + 4E^{T2}_{y_{3-j}})) - k_jL\right]}, \tag{13}$$

$$E_{y_j}^{T2} = \frac{\eta_j^2 T^2 E_{y_j}^{02}}{(1-\eta_j^2 R)^2 + 4\eta_j^2 R \sin^2\left[\frac{1}{2}(\zeta_j(9E_{x_j}^{T2} + 4E_{y_j}^{T2}) + \zeta_{3-j}(12E_{x_{3-j}}^{T2} + 4E_{y_{3-j}}^{T2})) - k_j L\right]} \tag{13a}$$

where $\zeta_j = \frac{-\omega_j^2 \mu_0 \chi(\eta_j^2 - 1)(1+\eta_j^2 R)}{32 T k_j \beta_j \eta_j^2}$.

As a result, we have obtained relations (13), (13a) which are necessary to describe intensity of transmitted waves.

4. Characteristics of transmitted intensity through the Fabry–Perot resonator

Intensity of waves transmitted through the Fabry–Perot resonator can be described by [3]

$$I_{T_j} = 2n_L^{NL}\left(\frac{\varepsilon_0}{\mu_0}\right)^{1/2} [E_{x_j}^{T2} + E_{y_j}^{T2}] \tag{14}$$

where $E_{a_j}^T$ for $a = x, y$ are Cartesian components of transmitted fields.

It can be seen ((13)–(14)) that the intensity of transmitted waves depends on:

1. Incident intensity of both waves via cross-modulation and self-modulation effects.

2. Frequencies of incident waves.

3. Resonator parameters – its thickness L as well as intensity transitivity T and reflectivity R.

4. Type of non-linear medium – constant χ and the linear absorption coefficient α.

It is important that the above mentioned parameters of incident waves and resonator influence transmitted intensities in two ways. We draw this conclusion from the influence of parameter changes on the symmetry of equation set (13)–(14) (change of the field indexes j, $3-j$). Only the parameters depending on wave frequencies change the symmetry of equation set. The following parameters: $|\chi|$, L, T and R do not change the shape of transmitted intensity curves because their influence can be compensated by the change of the range of incident intensity. Representing this group of parameters, as an example, we will consider the influence of thickness L. Besides, what is most important, the shape of curves depends on sign χ and frequency of incident waves. That is why the next part of this work comprises an analysis of the influence of those parameters on transmitted intensities. The coefficient α occurring in expressions η_j, β_j, which are in the final equations, influences the solutions in two ways. The influence of α, as it was for $|\chi|$, L and R, can be compensated by the change of the range of incident intensity. It will also play the role of the coefficient slightly changing the shape of curves resulting from the differences between η_1, β_1 and η_2, β_2.

In order to expose the influence of wave parameters and the sign χ on the character of curves, in numerical calculations, we have assumed lack of absorption of both waves. From Eqs. (13)–(14) we can see that the transmission maxima corresponding to non-linear resonance of cavity occur when sine in these equations takes the value of zero.

5. Results of numerical calculation

In order to better present the influence of cross-modulation effect, the charts show situations in which incident intensity of the first wave I_{0_1} is constant, while incident intensity of the second wave I_{0_2} increases above zero. We have done calculations for different values of incident intensity of constant wave taking the same range of changes of incident intensity of the second one. In order to show the dependence between the parameters of non-linear medium, resonator, waves and transmission characteristics, the charts have been done for non-linear media with $\chi > 0$ and $\chi < 0$ for resonators of different length and different incident wave frequencies. The incident total intensity $I_{0_1}+I_{0_2}$ has been marked (only I_{0_2} changes) on the axis of abscissae as the pictures show. Both axes of intensities are undimensional which is connected with the change of range.

Figure 2**a** presents the intensity of transmitted wave I_{T_1} at the constant incident intensity $I_{0_1} = I =$ const. as a function of incident total intensity $I_{0_1}+I_{0_2}$. Figure 2**b** presents the intensity of transmitted wave I_{T_2} at the incident intensity I_{0_2} as a function of incident total intensity $I_{0_1}+I_{0_2}$. Both charts have been done for $\chi > 0$, $L = 0.005$ m, $\lambda_1 = 632.8$ nm, $\lambda_2 = 1060$ nm.

