## Letters to the Editor

## Remarks on effects of aberrating layers in confocal scanning microscopes

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In paper [1], the condition for aberration-free immersion layer in aberration-free confocal scanning microscope (CSM) is given as dependent on $\lambda, \alpha, n_{1}, n_{2}$. A spherical aberration coefficient of the first order for the layer is equal to [1]

$$
\begin{equation*}
W_{40}=2 k t\left(n_{2}^{2}-n_{1}^{2}\right) \frac{n_{1}^{2}}{n_{2}^{3}} \sin ^{4}(\alpha / 2) . \tag{1}
\end{equation*}
$$

where: $\alpha$ - semi-angle of convergence, and $k=2 \pi / \lambda$, while $n_{1}, n_{2}, t$ and 0 are defined in Fig. 1.


Fig. 1. Ray incident on a dielectric slab

Assuming the Rayleigh criterion, according to which the maximum of phase aberration must be less than $\pi / 2$ which corresponds to the limiting resolution $\lambda / 4$, the condition for aberration-free layer thickness $t$ has been obtained [1]

$$
\begin{equation*}
t \leqslant \lambda n_{2}^{3} /\left\{2 n_{1}^{2}\left(n_{2}^{2}-n_{1}^{2}\right) \sin ^{4}(\alpha / 2)\right\} \tag{2}
\end{equation*}
$$

In this paper, a correcting term to the condition (2) has been determined as related to spherical aberration $\beta_{040}$ of CSM depending on $\Delta_{\text {limesm }}, k, \alpha, n_{1}, n_{2}$; where $\Delta_{\text {limCMs }}$ - limiting resolution of CSM, $n_{2}=n_{1}+\Delta n$. A correcting coefficient $W=\frac{\Delta_{\max }}{\Delta_{\text {min }}}$ for an apodized CSM system suffering from spherical aberration has been
introduced, where $\Delta_{\text {min }}$ - minimum value of limiting resolution in CSM with apodization and spherical aberration, $\Delta_{\max }$ - limiting resolution in CSM of uniform type. In further considerations, it has been assumed that the refraction index $n_{2}$ differs only slightly from $n_{1}$. The intensity distribution in the focal plane of CSM is defined by the relation [1]

$$
\begin{equation*}
I(u, v)=\left|\int_{0}^{\alpha} A(0) P(0) J_{0}\left(\frac{v \sin \theta}{\sin \alpha}\right) \exp \left(-\frac{1}{2} i u \frac{\sin ^{2}(0 / 2)}{\sin ^{2}(\alpha / 2)}\right) \sin 0 d 0\right|^{2} \tag{3}
\end{equation*}
$$

where $u, v$ - optical coordinates which are defined by the axial distance $z$ from the focus and radial distance $r$ from the optical axis in the following way: $u=4 k z \sin ^{2}(\alpha / 2), v=k r \sin \alpha, A\left(\theta_{1}\right)$ for aplanatic system is equal to $A\left(\theta_{1}\right)=\cos ^{1 / 2} 0_{1}$, $P\left(\theta_{1}\right)$ - wavefront aberration, $P\left(\theta_{1}\right)=e^{i \varphi}$. Basing on the formula (3), the limiting resolution of CSM denoted by $\Delta_{\mathrm{HIm} \text { CsM }}$ has been numerically evaluated. The total spherical wave aberration of first order for the combination CSM plus immersion layer fullills the condition

$$
\begin{aligned}
& \Phi_{\max }=\frac{k t}{2}\left(n_{2}^{2}-n_{1}^{2}\right) \frac{n_{1}^{2}}{n_{2}^{3}} \sin ^{4}(\alpha / 2)+\Delta_{\lim }=0, \\
& \Delta_{\lim }=\Delta_{\operatorname{limCSM}} .
\end{aligned}
$$

Hence

$$
\begin{equation*}
t \leqslant 2 \Delta_{\lim } n_{2}^{3} /\left\{k\left(n_{2}^{2}-n_{1}^{2}\right) n_{1}^{2} \sin ^{4}(\alpha / 2)\right\} . \tag{4}
\end{equation*}
$$

