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SOME ASPECTS OF KINEMATIC COALESCENCE OF DROPLETS IN ANNULAR-DISPERSED FLOW

An influence of differentiation of the droplet acceleration on the their mutual collisions and, consequently, on the processes of heat, mass and momentum transfer occurring during annular-dispersed flows has been analysed. A model of kinematic coalescence of a polydispersional set of droplets moving in a cocurrent gas stream of a considerable velocity has been proposed. Exemplary results of simulation research on frequency of collisions to which droplets in injection scrubbers are subject have been presented.

NOMENCLATURE

C, $\bar{C}$ – mass concentration of the liquid phase (kg/m^3),
d_K, d_i – droplet diameter, mean droplet diameter of the i-th size grade (m),
g – acceleration due to gravity (m/s^2),
i – number of the size grades of droplets,
Ko, $\bar{Ko}$ – frequency, mean frequency of droplet collisions (s^{-1}),
l – path of droplet movement (m),
m – number of size grades of droplets,
m_K – a single droplet mass (kg),
n_i – quantitative concentration of droplets of the i-th size grade (m^{-3}),
N – quantitative concentration of droplets of all size grades (m^{-3}),
$Nz = \dot{V}_c/\dot{V}_g$ – liquid to gas flow rate ratio (m^3/m^3),
P – probability,
t – time (s),
u_i – quantitative fraction of droplets of the i-th size grade,
U_{mi} – mass fraction of droplets of the i-th size grade,
$\dot{V}$ – flow rate (m^3/s),
w – velocity (m/s),
x – distance from the section of liquid injection (m).

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DIMENSIONLESS NUMBERS

Re – Reynolds number,
Stk – Stokes number.

GREEK SYMBOLS

δ_i – distance between droplets of the *i*-th size grade (m),
Δ – difference, increase,
η – viscosity (kg/(m · s)),
ρ – density (kg/m^3),
τ – time passing between subsequent collisions of droplets (s),
τ – parameter defined by eqn (9).

SUBSCRIPTS

c – liquid phase,
g – gas phase,
i – *i*-th size grade,
j – *j*-th size grade,
K – droplets,
o – initial conditions.

1. INTRODUCTION

In many cases it is very convenient to use the Sauter mean diameter that conventionally represents fineness of a polydispresional set of droplets, as it facilitates simulation of phenomena and processes without making their mathematical models excessively complicated. One can assume, without exaggeration, that modelling of heat, mass and momentum transfers between a stream of gas and a polydispersional set of droplets is so complicated that it questions sensibleness of such a type of research.

There are, however, some issues that would be very difficult to solve when using the monodispersional model, e.g., the case of kinematic coalescence of droplets. It results from the fact that droplets of different sizes are characterized by different accelerations in the stream of gas which, in turn, is the reason of their relative movement and mutual collisions.

Estimation of a local intensity of the process of kinematic coalescence of the droplets of a working liquid undoubtedly has a favourable influence on accuracy of modelling of heat and mass transfer processes taking place, e.g., during waste gases cleaning in injection srubbers [2].

Droplets of liquid moving during annular-dispersed flows are mostly small, predominantly of diameters ranging from 10^1 to 10^2μm. Thus they can be admitted to be almost rigid and practically devoid of internal circulation. This internal stability of droplets causes a relatively quick increase in gradients of temperature inside them, absorbate concentration or concentration of collected aerosol particles. The increase in these gradients brings about a temporary slowdowns in the waste gase cooling, absorption and particle collection processes. It happens so until collision of droplets occurs. The collision, admittedly, can lead both to coalescence of droplets (i.e., they join up forming larger droplets) as well as to their secondary decomposition into a series of small droplets when kinetic energy exceeds the enegry of surface tension.

Nevertheless, in each of the enumerated cases, violent deformations of droplets, their immediate internal mixing and disappearance of internal temperature and concentration gradients occur. Visible effects of these phenomena are temporary temperature and concentration decreases at the droplet surfaces as well as a temporary acceleration of heat transfer, absorption and particle collection.

Thus, we can assume that the coalescence process and, to be more precise, collisions of droplets added in order to renew the transfer surface affect kinetics of heat transfer, absorption (provided there is some relevant transfer resistance on the side of the liquid phase) and particle collection processes.

The above analysis allows the conclusion that it is necessary to take into account the polydispersity of a set of droplets only in order to estimate an average, locally representative (in the part of the considered apparatus section) frequencies of droplet collisions.