Figures 2**a**,**b** evidently show that at some levels of incident intensities bistable states of transmitted intensities of both waves I_{T_1} and I_{T_2} appear. Passes between different states I_{T_1} and I_{T_2} are marked with dashed line in all figures. They are

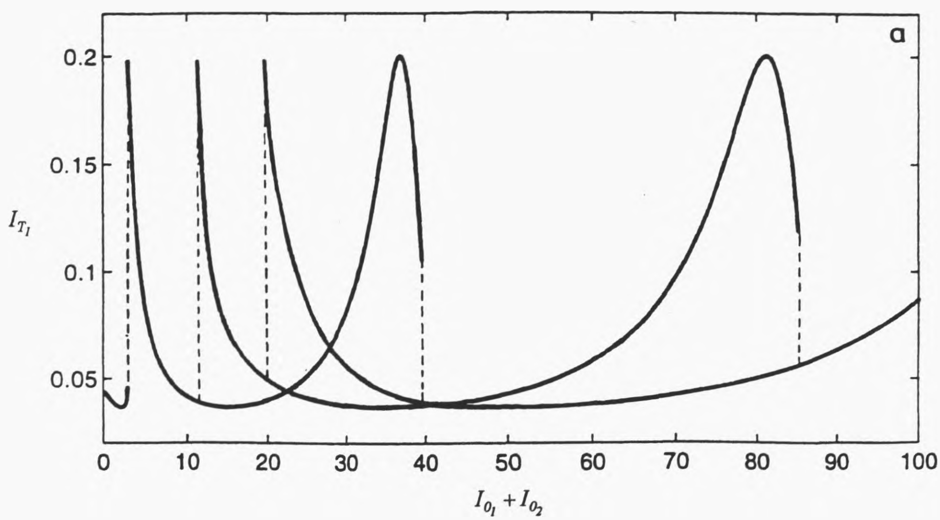


Fig. 2**a**

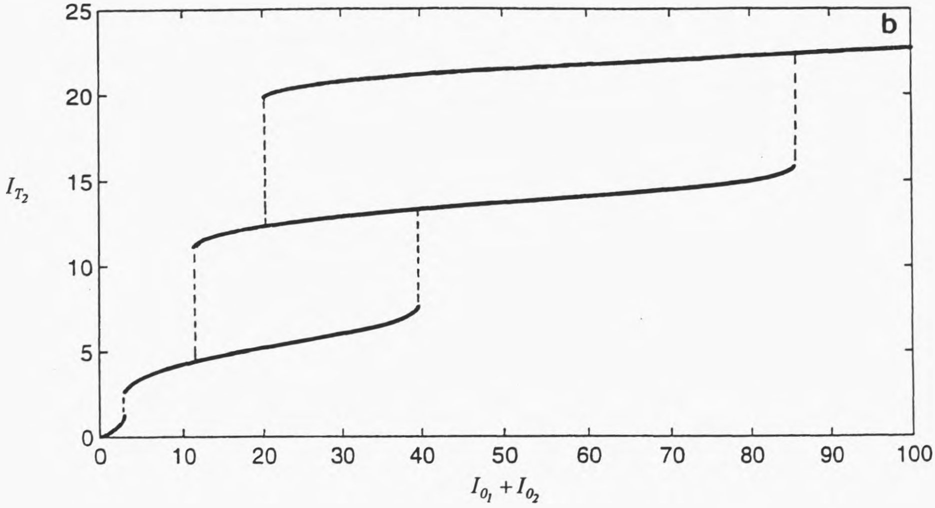


Fig. 2. Intensity of transmitted wave I_{T_1} at the constant incident intensity $I_{0_1} = I$ as a function of incident total intensity $I_{0_1}+I_{0_2}$ ($\chi > 0$, $L = 0.005$ m, $\lambda_1 = 632.8$ nm, $\lambda_2 = 1060$ nm) (a). Intensity of transmitted wave I_{T_2} as a function of incident total intensity $I_{0_1}+I_{0_2}$ ($\chi > 0$, $L = 0.005$ m, $\lambda_1 = 632.8$ nm, $\lambda_2 = 1060$ nm) (b)

related to the occurrence of non-linear resonances of cavity. Comparing these charts we can observe that bistable passes of both transmitted intensities happen at the same incident intensity $I_{0_1}+I_{0_2}$ confirming the symmetric influence of the cross-modulation effect on active waves.

The arrows placed on dashed lines in Figs. **2a,b** (and in all above charts) indicate directions of the passes between parts of curves I_{T_1} and I_{T_2} depending on increase and decrease of total intensity. We can easily observe that at the increase and decrease of $I_{0_1}+I_{0_2}$, the intensities I_{T_1} and I_{T_2} pass along different parts of curves. It is interesting that the passes of transmitted intensities I_{T_1} and I_{T_2} can be different according to the maximum value $I_{0_1}+I_{0_2}$ at which it will start to decrease. All above relations refer to charts mentioned below.

Figures 3**a**,**b** describe analogous situations for the same wave frequencies and the parameters of the cavity and non-linear medium. In this case the constant incident intensity is bigger than in previous situation $I_{0_1} = 20I = \text{const}$. The maximum value of $I_{0_1}+I_{0_2}$ is twice as small as the above in order to underline differences between Figs. 2**a**,**b** and Figs. 3**a**,**b**.

We can observe a different shape of curves and appearance of a new branch in Figs. 3**a**,**b**. The passes of transmitted intensities depend on the level of constant incident intensity I_{0_1}.

Figures 4**a**,**b** and 5**a**,**b** present corresponding situations (Figs. 2**a**,**b**, 3**a**,**b**) for the same wave frequencies and the parameters of the cavity but for the non-linear medium with $\chi < 0$.

