

## COMMUNICATION

P. GRAU, F. TUČEK, M. DOHÁNYOS, I. HOLATA\*

KINETIC ASPECTS OF BIOLOGICAL REACTORS WITH MIXED POPULATIONS  
FOR WASTEWATER TREATMENT

## 1. INTRODUCTION

The activated sludge process has been used traditionally as relatively inexpensive, highly effective and reliable operation for the removal of organic carbon, and partially also of nitrogen and phosphorus from municipal and industrial wastewaters.

The process is a continuous system in which an aerobic heterogenous population of microorganisms, (called activated sludge) spontaneously developed under operating conditions is mixed with wastewater and, after appropriate reaction time, separated and concentrated in a gravity clarifier in order to be partially recycled. Volumetric recirculation ratio is usually within the 20–100% range.

The activated sludge system is thus a biological reactor with biomass recycle. Its theoretical analysis is extremely complex and complicated. The variability of the substrate composition and concentration, the diversity of biota and the nature of biological processes together with common operating features, i.e. variable flow and temperature, introduce numerous problems which have not been satisfactorily solved yet.

The practical design technique is usually based on empiric complex criteria, as for example sludge loading, volume loading etc. For further development of the process and its better understanding it is desirable, however, to develop as rational mathematical models as possible.

## 2. KINETICS PRINCIPLES

The organic substances present in wastewater serve to the substrate for microorganisms. Depending on the type of wastewater, fast, medium and slowly removable substrates in various concentrations can be distinguished. Some are not degradable and in addition a minor portion of nondegradable metabolics is generated in the process.

Since it is not practical and sometimes not possible to analyze individually for all the constituents present, analytical techniques giving the total amount of substrate, such as biochemical oxygen demand (BOD), chemical oxygen demand (COD) or total organic carbon (TOC) have been developed and used.

---

\* Prague Institute of Chemical Technology, Department of Water Technology and Environmental Engineering, 166 28 Prague, Czechoslovakia.

The kinetics of total substrate removal can be expressed, by analogy to chemical reactions, e.g.

$$\frac{dS}{dt} = k_n S \quad (1)$$

where  $S$  is total substrate concentration  $\text{mg} (\text{dm}^3)^{-1}$ ,  $k_n$  is rate constant  $\text{h}^{-1}$ ,  $t$  is time  $\text{h}$ ,  $n$  is the formal order of the reaction.

Since it often happens that not only the substrate, but also non removable organic substances are included in the chemical analysis, the following corrections of COD or TOC for the substrate have been proposed

$$S' = \frac{L}{b}, \quad (2)$$

$$S = S' - y, \quad (3)$$

where  $L$  is the concentration of total organics (as COD, TOC etc),  $S'$  denotes the concentration of biodegradable fraction,  $b$  and  $y$  are correction factors. Factor  $b$  considers non-removable organics present in the waste, while factor  $y$  corrects formation of non-removable organics in the process.

Combination of both factors yields

$$S = \frac{L}{b} - y. \quad (4)$$

The magnitude of the rate constant  $k_n$  depends on the biomass concentration

$$k_n = k_{nj} X, \quad (5)$$

where  $k_{nj}$  is the specific rate constant of microbial kinetics  $\text{mg}^{-1} \text{dm}^3 \text{h}^{-1}$ ,  $x$  is biomass concentration on  $\text{mg} (\text{dm}^3)^{-1}$ .

Recently it has been shown [3] that multicomponent mixtures of substrates are removed according to differential equation

$$\frac{dS}{dt} = k_{ns} X \frac{S^n}{S_0} \quad (6)$$

where  $k_{ns}$  is the substrate removal rate constant,  $\text{mg}$  of substrate  $(\text{mg}$  of biomass) $^{-1} \text{h}^{-1}$ ,  $S_0$  is the initial substrate concentration,  $\text{mg} (\text{dm}^3)^{-1}$ .

