# Wave aberrations caused by misalignments of aspherics, and their elimination 

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#### Abstract

When testing aspherics and optical systems by interferometric means wave aberrations due to misalignments may screen the test results and impede the alignment process. In order to study the effect of misalignments on the interference pattern in five degrees of freedom holographical and mathematical simulation patterns were produced. The simulation results corresponds with one another. The wave aberrations, remaining after a best fit adjustment is performed, can be eliminated by least squares approximations. In simple cases the misalignment aberrations can be described by an analytical expression. In more complex test situations when an analytic description is impossible numerical methods have to be applied. The usefulness of the method is demonstrated on an interferogram with a lateral shift of $50 \mu \mathrm{~m}$.


## Introduction

In a previous publication [1] a simple interferometric set-up for testing aspherics in transmitted light was described. The aspheric wave aberrations caused by the aspheric element are compensated by a synthetic hologram. In particular, rotational symmetric aspherics can also be tested by means of rotational symmetric holograms (RSH). The calculation and plotting procedure of the RSH is described in [1]. For a survey of the literature on this subject we refer to [4]. The set-up used in our experiments consists of a Mach-Zehnder-Interferometer followed by a double diffraction arrangement containing a compensating hologram. This hologram has stored the deviations of the RSH-glass carrier from constant optical thickness. During the reconstruction process two waves impinge on the compensating hologram. When the undiffracted wave of the object arm and the first order of diffraction of the reference arm are chosen, then the resulting interference pattern is free from the RSH-carrier deviations.

## Simulation of misalignment patterns

This compensation technique can also be applied to the simulation of pure misalignment patterns. To this end an arrangement shown in fig. 1 is used. A Mach-Zehnder-Interferometer is the basic interferometric structure. The aspheric and RSH are jointly situated in one arm of the interferometer. The other arm provides a plane reference wave. If all
components are free from faults and the best adjustment is attained then the resulting intensity is constant over the whole field.

Unfortunately, in reality neither the aspherics nor the other interferometric components are ideal. This results in an irregular fringe pattern even in the case of best adjustment. As was shown in [1], however, the


Fig. 1. Scheme of the test interferometer. The interferometer consists of a Mach-Zehnder-Interferometer in combination with a common path-Interferometer containing an off-set-hologram (CH)
$F_{1}, F_{2}$ - spatial filters, $I P$ - image plane, $O$ - collimator objective of the camera, $M$ - tilted mirror providing the carrier frequency for CH
remaining disturbances due to deviations of optical components can be eliminated by adding a second hologram (see fig. 1). In contrast to the technique described in [1], the aspheric is not removed from the Mach-Zehnder-Interferometer when the second hologram (CH) is produced. In this way during the reconstruction of CH all the deviations, including those of the aspherics from the ideal shape are eliminated. Regarding the mathematical description involved we refer to [1].

Following the procedure mentioned above the whole interferometric set-up is compensated and ready for simulation experiments. Alignments performed after the repositioning of the developed hologram CH show up as fringes. In general, the deviations of the aspherics from the ideal shape are small. Therefore the effect of these small deviations on the alignment pattern can be neglected, because the first partial derivatives of the ideal surface contribute much stronger to the aberrations.

The results of the optical simulation were confirmed by computer simulations. For this purpose a Mach-Zehnder-Interferometer with identical aspherics in either interferometer arm was assumed. The degrees of freedom for alignment were split up into two sets. One set comprised
the translational degrees of freedom and the other the rotational ones. The wave aberrations corresponding to the interference pattern were calculated by ray tracing. The coincidence of the rays in the output plane was obtained by holding fixed the ray of the one arm and iterating the other. Starting from the input plane the optical path difference was calculated and plotted for whole multiples of the assumed wavelength. In the calculation small rotations were dealt with a vector approximation [2] to avoid matrix operations. The calculational error due to this approximation is of the second order in the alignment parameters and can be neglected in this approximation (for further details see [3]).

Fig. 2 shows the result for a lateral shift of one aspheric*) with respect to the other in the set-up of fig. 1. The experimental and computational simulations correspond with each other. In this way all degrees of freedom


Fig. 2. Simulation pattern for lateral shift by $50 \mu \mathrm{~m}$ : left - interferogram, right computer simulation
were studied yielding the influence of several alignment parameters on the wave aberrations. So it was possible to find out the critical adjustment elements in the test interferometer. Furthermore, the kind of existing misalignments can be concluded from the interference pattern. This enables a faster and more guided approach to the best alignment.

## Mathematical elimination of misalignment aberrations

So far we have considered only pure misalignments. When testing aspherics, however, the wave aberrations showing up are the sum of the effects of misalignments and relevant surface deviations, provided that

[^0]the RSH-carrier aberrations are eliminated holographically. In general, the restriction to pure alignment steps makes impossible the separation of these influences.

Therefore, to reach this aim a posteriori mathematical procedures should be applied. It is based on the assumption that only those wave aberrations are considered to be relevant that cannot be compensated for by adjustments. This assumption seems to be justified, because aberrations caused by surface deviations cannot be distinguished from misalignment aberrations if they both yield the same interference pattern. Therefore only those aberrations which cannot be interpreted as being also misalignment aberrations are considered to originate from the surface. For this reason the measured values are subject to a least squares procedure taking misalignment aberrations into account.