Comparing Figures 2**a**,**b**, 3**a**,**b** and Figures 4**a**,**b**, 5**a**,**b**, we can observe that in both situations behaviour of transmitted intensities is similar but shapes of curves are

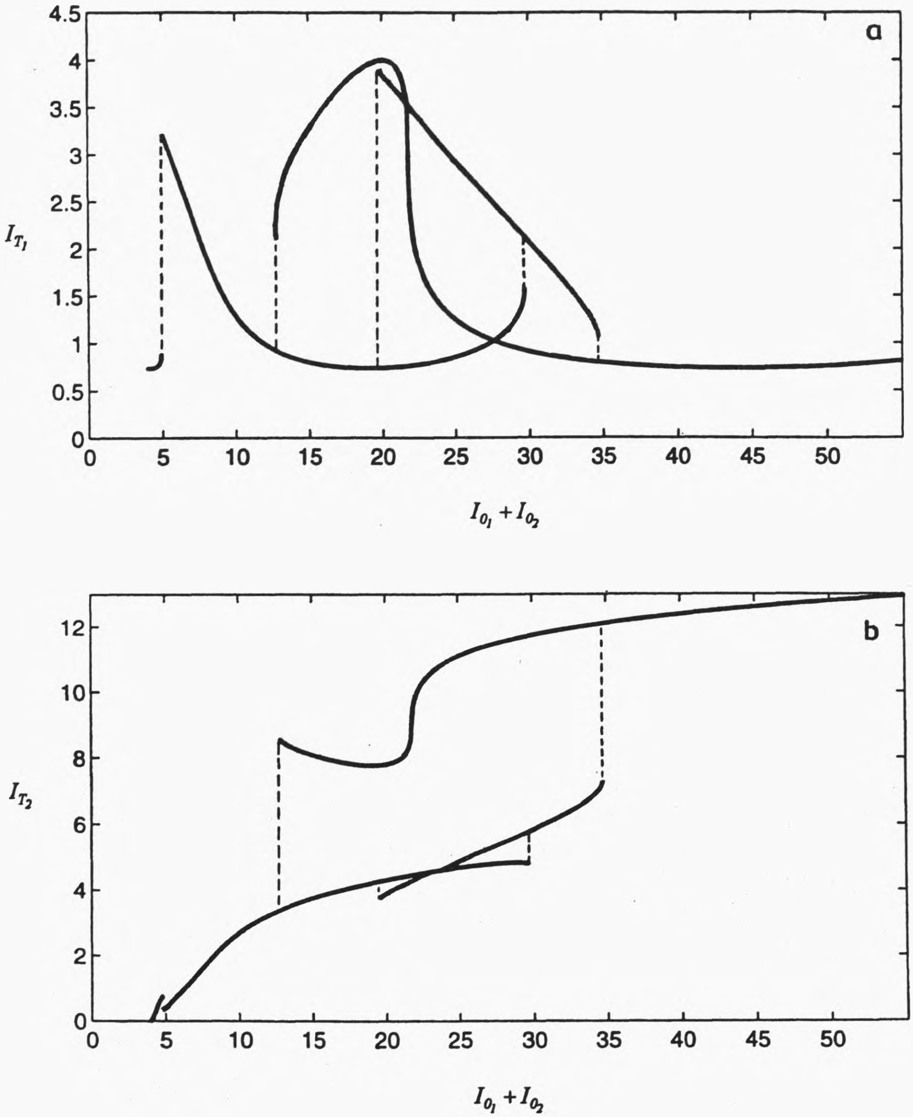


Fig. 3. Intensity of transmitted wave I_{T_1} at the constant incident intensity $I_{0_1} = 20I$ as a function of incident total intensity $I_{0_1}+I_{0_2}$ ($\chi > 0$, $L = 0.005$ m, $\lambda_1 = 632.8$ nm, $\lambda_2 = 1060$ nm) (a). Intensity of transmitted wave I_{T_2} as a function of incident total intensity $I_{0_1}+I_{0_2}$ ($\chi > 0$, $L = 0.005$ m, $\lambda_1 = 632.8$ nm, $\lambda_2 = 1060$ nm) (b)