For $n_{2}=n_{1}+\Delta n$ we have

$$
\frac{\left(n_{1}+\Delta n\right)^{3}}{\left(2 n_{1} \Delta n+\Delta n^{2}\right) n_{1}^{2}} \simeq \frac{n_{1}+3 \Delta n}{2 n_{1} \Delta n},
$$

and, consequently,

$$
\begin{equation*}
t \leqslant\left[2 \Delta_{\lim } / k \sin ^{4}(\alpha / 2)\right]\left[\frac{n_{1}+3 \Delta n}{2 n_{1} \Delta n}\right] \simeq \frac{2 W\left(n_{1}+3 \Delta n\right)}{2 n_{1} \Delta n k \sin ^{4}(\alpha / 2)}, \tag{5}
\end{equation*}
$$

The limiting value of resolution in a nonapodized and aberration-free CSM amounts to $\Delta_{\mathrm{lim}}=2.89$, which was shown in paper [2]. In CSM equipped with an apodized collector and objective with the apodizer of $r^{2}$ type, the limiting value of $\Delta_{\text {lim }}$ is equal to 1.31. For CMS charged with spherical aberration $\beta_{040}=0.5-1.5$ the limiting value $\Delta_{\text {lim }}$ does not exceed $\Delta_{\text {lim }}$ csm for the uniform case (Tab. 1). With the increase of $\alpha$ from 0.1 to $1.6, \Delta_{\mathrm{lim}} / \sin ^{4}(\alpha / 2)$ diminishes (Fig. 2). The correcting coefficient $W=\frac{\Delta_{\text {max }}}{\Delta_{\text {min }}}$ has been calculated again from formula (3) (Tab. 2) for the respective two cases.

For the classic optical system, for which $\Delta=3.83$ the correcting coefficient $W$ is equal to $\sim 2.9$. For the sake of comparison, the same intervals for refractive index were assumed as those used in paper [1]. In CSM with $\lambda=633 \mathrm{~nm}$, two cases were

Table 1. Dependence of the limiting resolution $\Delta_{\mathrm{Um}}$ on $\alpha$ in CSM with apodization of $r^{2}$ type, annular $\varepsilon$ and spherical aberration $\beta_{040}$ ( $A_{0}$ - objective, $A_{c}$ collector, $\varepsilon$ - circular central obstruction)

| $A_{0}$ | $A_{c}$ | $\Delta_{1 \mathrm{~lm}}$ | $\boldsymbol{\alpha}$ | $\Delta_{\text {umm }} / \sin ^{4}(\alpha / 2)$ |
| :---: | :---: | :---: | :---: | :---: |
| Uniform | Uniform | 2.976 | 0.1 | 476954 |
| $r^{2}$ | $r^{2}$ | 1.31 |  | 209950 |
| $8=0$ | $\varepsilon=0.25$ | 1.48 |  | 237195 |
| $\varepsilon=0.5$ | $\varepsilon=0.5$ | 1.425 |  | 228380 |
| $\varepsilon=0.9$ | $\varepsilon=0.9$ | 1.40 |  | 224374 |
| $\beta_{040}=0$ | $\beta_{040}=0$ | 2.98 |  | 477595 |
| $\beta_{040}=0.5$ | $\beta_{040}=0.5$ | 2.96 |  | 474390 |
| $\beta_{040}=1$ | $\beta_{040}=1$ | 2.98 |  | 477595 |
| $\beta_{040}=1.5$ | $\beta_{040}=1.5$ | 2.87 |  | 459966 |
| Uniform | Uniform | 2.976 | 0.5 | 794.34 |
| $r^{2}$ | $r^{2}$ | 1.31 |  | 349.659 |
| $\varepsilon=0$ | $\varepsilon=0.25$ | 1.48 |  | 395.035 |
| $\varepsilon=0.5$ | $\varepsilon=0.5$ | 1.425 |  | 380.35 |
| $\varepsilon=0.9$ | $\varepsilon=0.9$ | 1.40 |  | 373.681 |
| $\beta_{040}=0$ | $\beta_{040}=0$ | 2.98 |  | 795.408 |
| $\beta_{040}=0$ | $\beta_{040}=1.5$ | 2.88 |  | 768.716 |
| $\beta_{040}=0.5$ | $\beta_{040}=0.5$ | 2.96 |  | 790.069 |
| $\beta_{040}=1.5$ | $\beta_{040}=1.5$ | 2.87 |  | 766.047 |
| Uniform | Uniform | 2.976 | 1 | 56.331 |
| $r^{2}$ | $r^{2}$ | 1.31 |  | 24.7963 |
| $\varepsilon=0$ | $\varepsilon=0.25$ | 1.48 |  | 28.014 |
| $\varepsilon=0.5$ | $\varepsilon=0.5$ | 1.425 |  | 26.973 |
| $\varepsilon=0.9$ | $\boldsymbol{\varepsilon}=0.9$ | 1.40 |  | 26.499 |
| $\beta_{040}=0$ | $\beta_{040}=0$ | 2.98 |  | 56.407 |
| $\beta_{040}=0$ | $\beta_{040}=1.5$ | 2.88 |  | 54.514 |
| $\beta_{040}=0.5$ | $\beta_{040}=0.5$ | 2.96 |  | 56.028 |
| $\beta_{040}=1.5$ | $\beta_{040}=1.5$ | 2.87 |  | 54.325 |
| Uniform | Uniform | 2.976 | 1.5 | 13.785 |
| $r^{2}$ | $r^{2}$ | 1.31 |  | 6.028 |
| $\varepsilon=0$ | $\varepsilon=0.25$ | 1.48 |  | 6.856 |
| $\varepsilon=0.5$ | $\varepsilon=0.5$ | 1.425 |  | 6.60 |
| $\varepsilon=0.9$ | $\varepsilon=0.9$ | 1.40 |  | 6.48 |
| $\beta_{040}=0$ | $\beta_{040}=0$ | 2.98 |  | 13.804 |
| $\beta_{040}=0$ | $\beta_{040}=1.5$ | 2.88 |  | 13.3406 |
| $\beta_{040}=0.5$ | $\beta_{040}=0.5$ | 2.96 |  | 13.7116 |
| $\beta_{040}=1.5$ | $\beta_{040}=1.5$ | 2.87 |  | 13.2943 |