2. MATHEMATICAL MODEL OF KINEMATIC COALESCENCE OF DROPLETS

Assuming that collision of droplets due to differentiated accelerations in the stream of gas is the dominating mechanism of their coalescence, the efficiency of the coalescence process will mainly depend on:

- size distribution of droplets,
- velocity of gas,
- concentration of droplets in the gas phase core.

If the quantitative fractions u_i of particular size grades of droplets are defined by means of the Ueda equation [1], the quantitative concentration of droplets can be calculated as

$$N = \frac{6 C_K}{\pi \varrho_c \sum_{i=1}^{m} u_i d_i^3}, \tag{1}$$

and fractional quantitative concentrations as

$$n_i = N \cdot u_i . \tag{2}$$

Mass fractions of particular size grades can be therefore calculated according to equation

$$U_{mi} = \frac{n_i d_i^3}{\sum_{i=1}^{m} n_i d_i^3}. \tag{3}$$

Basing on an assumption that droplets of a particular size grade are distributed uniformly in a local space (in a sufficiently short segment) of the apparatus according to their local concentrations, the average local distances between such droplets can be reckoned as follows:

$$\delta_i = d_i \sqrt[3]{\frac{\pi \rho_c}{6 \cdot C_K \cdot U_{mi}}}. \tag{4}$$

Analysing the movement of droplets differing in their diameters in the stream of gas, one can ascertain that a basic mechanism of kinematic coalescence is a result of "catching" of the part of smaller droplets (of the i-th size grade) with larger ones (of the j-th size grade). Thus, the probable initial distance δ_{ij} separating the droplets of i-th and j-th size grades is included in the interval

$$0 \le \delta_{ij} \le \delta_i. \tag{5}$$

In order to cause a collision between a "catching" droplet of the i-th size grade and a "being catched" droplet of the j-th size grade, the droplet of an i-th size grade should cover a distance

$$l_i = \delta_{ij} + \Delta x \tag{6}$$

where Δx is a distance that the droplet of the j-th size grade will manage to cover while "being caught with" by the droplet of an i-th size grade, which happens in time Δt_{ij}. Dependence between the distance covered by both kinds of droplets and time indispensable to a possible collision can be represented by the following equations [2]:

$$\delta_{ij} + \Delta x = \left(w_g - g\frac{m_{Ki}}{\tau_i}\right)\Delta t_{ij} + \left(\frac{m_{Ki}}{\tau_i}\right)^2 \times \left[(w_g - (w_K)_0)\cdot\frac{\tau_i}{m_{Ki}} - g\right]\left[\exp\left(-\frac{\tau_i}{m_{Ki}}\cdot\Delta t_{ij}\right) - 1\right], \tag{7}$$

$$\Delta x = \left(w_g - g\frac{m_{Kj}}{\tau_j}\right)\Delta t_{ij} + \left(\frac{m_{Kj}}{\tau_j}\right)^2 \times \left[(w_g - (w_K)_0)\cdot\frac{\tau_j}{m_{Kj}} - g\right]\left[\exp\left(-\frac{\tau_j}{m_{Kj}}\cdot\Delta t_{ij}\right) - 1\right], \tag{8}$$

where

$$\tau = \pi \alpha \eta_g d_K / 8. \tag{9}$$

As we base our considerations on the assumption that the droplets of a particular size grade are uniformly distributed in the local space of the apparatus, the hypothesis about a uniform distribution of the random variable δ_{ij} should be accepted as probable, which is expressed in the form of the following conditions

$$f(\delta_{ij}) = \begin{cases} 0 & \text{when } \delta_{ij} < 0, \\ \dfrac{1}{\delta_i} & \text{when } 0 \leq \delta_{ij} \leq \delta_i, \\ 0 & \text{when } \delta_{ij} > \delta_i. \end{cases} \tag{10}$$

Additionally, if we take account of the fact that the aim of this analysis is to estimate an order of magnitude of the average frequency of the droplet collision, then treating the value $\delta_{ij} = 0.5\ \delta_i$ as representative seems to provide a sufficient accuracy of the estimation process.