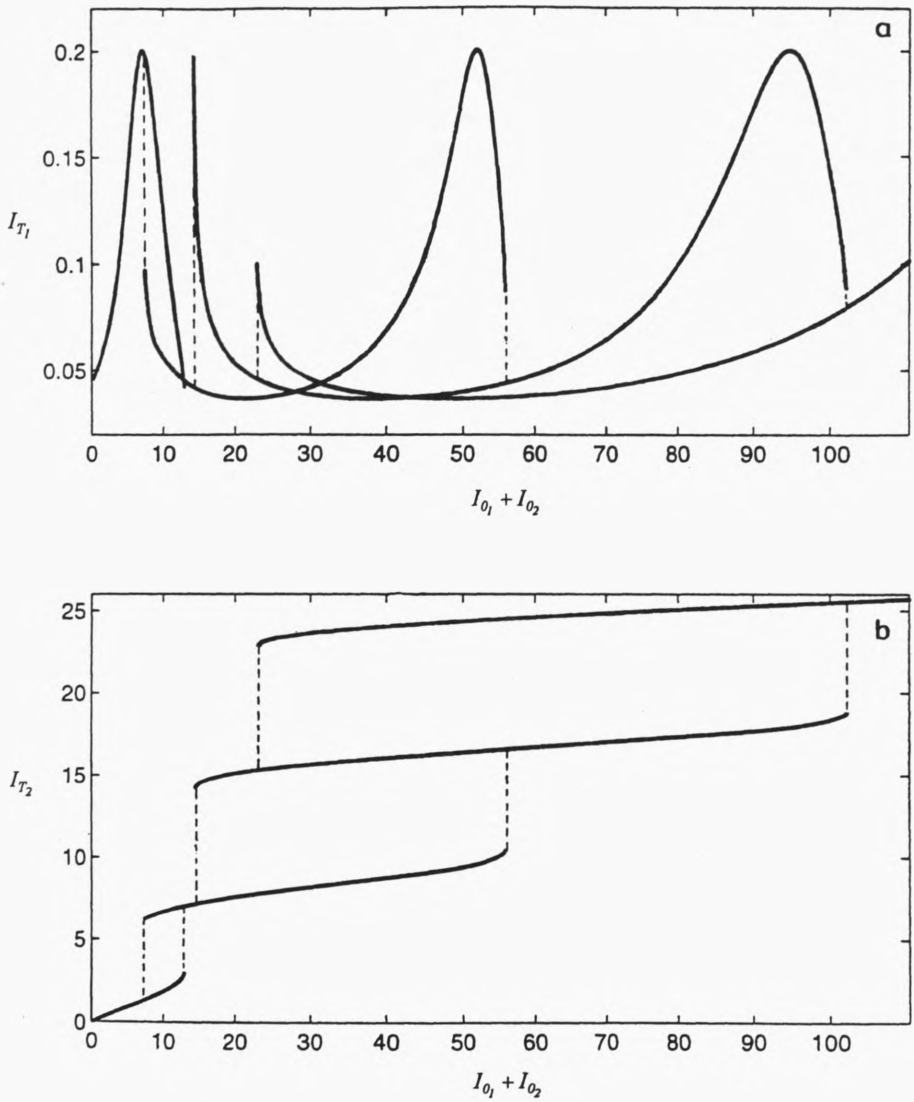
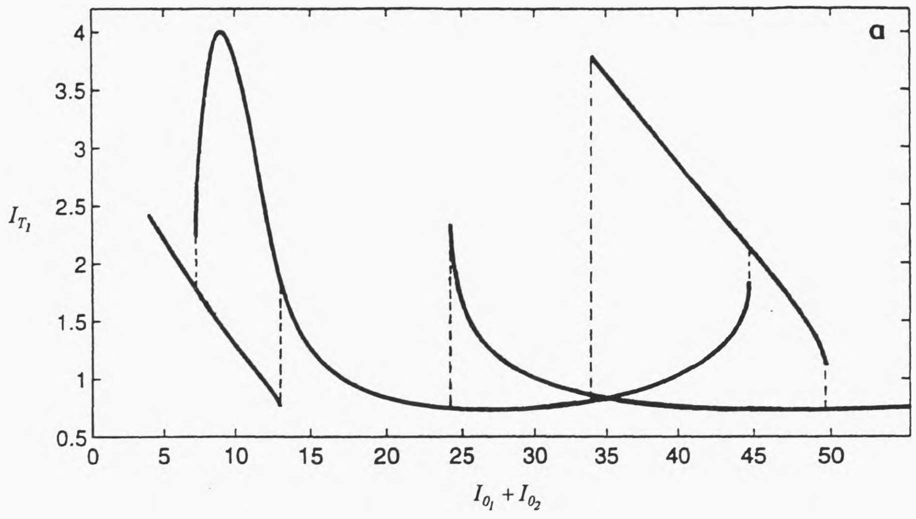


Fig. 4. Intensity of transmitted wave I_{T_1} at the constant incident intensity $I_{0_1} = I$ as a function of incident total intensity $I_{0_1} + I_{0_2}$ ($\chi < 0$, $L = 0.005$ m, $\lambda_1 = 632.8$ nm, $\lambda_2 = 1060$ nm) (a). Intensity of transmitted wave I_{T_2} as a function of incident total intensity $I_{0_1} + I_{0_2}$ ($\chi < 0$, $L = 0.005$ m, $\lambda_1 = 632.8$ nm, $\lambda_2 = 1060$ nm) (b)



