# Aberrations of third and fifth orders of holograms made on rotational surfaces of second degree * 

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#### Abstract

On the basis of the classical Seidel method, the formulae are derived for the third order and fifth order aberrations of holograms made on aspherical and spherical surfaces, respectively. The considerations are limited to the monochromatic case.


## Introduction

Aberrations belong to the main factors restricting the quality of the holographic imaging. The formulae for third order aberrations for the plane holograms were given by Meier [1] and Champagne [2].

There appears sometimes a necessity of using holograms on the nonplane surfaces. For instance, Welford [3] showed that in order to fulfill the aplanaticity condition of imaging it is necessary to use the hologram recorded on spherical surface. The formulae defining the coefficients of third order aberrations for spherical holograms were determined by Mustafin [4].

The goal of this paper is to derive the formulae defining the aberrations of third order for holograms made on aspherical surfaces of rotational symmetry and to determine the fifth order aberrations and their coefficients for holograms on spherical substrate. The considerations are limited to the monochromatic case.

## Theory

Let us consider the hologram made on aspherical surface $S$. Let us assume the Cartesian coordinate system with the origin in $Q$, as seen in the figure, where

$$
\begin{aligned}
P(x, y, z) & - \\
\varrho & \text { considered point of hologram, } \\
& \text { curvature radius of the hologram substrate at the } \\
& \text { given point, } \\
\mathrm{O}\left(x_{o}, y_{O}, z_{o}\right) & \text { object position, } \\
R_{O}- & \text { distance of the object from the hologram vertex, } \\
r_{o} & \text { - distance of the object from the point } P,
\end{aligned}
$$

[^0]\[

$$
\begin{aligned}
R\left(x_{R}, y_{R}, z_{R}\right) & \text { - position of the reference wavefront source, } \\
C\left(x_{C}, y_{C}, z_{C}\right) & \text { - position of the reconstruction wavefront source, } \\
\mathrm{O}^{\prime}\left(x_{O^{\prime}}, y_{O^{\prime}}, z_{o^{\prime}}\right) & \text { - image position, } \\
r_{R} & \text { - distance of the source } R \text { from the point } P, \\
r_{C} & \text { - distance of the source } C \text { from the point } P, \\
r_{O^{\prime}} & \text { - distance of the image from the point } P .
\end{aligned}
$$
\]



Recording and reconstruction of holograms made on the second degree surface of rotational symmetry

From the figure it may be seen that

$$
\begin{equation*}
r_{O}=\left[\left(x-x_{O}\right)^{2}+\left(y-y_{o}\right)^{2}+\left(z-z_{o}\right)^{2}\right]^{1 / 2} \tag{1}
\end{equation*}
$$

After the development into series

$$
\begin{align*}
r_{O}=R_{O} & +\frac{1}{2 R_{O}}\left[x^{2}+y^{2}+z^{2}-2\left(x x_{O}+y y_{O}+z z_{O}\right)\right]-\frac{1}{8 R_{O}^{3}}\left[\left(x^{2}+y^{2}+z^{2}\right)^{2}\right. \\
& \left.-4\left(x^{2}+y^{2}+z^{2}\right)\left(x x_{O}+y y_{o}+z z_{O}\right)+4\left(x x_{O}+y y_{O}+z z_{o}\right)^{2}\right] \\
& +\frac{1}{16 R_{O}^{5}}\left[\left(x^{2}+y^{2}+z^{2}\right)^{3}-6\left(x^{2}+y^{2}+z^{2}\right)^{2}\left(x x_{O}+y y_{o}+z z_{o}\right)\right. \\
& \left.+12\left(x^{2}+y^{2}+z^{2}\right)\left(x x_{O}+y y_{O}+z z_{O}\right)^{2}-8\left(x x_{O}+y y_{O}+z z_{O}\right)^{3}\right]+\ldots \tag{2}
\end{align*}
$$

The surface $S$ was generated by rotation of the curve of second order around the $z$ axis and has the curvature $1 / \varrho$ at the point $P$. They may be represented by the equation [5]:

$$
\begin{equation*}
z=\frac{1}{2 \varrho}\left(x^{2}+y^{2}+\varepsilon z^{2}\right) \tag{3}
\end{equation*}
$$