Table 2. Aberration correcting coefficient in CSM with apodization optimal in uniform CSM ( $\Delta_{\text {mla }}$ - limiting resolution in CSM with $\boldsymbol{r}^{2}$ apodization, $\Delta_{\max }$ - limiting resolution in uniform CSM)

| $\alpha$ | $\Delta_{\mathrm{uma}}$ | $\Delta_{\mathrm{uma}} / \sin ^{4}(\alpha / 2)$ | $W=\Delta_{\max } / \Delta_{\min }$ |
| :--- | :--- | :--- | :--- |
| 1 | 2 | 3 | 4 |
| 0.1 | $\Delta_{\min }=1.31$ | 209950 | $\sim 2.27$ |
|  | $\Delta_{\max }=2.98$ | 477595 |  |


| 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- |
| 0.5 | $\Delta_{\min }=1.31$ | 349.659 | $\sim 2.27$ |
|  | $\Delta_{\max }=2.98$ | 795.408 |  |
| 1.5 | $\Delta_{\max }=1.31$ | 6.068 | $\sim 2.27$ |
|  | $\Delta_{\max }=2.98$ | 13.804 |  |
| 1.6 | $\Delta_{\min }=1.31$ | 4.947 | $\sim 2.27$ |
|  | $\Delta_{\max }=2.98$ | 11.2532 |  |



Fig. 2. Intensity in focal region in CSM as dependent on $\alpha$ (semi-angle of convergence), curve $1-\alpha=0.5$, curve $2-\alpha=1$, curve $3-\alpha=1.5$
calculated. Case 1: $\Delta n=0.01$ (while $n_{2}$ ranging within the interval 1.513-1.523), $n_{1}=1, t_{\text {optcsm }}=19.32 \mu \mathrm{~m}$. Case $2: \Delta n=0.033$ (while $n_{2}$ ranging within the interval $1.514-1.481), n_{1}=1, t_{\text {optcsm }} \simeq 6.25 \mu \mathrm{~m}$. Optimal thickness of the immersion layer should be adjusted to the resolution of CSM which in the first case corresponds to the value $19.32 \mu \mathrm{~m}$, while in the second case to the value $6.25 \mu \mathrm{~m}$.

## References

[1] Sheppard C. J. R., Cogswell C. J., Optik 87 (1991), 34.
[2] Magirra A., Atti Fond. Giorgio Ronchi 45 (1990), 873.