Let $W$ be the wave aberrations measured and $\bar{W}$ the aberrations due to misalignments, then one gets corrected aberrations $\tilde{W}$ :

$$
\begin{equation*}
\tilde{W}=W-\bar{W} \tag{1}
\end{equation*}
$$

For small misalignments corresponding to the three translationa degrees of freedom an analytical expression can be derived:

$$
\begin{equation*}
\bar{W}\left(x, y, z_{H}, u, v, w, e, f, g\right)=\frac{\partial L}{\partial x} u+\frac{\partial L}{\partial y} v+\frac{\partial L}{\partial z_{H}} w+e+f x+g y \tag{2}
\end{equation*}
$$

where $L=L\left(x, y, z_{H}, u, v, w\right)$ are the optical path differences due to a ray trace through the aspheric as member of an optical system*), $u, v, w$, the translations parallel to the $x$-, $y$-, $z$-axis, $z_{H}$ the distance aspheric-RSH (or the distance to the next surface within a system), and $e, f, g$ constants due to a plane reference wavefront. The aberrations are determined in a net of points $\left(x_{i}, y_{j}\right)$ in the exit pupil of the interferometer.

The sum

$$
\begin{equation*}
S=\sum_{i}^{I} \sum_{j}^{J}\left[W\left(x_{i}, y_{i}\right)-\bar{W}\left(x_{i}, y_{j} ; z_{H}, u, v, w, e, f, g\right)\right]^{2} \tag{3}
\end{equation*}
$$

is made minimum by the variation of this alignment parameters resulting in a system of 6 linear equations for the unknown quantities $u, v, w, e$, $f, g$ :

$$
\begin{equation*}
\frac{\partial S}{\partial u}=\frac{\partial S}{\partial v}=\frac{\partial S}{\partial w}=\ldots=\frac{\partial S}{\partial g}=0 \tag{4}
\end{equation*}
$$

After having solved the equation system (4) the found 6 quantities are inserted into eqs. (2) and (1) to obtain the corrected wave aberrations. In this way we get rid of irrelevant aberrations caused by misalignments.

[^1]Only in few selected cases, however, the geometry of the testing set-up coincides with the optical system, of which the aspheric is a part. Therefore in any particular application the function $L\left(x, y, z_{H}, u, v, w\right)$ should be adapted to the prevailing geometry. On the one hand, while applying the testing set-up and the aspheric as optical component of an optical system, essential differences between the misalignment aberrations may appear. The aberrations, however, will maintain their character, because the optical path differences depend only on the cosines of the inclination of the rays relative to the optical axis. On the other hand, the aberrations due to misalignments depend strongly on the dimension of asphericity of the surfaces for small asphericities. Therefore the working principle of the elimination method could be studied at the interferometer geometry without the restriction of general validity.

The input data for the elimination procedure were extracted from an interferogram of a real aspheric having a maximum deviation from the tangential plane at the vertex of the surface of 2.5 mm . The aspheric was intentionally shifted from the best fit position by $50 \mu \mathrm{~m}$ laterally*) to the optical axis. The evaluation of the interferogram was made by a graphical method yielding the actual phase in a rectangular net of points. Fig. 3 shows the initial interferogram (on the left) and the resulting least squares contour map of the wave aberrations due to surface deviations (on the right, contour line distance corresponding to a level difference of $1.2 \mu \mathrm{~m}$ ).

The rms-error of the graphical evaluation amount to half a fringe at the rim of the interferogram. This relatively big error is caused by the


Fig. 3. Example for the elimination of misalignment aberrations: left - real aspheric deviations superposed with aberrations due to lateral shift ca. $50 \mu \mathrm{~m}$; right - contour map after the application of the "least squares" procedure
*) The choice of a lateral displacement is justified by the fact that axial translations can be excluded by additional measurements.
severe gradient of the deviations in the outer parts of the surface. Error reductions could easily be attained by an improved evaluation technique of the interferograms.

## References

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## Волновые аберрации, вызванные несоосностью асферических линз и их устранение

При испытании асферических линз и оптических систем интерферометрическими методами волновые аберрации, вызванные несоосностью, могут прикрыть результаты испытаний и затруднить процесс центрирования. Для исследования влияния несоосности на интерференционную систему при пяти степенях свободы выполнены голографические и математические имитационные модели; результаты имитирования соответствуют друг другу. Волновые аберрации, возникающие после выполнения наилучшей припасовки, могут быть устранены приближением с помощыо метода наименьших квадратов; в несложных случаях аберрации несоосности могут быть описаны аналитически, а при более сложных испытаниях, где не может применяться аналитическое описание, использовались численные методы. Пригодность метода доказана на интерферограмме с боковым смещением в 50 мкм.


[^0]:    *) The interferogram shown is typical for aspheric wavefronts in contrast to spherical wavefronts where lateral shear results in straight and parallel "tilt" fringes.

[^1]:    *) Here the optical system is the test interferometer.