The parameter $\varepsilon$ describes the asphericity and takes the following values:

$$
\begin{aligned}
& \varepsilon>1, \text { and } 0<\varepsilon<1 \text { for the ellipsoid, } \\
& \varepsilon=1 \text { for the sphere, } \\
& \varepsilon=0 \text { for the paraboloid, } \\
& \varepsilon<0 \text { for the hyperboloid. }
\end{aligned}
$$

After the surface equation (3) is inserted into (2) the latter formula takes the form:

$$
\begin{align*}
& r_{O}=R_{O}+\frac{x^{2}+y^{2}+z^{2}}{2 R_{O}}-\frac{x x_{O}+y y_{O}}{R_{O}}-\frac{z_{O}\left(x^{2}+y^{2}+\varepsilon z^{2}\right)}{2 R_{O} \varrho} \\
& -\frac{\left(x^{2}+y^{2}+z^{2}\right)^{2}}{8 R_{O}^{3}}+\frac{\left(x^{2}+y^{2}+z^{2}\right)\left(x x_{O}+y y_{O}\right)}{2 R_{O}^{3}} \\
& +\frac{z_{O}\left(x^{2}+y^{2}+z^{2}\right)\left(x^{2}+y^{2}+\varepsilon z^{2}\right)}{4 R_{O}^{3} \varrho}-\frac{\left(x x_{O}+y y_{O}\right)^{2}}{2 R_{O}^{3}} \\
& -\frac{z_{O}\left(x x_{O}+y y_{O}\right)\left(x^{2}+y^{2}+\varepsilon z^{2}\right)}{2 R_{O}^{3} \varrho}-\frac{z_{O}^{2}\left(x^{2}+y^{2}\right)^{2}}{8 R_{O}^{3} \varrho^{2}}-\frac{z_{O}^{2} \varepsilon^{2} z^{4}}{8 R_{O}^{3} \varrho^{2}} \\
& +\frac{\left(x^{2}+y^{2}+z^{2}\right)^{3}}{16 R_{O}^{5}}-\frac{3\left(x^{2}+y^{2}+z^{2}\right)^{2}\left(x x_{O}+y y_{O}\right)}{8 R_{O}^{5}} \\
& -\frac{3 z_{O}\left(x^{2}+y^{2}+z^{2}\right)^{2}\left(x^{2}+y^{2}+\varepsilon z^{2}\right)}{16 R_{O}^{5} \varrho}+\frac{3\left(x^{2}+y^{2}+z^{2}\right)\left(x x_{O}+y y_{O}\right)^{2}}{4 R_{O}^{5}} \\
& -\frac{3 z_{O}^{2}\left(x^{2}+y^{2}+z^{2}\right)\left(x^{2}+y^{2}+\varepsilon z^{2}\right)^{2}}{16 R_{O}^{5} \varrho^{2}} \\
& +\frac{3 z_{O}\left(x^{2}+y^{2}+z^{2}\right)\left(x x_{O}+y y_{O}\right)\left(x^{2}+y^{2}+\varepsilon z^{2}\right)}{4 R_{O}}-\frac{\left(x x_{O}+y y_{O}\right)^{3}}{2 R_{O}^{5}} \\
& -\frac{3 z_{O}\left(x x_{O}+y y_{O}\right)^{2}\left(x^{2}+y^{2}+\varepsilon z^{2}\right)}{4 R_{O}^{5} \varrho} \\
& -\frac{3 z_{O}^{2}\left(x x_{O}+y y_{O}\right)\left(x^{2}+y^{2}+\varepsilon z^{2}\right)^{2}}{8 R_{O}^{5} \varrho^{2}}-\frac{z_{O}^{3}\left(x^{2}+y^{2}+\varepsilon z^{2}\right)^{3}}{16 R_{O}^{5} \varrho^{3}}+\ldots \tag{4}
\end{align*}
$$

In the formulae (2) and (4) the expansion into series was cut out after the fifth expansion term. In an analogical way we may calculate the distance of the reference wave source $r_{R}$, and that of the reconstructing wave source $r_{C}$ from the point $P$ on the hologram and the distance $r_{O^{\prime}}$ of the image from the point.

Next we will derive the characteristic function called eikonal and defined as follows [6]:

$$
\begin{equation*}
E[x, y, z(x, y)]=r_{G}-r_{o^{\prime}} \pm \mu\left(r_{o}-r_{R}\right) \tag{5}
\end{equation*}
$$

In this formula all the distances denoted in the figure take the form analogical to (4). The introduced parameters $\mu$ represents the reconstruc-ting-to-recording-wavelength-ratio.