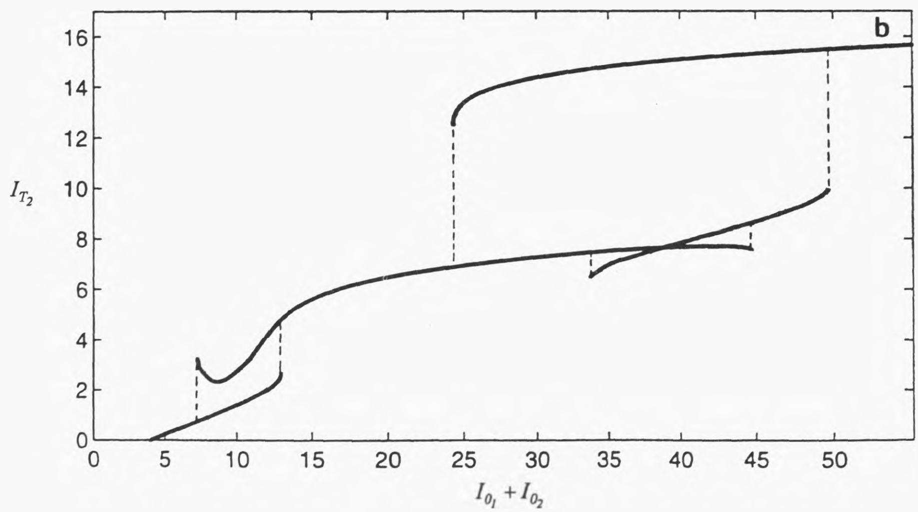


Fig. 5. Intensity of transmitted wave I_{T_1} at the constant incident intensity $I_{0_1} = 20I$ as a function of incident total intensity $I_{0_1}+I_{0_2}$ ($\chi < 0$, $L = 0.005$ m, $\lambda_1 = 632.8$ nm, $\lambda_2 = 1060$ nm) (a). Intensity of transmitted wave I_{T_2} as a function of incident total intensity $I_{0_1}+I_{0_2}$ ($\chi < 0$, $L = 0.005$ m, $\lambda_1 = 632.8$ nm, $\lambda_2 = 1060$ nm) (b)