Thus, on the basis of equations (7) and (8) one can numerically establish an average "catching" time Δt_{ij} when

$$\Delta t_{ij} = f(0.5\ \delta_i + \Delta x), \tag{11}$$

and then calculate the velocities reached by the droplets of both size grades analysed during this time

$$w_{Ki} = w_g - g\frac{m_{Ki}}{\tau_i} + \left[g\frac{m_{Ki}}{\tau_i} - [w_g - (w_K)_0] \right] \exp\left(-\frac{\tau_i}{m_{Ki}} \cdot \Delta t_{ij} \right), \tag{12}$$

$$w_{Kj} = w_g - g\frac{m_{Kj}}{\tau_j} + \left[g\frac{m_{Kj}}{\tau_j} - [w_g - (w_K)_0] \right] \exp\left(-\frac{\tau_j}{m_{Kj}} \cdot \Delta t_{ij} \right). \tag{13}$$

The collision frequency Ko_{ij} of the droplets of the i-th size grade with the droplets of the j-th size grade would be equal to an iverse of the time Δt_{ij} if all the movement trajectories of the droplets of i-th size grade crossed the surfaces of the droplets of j-th size grade. Because it is not the case, the frequency should be estimated as

$$Ko_{ij} = \frac{P_{F_{ij}} \cdot P_{T_{ij}}}{\Delta t_{ij}} \tag{14}$$

where:

$P_{F_{ij}}$ – probability of the occurrence of the i-th size grade droplets in a gas stream of the diameter d_{Kj},

$P_{T_{ij}}$ – probability of the hitting the j-th size grade droplet surface by the i-th size grade droplets present in a gas stream of the diameter d_{Kj}.

Probability $P_{F_{ij}}$ results from a ratio of projection surfaces of all the j-th size grade droplets present in the apparatus cross-section to the surface of this section, and finally equals to

$$P_{F_{ij}} = \frac{\pi d_{Kj}^2}{4} n_j \delta_j, \tag{15}$$

whereas the probability of the collisions $P_{T_{ij}}$ is estimated on the basis of Langmuir's theory by applying the following dependencies

$$P_{T_{ij}} = \frac{P_1 + P_2 \cdot \dfrac{Re_{ij}}{60}}{1 + \dfrac{Re_{ij}}{60}} \tag{16}$$

where

$$P_1 = 1 + \frac{0.75 \ln (2Stk)^{-2}}{Stk - 1.214}, \tag{17}$$

$$P_2 = \left(\frac{Stk}{Stk + 0.25}\right)^2, \tag{18}$$

$$Re_{ij} = \frac{d_{Kj}(w_{Ki} - w_{Kj})\rho_g}{\eta_g} \tag{19}$$

and

$$Stk = \frac{d_{Ki}^2 \rho_c (w_{Ki} - w_{Kj})}{18 d_{Kj} \eta_g}, \tag{20}$$

provided that if $Stk \leq 1.214$, then $P_1 = 0$.

Bearing in mind that collision frequency of smaller droplets (i.e., of size grades in the $1 \leq i \leq j-1$ range) with the droplets of the j-th size grade is a sum of collision frequencies of particular i-th size grade with the j-th size grade droplets, one can write

$$Ko_j = \sum_{i=1}^{j-1} \frac{P_{F_{ij}} \cdot P_{T_{ij}}}{\Delta t_{ij}}. \tag{21}$$