The coordinate $z$ being the function of the independent variables $(x, y)$ our function is the composite function of the term $E[x, y, z(x, y)]$. Hence, its partial derivatives will be expressed by formulae [7]:

$$
\begin{align*}
& \frac{\partial E}{\partial x}=E_{x}^{\prime}[x, y, z(x, y)]+E_{z}^{\prime}[x, y, z(x, y)] z_{x}^{\prime}(x, y)  \tag{6a}\\
& \frac{\partial E}{\partial y}=E_{y}^{\prime}[x, y, z(x, y)]+E_{z}^{\prime}[x, y, z(x, y)] z_{y}^{\prime}(x, y) \tag{6~b}
\end{align*}
$$

In order to find the conditions determining the position of the reconstructed images in the paraxial region we will calculate the derivatives of the first two terms in the expansion of the characteristic function and equate them to zero. The following conditions are then obtained:

$$
\begin{align*}
& \frac{1}{R_{O^{\prime}}}-\frac{(1+\varepsilon) z_{O}}{R_{O^{\prime}} \varrho}=\frac{1}{R_{C}} \pm \mu\left(\frac{1}{R_{O}}-\frac{1}{R_{R}}\right)-\frac{1+\varepsilon}{\varrho}\left[\frac{z_{C}}{R_{C}} \pm \mu\left(\frac{z_{O}}{R_{O}}-\frac{z_{R}}{R_{R}}\right)\right]  \tag{7}\\
& \frac{x_{O^{\prime}}}{R_{O^{\prime}}}=\frac{x_{C}}{R_{C}} \pm \mu\left(\frac{x_{O}}{R_{O}}-\frac{x_{R}}{R_{R}}\right)  \tag{8a}\\
& \frac{y_{O^{\prime}}}{R_{O^{\prime}}}=\frac{y_{C}}{R_{C}} \pm \mu\left(\frac{y_{O}}{R_{O}}-\frac{y_{R}}{R_{R}}\right) \tag{8b}
\end{align*}
$$

The dependence upon the shape of the hologram surface occurs only in the formula (7) determining the focussing properties of the hologram, but it does not exist in equations (8a) and (8b) defining the directional cosines.

For the case $\varepsilon=1$ the formula (7) takes the form:

$$
\begin{equation*}
\frac{1}{R_{C}}-\frac{1}{R_{O^{\prime}}} \pm \mu\left(\frac{1}{R_{O}}-\frac{1}{R_{R}}\right)-\frac{2}{\varrho}\left[\frac{z_{C}}{R_{C}}-\frac{z_{O^{\prime}}}{R_{O^{\prime}}} \pm \mu\left(\frac{z_{O}}{R_{O}}-\frac{z_{R}}{R_{R}}\right)\right]=0 \tag{7a}
\end{equation*}
$$

This is the formula obtained by Mustafin for the spherical hologram [4]. The formulae (8a), and (8b) have the same form as that of the spherical and plane holograms.

In order to find the coefficients corresponding to particular Seidel sums we calculate the derivatives of the third order terms of the characteristic function expansion, and group the term associated with the respective powers of the variable. The obtained aberration coefficient is the following:

$$
\begin{gather*}
S=\frac{1}{R_{C}^{3}}-\frac{1}{R_{O^{\prime}}^{3}} \pm \mu\left(\frac{1}{R_{O}^{3}}-\frac{1}{R_{R}^{3}}\right)-\frac{2(1+\varepsilon)}{\varrho} \\
{\left[\frac{z_{C}}{R_{C}^{3}}-\frac{z_{O^{\prime}}}{R_{O^{\prime}}^{3}} \pm \mu\left(\frac{z_{O}}{R_{O}^{3}}-\frac{z_{R}}{R_{R}^{3}}\right)\right]+\frac{(1+\varepsilon)^{2}}{\varrho^{2}}} \\
\quad\left[\frac{z_{C}^{2}}{R_{C}^{3}}-\frac{z_{O^{\prime}}^{2}}{R_{O^{\prime}}^{3}} \pm \mu\left(\frac{z_{O}^{2}}{R_{O}^{3}}-\frac{z_{R}^{2}}{R_{R}^{3}}\right)\right] . \tag{9}
\end{gather*}
$$

It may be seen that the spherical aberration depends upon the curvature radius of the hologram surface and upon the asphericity coefficient.