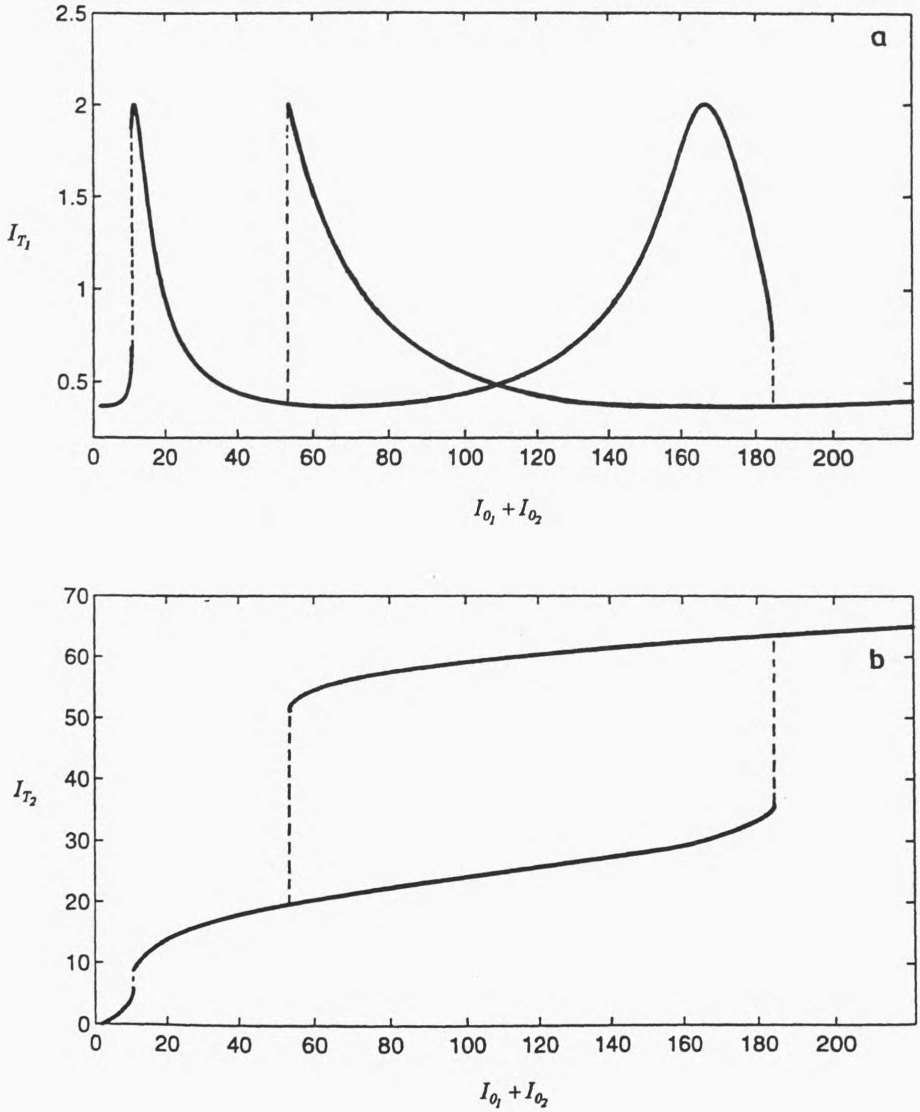


Fig. 6. Intensity of transmitted wave I_{T_1} at the constant incident intensity $I_{0_1} = 10I$ as a function of incident total intensity $I_{0_1}+I_{0_2}$ ($\chi > 0$, $L = 0.001$ m, $\lambda_1 = 632.8$ nm, $\lambda_2 = 1060$ nm) (a). Intensity of transmitted wave I_{T_2} as a function of incident total intensity $I_{0_1}+I_{0_2}$ ($\chi > 0$, $L = 0.001$ m, $\lambda_1 = 632.8$ nm, $\lambda_2 = 1060$ nm) (b)

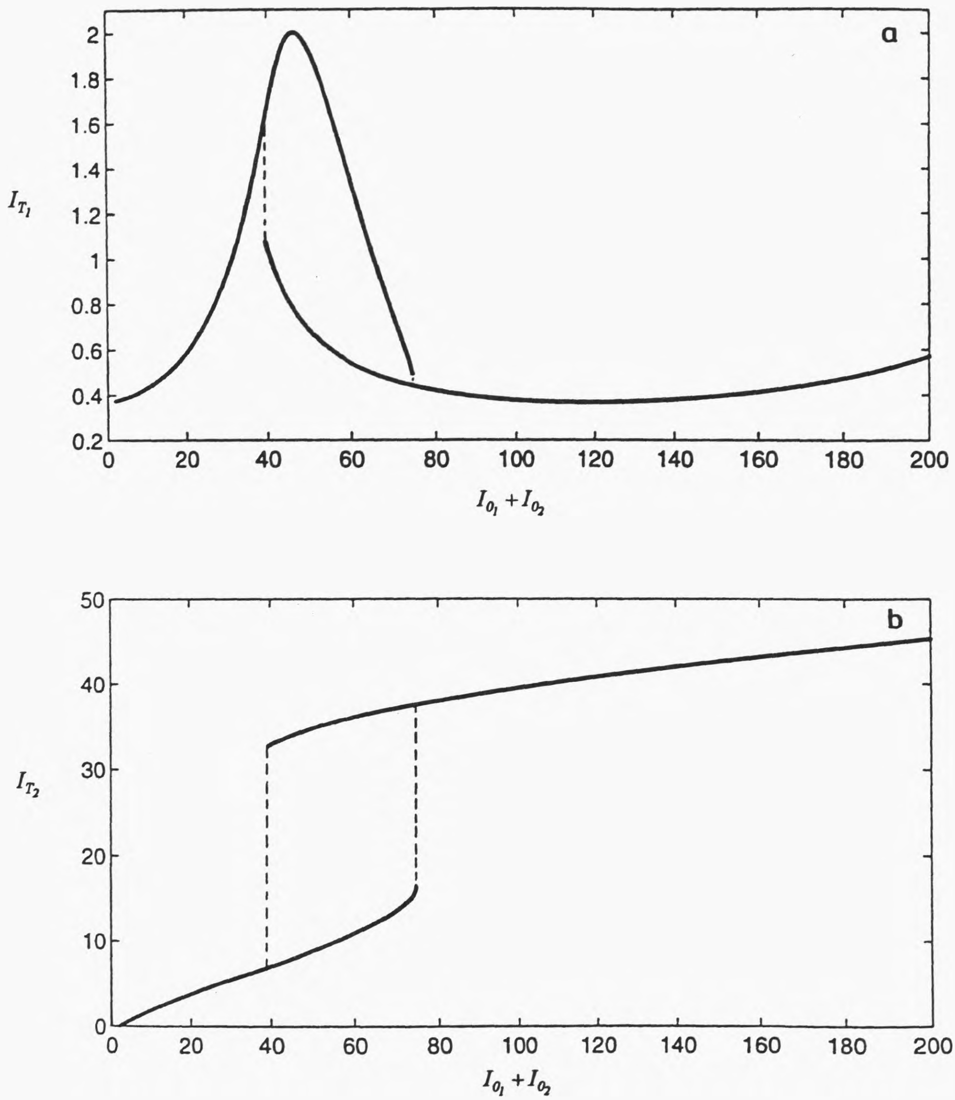
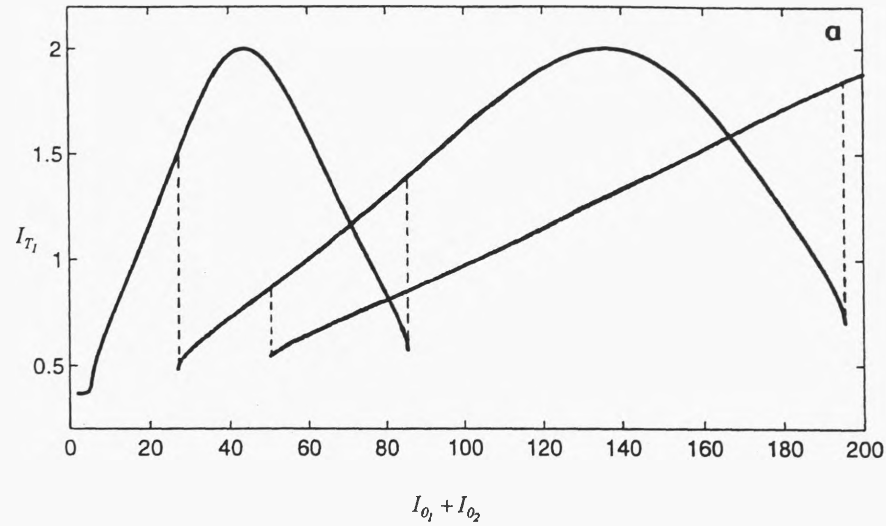