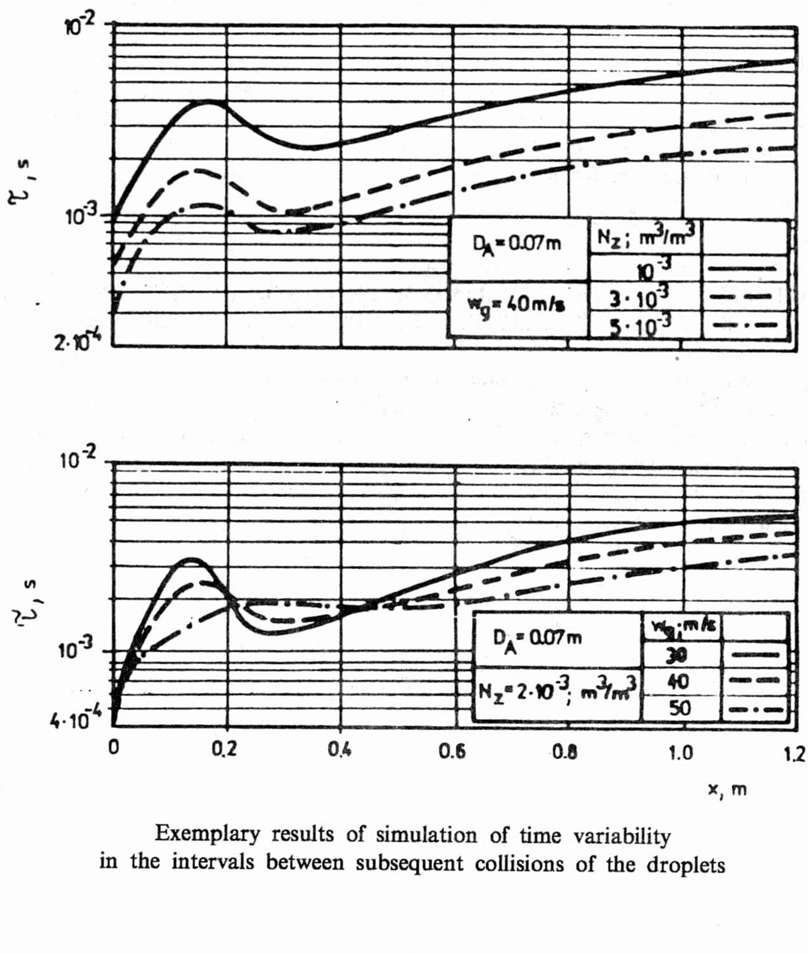
Moreover, if we take into account the facts that heat, momentum and mass transfer processes are subject to modelling on the basis of a conventional set of monodispersional droplets and that collision frequency influences unification of temperature and concentration in the whole volume (mass) of droplets, it seems indispensable to estimate average droplet collision frequency of the conventional set as a weighted average referred to mass fraction of the j-th size grade

$$\bar{K}o = \sum_{j=2}^{m} u_{mj} \sum_{i=1}^{j-1} \frac{P_{F_{ij}} \cdot P_{T_{ij}}}{\Delta t_{ij}}. \tag{22}$$

In this case an inverse of $\bar{K}o$ constitutes the estimated mean time passing between subsequent collisions to which each of the droplets of the conventional set is subject to. Consequently, this time is the average time of both droplet existence in an undisturbed state and increase in temperature and concentration gradients inside of them. After this time it can be assumed that every droplet will undergo ideal internal mixing.

3. SIMULATION RESULTS

Basing on the proposed mathematical model of kinematic coalescence of droplets, it is possible to carry out the simulation of this process for the injection scrubber throat. Exemplary result are shown in the figure.



Exemplary results of simulation of time variability
in the intervals between subsequent collisions of the droplets

The simulation experiments carried out allow us to obtain an extremely important information which reveals that under the conditions inside the injection scrubber throats, an average droplet collision frequency ranges from 10^2 to $10^4 s^{-1}$.

Due to that information we can apply a proper time step in simulation experiments of heat transfer, gas impurities absorption and particle collection as well as understand better the mechanisms of these processes.

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NIEKTÓRE ASPEKTY KOALESCENCJI KINEMATYCZNEJ KROPEL PODCZAS PRZEPŁYWÓW PIERŚCIENIOWO-DYSPERSYJNYCH

Przeanalizowano wpływ zróżnicowania przyspieszeń kropel na ich wzajemne zderzenia, a w konsekwencji – na przebieg procesów wymiany ciepła, masy i pędu zachodzących podczas przepływów pierścieniowo-dyspersyjnych. Przedstawiono model kinematycznej koalescencji polidyspersyjnego zbioru kropel poruszających się we współprądowym strumieniu gazu o znacznej prędkości. Zaprezentowano przykładowe wyniki badań symulacyjnych częstotliwości zderzeń, którym podlegają krople w iniekcyjnych płuczkach.

НЕКОТОРЫЕ АСПЕКТЫ КИНЕМАТИЧЕСКОЙ КОАЛЕСЦЕНЦИИ КАПЕЛЬ ВО ВРЕМЯ ЦИКЛИЧНО-ДИСПЕРСИОННЫХ ТЕЧЕНИЙ

Проведен анализ влияния дифференцирования ускорения капель на их столкновения, а в последствии – на протекание процессов тепло- и массообмена, происходящих во время циклично-дисперсионных течений. Представлена модель кинематической коалесценции полидисперсионного множества капель, продвигающихся в прямоточном потоке газа значительной скорости. Представлены примерные результаты имитационных исследований частоты столкновений, которым подвергаются капли в инъекционных скрубберах.