The derived coefficients determining the coma are:

$$
\begin{align*}
& C_{x}=\frac{x_{C}}{R_{C}^{3}}-\frac{x_{O^{\prime}}}{R_{O^{\prime}}^{3}} \pm \mu\left(\frac{x_{O}}{R_{O}^{3}}-\frac{x_{R}}{R_{R}^{3}}\right)-\frac{1+\varepsilon}{\varrho} \\
& {\left[\frac{x_{C} z_{C}}{R_{C}^{3}}-\frac{x_{O^{\prime}} z_{O^{\prime}}}{R_{O^{\prime}}^{3}} \pm \mu\left(\frac{x_{O} z_{O}}{R_{O}^{3}}-\frac{x_{R} z_{R}}{R_{R}^{3}}\right)\right]}  \tag{10a}\\
& C_{v}=\frac{y_{C}}{R_{C}^{3}}-\frac{y_{O^{\prime}}}{R_{O^{\prime}}^{3}} \pm \mu\left(\frac{y_{O}}{R_{O}^{3}}-\frac{y_{R}}{R_{R}^{3}}\right)-\frac{1+\varepsilon}{\varrho} \\
& {\left[\frac{x_{C} z_{C}}{R_{C}^{3}}-\frac{y_{O^{\prime}} z_{O^{\prime}}}{R_{O^{\prime}}^{3}} \pm \mu\left(\frac{y_{O} z_{O}}{R_{O}^{3}}-\frac{y_{R} z_{R}}{R_{R}^{3}}\right)\right] .} \tag{10b}
\end{align*}
$$

Also the coma is the function of the hologram curvature radius and the coefficient of asphericity.

The obtained coefficient determining the astigmatism are the following:

$$
\begin{gather*}
A_{x}=\frac{x_{C}^{2}}{R_{C}^{3}}-\frac{x_{O^{\prime}}^{2}}{R_{O^{\prime}}^{3}} \pm \mu\left(\frac{x_{O}^{2}}{R_{O}^{3}}-\frac{x_{R}^{2}}{R_{R}^{3}}\right),  \tag{11a}\\
A_{y}=\frac{y_{C}^{2}}{R_{C}^{3}}-\frac{y_{O^{\prime}}^{2}}{R_{O^{\prime}}^{3}} \pm \mu\left(\frac{y_{O}^{2}}{R_{O}^{3}}-\frac{y_{R}^{2}}{R_{R}^{3}}\right),  \tag{11b}\\
A_{x y}=\frac{x_{C} y_{C}}{R_{C}^{3}}-\frac{x_{O^{\prime}} y_{O^{\prime}}}{R_{O^{\prime}}^{3}} \pm \mu\left(\frac{x_{O} y_{o}}{R_{O}^{3}}-\frac{x_{R} y_{R}}{R_{R}^{3}}\right) . \tag{11c}
\end{gather*}
$$

The value of astigmatism does not depend upon the surface shape. The formulae obtained for the coefficients of astigmatism are the same as those for the spherical [4] and plane [2] holograms. In this approach the distortion and field curvature do not appear at all, since these aberrations do not influence the image sharpness though they affect its position. In this approach the object and the image were assumed to be the point-sources of the waves and their relative position were of no importance.

If in the formulae (9), (10a), and (10b) we take $\varepsilon=1$ (the case of spherical surface) we obtain