For continuous stirred tank reactors with maxing mixedness (CSTR - MM) the effluent concentration is

$$\bar{S} = \int_{t=0}^{t=\infty} S \frac{e^{-t/\bar{t}}}{\bar{t}} dt \quad (7)$$

where  $\bar{S}$  is the mean substrate concentration in the outlet of the reactor,  $\bar{t}$  is the mean detention time (volume to flow). By solving equation (7) we arrive at the expressions:

For  $n = 1$

$$\frac{S}{S_0} = \frac{1}{1 + \frac{k_{1s} X t}{S_0}} \quad (8)$$

For  $n = 2$

$$\frac{S}{S_0} = \frac{-1 \pm \sqrt{1 + 4k_{2s} X t}}{2k_{2s} X t} \quad (9)$$

For  $n$

$$\frac{S^n}{S_0^{n+1}} k_{ns} X t = \frac{S_0 - S}{S} \quad (10)$$

### 3. RESULTS AND DATA ANALYSIS

Using two sets of the published data, namely those by ADAMS et al. [1], and combined data by WUHRMAN [6], KEHR and von der EMDE [5] values of  $S$  have been computed according to the equations (1), (5) and (6). Data by Adams have been corrected according to the equation (3);  $y = 10 \text{ mg}(\text{dm}^3)^{-1}$  TOC. No correction factors have been applied to the other data [5], [6].

Treatment efficiency  $E$  has been calculated according to the formula:

$$E_c = \frac{S_0 - S_t}{S_0} 100 \quad (11)$$

and compared to the measured values  $E_m$ .

Residual standard deviation has been used as a criterion of good fit:

$$s_E = \frac{\sqrt{\sum (E_m - E_c)^2}}{m - p} \quad (12)$$

where  $s_E$  is the residual standard deviation of treatment efficiency in %,  $m$  is the number of observations,  $p$  is the number of the constants determined. The results of these computations are presented in tables 1 and 2.

Table 1

Computed constants and the residual standard deviation of treatment efficiency (combined data [5] and [6])

Type of equation	$k_n$	$n$	$s_E$
	*	—	%
Chemical kinetics eq. (1)	6.8904	1.00	5.04
	1.4619	1.50	3.95
	0.3240	2.00	3.99
	0.8716	1.67	3.89
Microbial kinetics eq. (5)	0.00135	1.00	5.27
	0.00029	1.50	3.80
	0.00007	2.00	3.85
	0.00017	1.68	3.72
Substrate removal kinetics eq. (6)	0.2102	1.00	5.43
	0.5713	1.50	4.59
	1.6296	2.00	4.96
	0.5484	1.48	4.59

\*The dimension depends on equation

Table 2

Computed constants and the residual standard deviation of  
of treatment efficiency (data [1])

Type of equation	$kn$	$n$	$sE$
	*	—	%
Chemical kinetics eq. (1)	5.5185	0.50	3.14
	2.3350	1.00	3.12
	1.3291	1.50	3.13
	6.8128	0.40	3.12
Microbial kinetics eq. (5)	0.00090	1.00	3.30
	0.00053	1.50	3.23
	0.00038	2.00	3.32
	0.00062	1.32	3.22
Substrate removal kinetics eq. (6)	0.0322	0.50	1.54
	0.1707	1.00	0.64
	0.9612	1.50	0.79
	0.2023	1.05	0.64

\*The dimension depends on equation.

#### 4. CONCLUSIONS

Application of chemical engineering concepts to the calculation of the completely mixed activated sludge reactors yielded satisfactory results, for a wide variation range of the substrate.

The residual standard deviation of the treatment efficiency found for actual plant data [5], [6] was in general below 5% for optimum order of the kinetics. Since the substrate concentration varied in relatively narrow limits, all three types of equations compared yielded similar precision and accuracy.

The residual standard deviation of the treatment efficiency found for a wide variation of the substrate concentration in a laboratory plant operated under strictly controlled conditions [1] dropped below 1% for substrate removal kinetics, being slightly over 3% for the other types of kinetics equations.

#### REFERENCES

- [1] ADAMS C. E., ECKENFELDER W.W. Jun., HOVIUS J. C., *Water Res.* **9**, 37–52, 1975.
- [2] BENEFIELD L. D., RANDALL C. W., *J. Wat. Pollut. Control Fed.* **7**, 1636–1641, 1977.
- [3] GRAU P., DOHÁNYOS M., CHUDOBA J., *Kinetics of multicomponent substrate removal by activated sludge*, *Water Res.* **9**, 637–642, 1975.
- [4] GRAU P., TUČEK F., DOHÁNYOS M., *Vodni hospodárství* **7**, 181–184, 1975.
- [5] KEHR D., EMDE VON DER W., *J. Wat. Pollut. Control Fed.* **32**, 1066–1080, 1960.
- [6] WUHRMANN K., *Factors effecting efficiency and solids production in the activated sludge process*, [In:] *Biological Treatment of Sewage and Industrial Wastes*, Vol. 1, p. 49, Reinhold P. Co., New York 1956.