Fig. 7. Intensity of transmitted wave I_{T_1} at the constant incident intensity $I_{0_1} = 10I$ as a function of incident total intensity $I_{0_1}+I_{0_2}$ ($\chi < 0$, $L = 0.001$ m, $\lambda_1 = 632.8$ nm, $\lambda_2 = 1060$ nm) (a). Intensity of transmitted wave I_{T_2} as a function of incident total intensity $I_{0_1}+I_{0_2}$ ($\chi < 0$, $L = 0.001$ m, $\lambda_1 = 632.8$ nm, $\lambda_2 = 1060$ nm) (b)



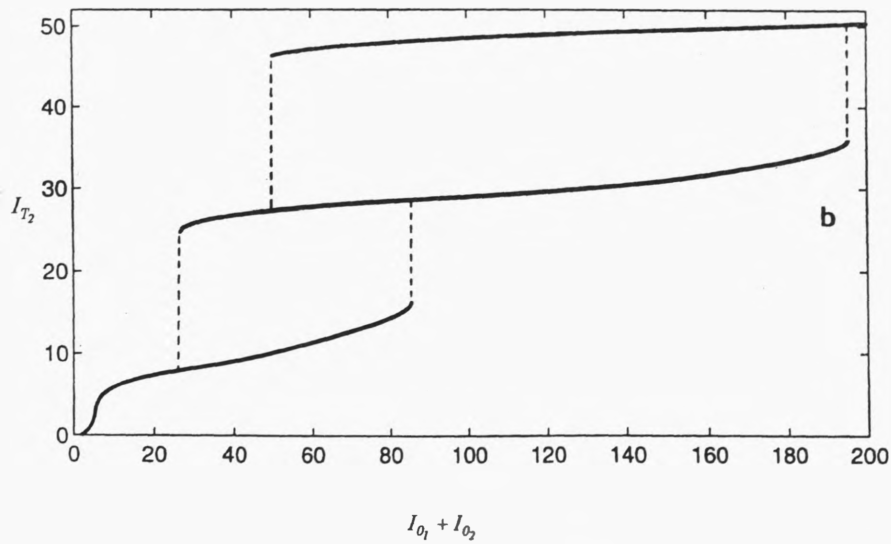


Fig. 8. Intensity of transmitted wave I_{T_1} at the constant incident intensity $I_{0_1} = 10I$ as a function of incident total intensity $I_{0_1}+I_{0_2}$ ($\chi > 0$, $L = 0.001$ m, $\lambda_1 = 632.8$ nm, $\lambda_2 = 514$ nm) (**a**). Intensity of transmitted wave I_{T_2} as a function of incident total intensity $I_{0_1}+I_{0_2}$ ($\chi > 0$, $L = 0.001$ m, $\lambda_1 = 632.8$ nm, $\lambda_2 = 514$ nm) (**b**)