$$
\begin{gathered}
S=\frac{1}{R_{C}^{3}}-\frac{1}{R_{O^{\prime}}^{3}} \pm \mu\left(\frac{1}{R_{O}^{3}}-\frac{1}{R_{R}^{3}}\right)-\frac{4}{\varrho}\left\{\frac{z_{C}}{R_{C}^{3}}-\frac{z_{O^{\prime}}}{R_{O^{\prime}}^{3}}\right. \\
\left. \pm \mu\left(\frac{z_{O}}{R_{O}^{3}}-\frac{z_{R}}{R_{R}^{3}}\right)+\frac{1}{\varrho^{2}}\left[\frac{z_{C}^{2}}{R_{C}^{3}}-\frac{z_{O^{\prime}}^{2}}{R_{O^{\prime}}^{3}} \pm \mu\left(\frac{z_{O}^{2}}{R_{O}^{3}}-\frac{z_{R}^{2}}{R_{R}^{3}}\right)\right]\right\}, \\
C_{x}=\frac{x_{C}}{R_{C}^{3}}-\frac{x_{O^{\prime}}}{R_{O^{\prime}}^{3}} \pm \mu\left(\frac{x_{O}}{R_{O}^{3}}-\frac{x_{R}}{R_{R}^{3}}\right)-\frac{2}{\varrho^{2}}\left[\frac{x_{C} z_{C}}{R_{C}^{3}}-\frac{\left.x_{O^{\prime} z_{O^{\prime}}}^{R_{O^{\prime}}^{3}} \pm \mu\left(\frac{x_{O} z_{O}}{R_{O}^{3}}-\frac{x_{R} z_{R}}{R_{R}^{3}}\right)\right],}{C_{y}=\frac{y_{C}}{R_{C}^{3}}-\frac{y_{O^{\prime}}^{3}}{R_{O^{\prime}}^{3}} \pm \mu\left(\frac{y_{O}}{R_{O^{\prime}}^{3}}-\frac{y_{R}}{R_{R}^{3}}\right)-\frac{2}{\varrho}\left[\frac{y_{C} z_{C}}{R_{C}^{3}}-\frac{y_{O^{\prime}} z_{O^{\prime}}}{R_{O^{\prime}}^{3}}\right.}\right. \\
\left. \pm \mu\left(\frac{y_{O_{O}} z_{O}}{R_{O}^{3}}-\frac{y_{R} z_{R}}{R_{R}^{3}}\right)\right] .
\end{gathered}
$$

These are the formulae for the aberration coefficients for the case of spherical holograms obtained by Mustafin [4] in another way.

If, however, in the formulae (9), (10a), and (10b) we take $\varrho \rightarrow \infty$ (the case for plane surface) we obtain

$$
\begin{aligned}
S & =\frac{1}{R_{C}^{3}}-\frac{1}{R_{O^{\prime}}^{3}} \pm \mu\left(\frac{1}{R_{O}^{3}}-\frac{1}{R_{R}^{3}}\right), \\
C_{x} & =\frac{x_{C}}{R_{C}^{3}}-\frac{x_{O^{\prime}}}{R_{O^{\prime}}^{3}} \pm \mu\left(\frac{x_{O}}{R_{O}^{3}}-\frac{x_{R}}{R_{R}^{3}}\right), \\
C_{\nu} & =\frac{y_{C}}{R_{C}^{3}}-\frac{y_{O^{\prime}}}{R_{O^{\prime}}^{3}} \pm \mu\left(\frac{y_{O}}{R_{O}^{3}}-\frac{y_{R}}{R_{R}^{3}}\right),
\end{aligned}
$$

which are identical with those reported by Champagne for plane holograms [2].

It would be interesting to determine the fifth order aberration and their coefficients. For the systems of small focal length they are essential and cannot be omitted. In order to obtain the formulae determining these aberrations we calculate the derivatives of the fifth order terms in the expansion of the characteristic function. For the sake of simplicity we consider the case of spherical surface ( $\varepsilon=1$ ). After the necessary trnasformations the following coefficient of fith order aberrations in holograms made on spherical surface are obtained