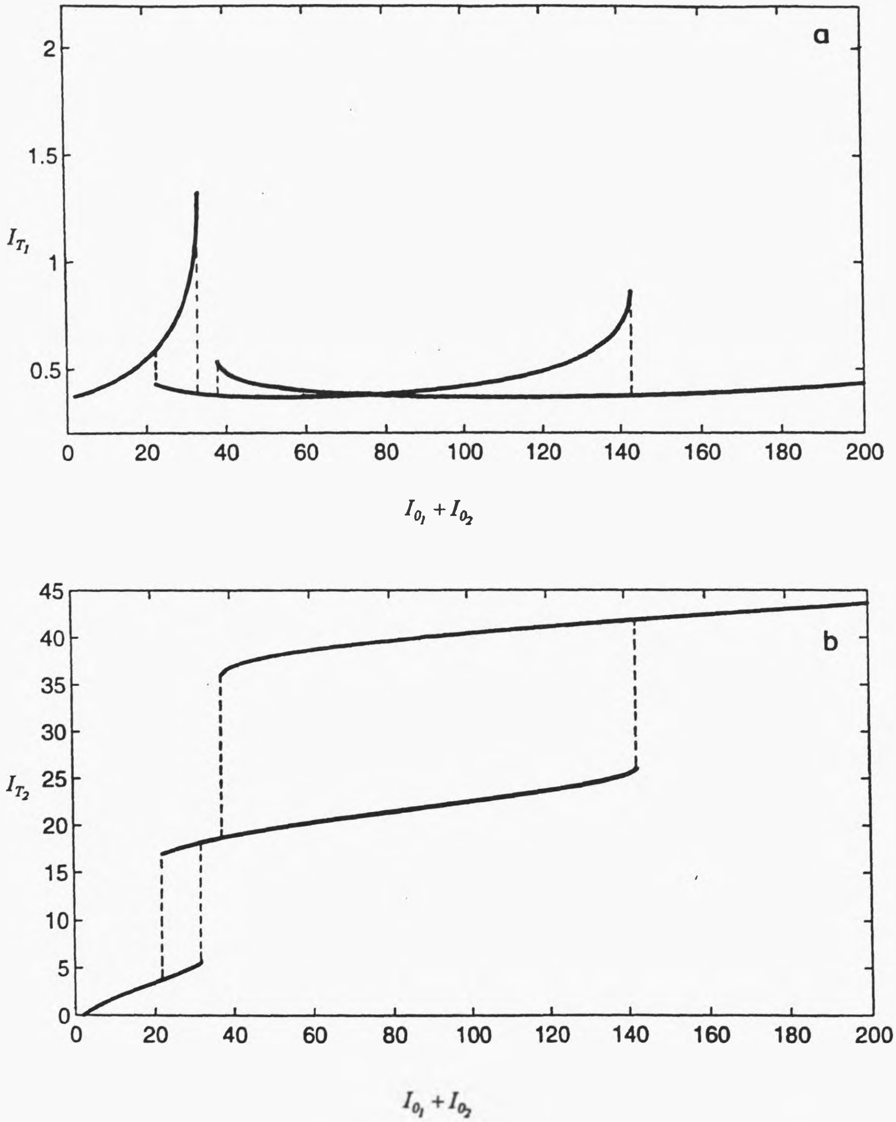


Fig. 9. Intensity of transmitted wave I_{T_1} at the constant incident intensity $I_{0_1} = 10I$ as a function of incident total intensity $I_{0_1}+I_{0_2}$ ($\chi < 0$, $L = 0.001$ m, $\lambda_1 = 632.8$ nm, $\lambda_2 = 514$ nm) (**a**). Intensity of transmitted wave I_{T_2} as a function of incident total intensity $I_{0_1}+I_{0_2}$ ($\chi < 0$, $L = 0.001$ m, $\lambda_1 = 632.8$ nm, $\lambda_2 = 514$ nm) (**b**)

different. Moreover, the χ sign influences levels of $I_{0_1}+I_{0_2}$ at which the passes between parts of curves appear. Those passes correspond to non-linear resonances of cavity.

Figures 6**a**,**b** and 7**a**,**b** show the influence of cavity thickness L on transmission characteristics. The charts present a similar situation to that in Figs. 2**a**,**b**, 3**a**,**b** but the cavity thickness is less and it takes the value $L = 0.001$ m, and the incident intensity is $I_{0_1} = 10I = \text{const}$. Figures 6**a**,**b** describe the case $\chi > 0$, and Figures 7**a**,**b** – $\chi < 0$.

As the comparison of charts shows, at this thickness we need stronger incident fields in order to obtain bistable passes than in the case of $L = 0.005$ m in length. At the same time, as we compare Figs. 2**a**,**b**, 3**a**,**b** and Figs. 6**a**,b, 7**a**,**b**, it is easy to observe that the change of thickness and increase in some range I_{0_1} do not change the shapes of curves. The thinner cavity is less sensitive to the change of input parameters.

Figures 8**a**,**b** and 9**a**,**b** describe the influence of incident wave frequencies. All the charts have been done for $L = 0.001$ m and $I_{0_1} = 10I = \text{const}$. In Figures 8**a**,**b** and Figures 9**a**,**b**, we can see the situations in which the wavelength at constant intensity is the same as in all previous cases with $\lambda_1 = 632.8$ nm, where the length of the second wave takes the value $\lambda_2 = 514$ nm.

The comparison of the above charts with Figs. 6**a**,**b**, 7**a**,**b** indicates that the change of the difference of wavelengths influences the character of curves and levels of $I_{0_1} + I_{0_2}$ at which non-linear resonances of cavity appear as well as the passes between parts of curves.

6. Conclusions

It can be seen that the cross-modulation effect has the influence on:

– intensities of waves transmitted through the Fabry–Perot resonator and occurrence of bistability and multistability of transmitted intensity of both waves,

– courses of intensities of transmitted waves depending on maximum value of incident intensities.

Our results lead to the conclusion that incident intensity of one wave can control transmission state of the second one and vice versa. These phenomena can be used in designing optical logic elements and various devices.

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