$$
\begin{align*}
& A=\frac{1}{R_{C}^{5}}-\frac{1}{R_{O^{\prime}}^{5}} \pm \mu\left(\frac{1}{R_{O}^{5}}-\frac{1}{R_{R}^{5}}\right)-\frac{3}{\varrho}\left[\frac{z_{C}}{R_{C}^{5}}-\frac{z_{O^{\prime}}}{R_{O^{\prime}}^{5}}\right. \\
& \left. \pm \mu\left(\frac{z_{O}}{R_{O}^{5}}-\frac{z_{R}}{R_{R}^{5}}\right)\right]+\frac{3}{\varrho^{2}}\left[\frac{z_{C}^{2}}{R_{C}^{5}}-\frac{z_{O^{\prime}}^{2}}{R_{O}^{5}} \pm \mu\left(\frac{z_{O}^{2}}{R_{O}^{5}}-\frac{z_{R}^{2}}{R_{R}^{5}}\right)\right] \\
& -\frac{1}{2 \varrho^{3}}\left[\frac{z_{C}^{3}}{R_{C}^{5}}-\frac{z_{O^{\prime}}^{3}}{R_{O^{\prime}}^{5}} \pm \mu\left(\frac{z_{O}^{3}}{R_{O}^{5}}-\frac{z_{R}^{3}}{R_{R}^{5}}\right)\right],  \tag{12}\\
& B=\frac{x_{C}}{R_{C}^{5}}-\frac{x_{O^{\prime}}}{R_{O^{\prime}}^{5}} \pm \mu\left(\frac{x_{O}}{R_{O}^{5}}-\frac{x_{R}}{R_{R}^{5}}\right) \\
& +\frac{1}{\varrho}\left[\frac{x_{C} z_{C}}{R_{C}^{5}}-\frac{x_{O^{\prime}} z_{O^{\prime}}}{R_{O^{\prime}}^{5}} \pm \mu\left(\frac{x_{O} z_{O}}{R_{O}^{5}}-\frac{x_{R} z_{R}}{R_{R}^{5}}\right)\right] \\
& -\frac{1}{\varrho^{2}}\left[\frac{x_{C} z_{C}^{2}}{R_{C}^{5}}-\frac{x_{O^{\prime}} z_{O^{\prime}}^{2}}{R_{O^{\prime}}^{5}} \pm \mu\left(\frac{x_{O} z_{O}^{2}}{R_{O}^{5}}-\frac{x_{R} z_{R}^{2}}{R_{R}^{5}}\right)\right],  \tag{13}\\
& C=\frac{y_{C}}{R_{C}^{5}}-\frac{y_{O^{\prime}}}{R_{O^{\prime}}^{5}} \pm \mu\left(\frac{y_{O}}{R_{O}^{5}}-\frac{y_{R}}{R_{R}^{5}}\right)+\frac{1}{\varrho}\left[\frac{y_{C} z_{C}}{R_{C}^{5}}-\frac{y_{O^{\prime}} z_{O^{\prime}}}{R_{O^{\prime}}^{5}}\right. \\
& \left. \pm \mu\left(\frac{y_{O} z_{O}}{R_{O}^{5}}-\frac{y_{R^{\prime}} z_{R}}{R_{R}^{5}}\right)\right]-\frac{1}{\varrho^{2}}\left[\frac{y_{C} z_{C}^{2}}{R_{C}^{5}}-\frac{y_{O^{\prime}} z_{O^{\prime}}^{2}}{R_{O^{\prime}}^{5}} \pm \mu\left(\frac{y_{O} z_{O}^{2}}{R_{O}^{5}}-\frac{y_{R} z_{R}^{2}}{R_{R}^{5}}\right)\right], \tag{14}
\end{align*}
$$

$$
\begin{align*}
& D=\frac{x_{C}^{2}}{R_{C}^{5}}-\frac{x_{O^{\prime}}^{2}}{R_{O^{\prime}}^{5}} \pm \mu\left(\frac{x_{O}^{2}}{R_{O}^{5}}-\frac{x_{R}^{2}}{R_{R}^{5}}\right)-\frac{1}{\varrho}\left[\frac{x_{C}^{2} z_{C}}{R_{C}^{5}}\right. \\
& \left.-\frac{x_{O^{\prime}}^{2} z_{O^{\prime}}}{R_{O^{\prime}}^{5}} \pm \mu\left(\frac{x_{O}^{2} z_{O}}{R_{O}^{5}}-\frac{x_{R}^{2} z_{R}}{R_{R}^{5}}\right)\right],  \tag{15}\\
& E=\frac{y_{C}^{2}}{R_{C}^{5}}-\frac{y_{O^{\prime}}^{2}}{R_{O^{\prime}}^{5}} \pm \mu\left(\frac{y_{O}^{2}}{R_{O}^{5}}-\frac{y_{R}^{2}}{R_{R}^{5}}\right)-\frac{1}{\varrho}\left[\frac{y_{C}^{2} z_{C}}{R_{C}^{5}}-\frac{y_{O^{\prime}}^{2} z_{O^{\prime}}}{R_{O^{\prime}}^{5}}\right. \\
& \left. \pm \mu\left(\frac{y_{O}^{2} z_{O}}{R_{O}^{5}}-\frac{y_{R}^{2} z_{R}}{R_{R}^{5}}\right)\right],  \tag{16}\\
& F=\frac{x_{C} y_{C}}{R_{C}^{5}}-\frac{x_{O^{\prime}} y_{O^{\prime}}}{R_{O^{\prime}}^{5}} \pm \mu\left(\frac{x_{o} y_{o}}{R_{O}^{5}}-\frac{x_{R} y_{R}}{R_{R}^{5}}\right)-\frac{1}{\varrho}\left[\frac{x_{C} y_{C} z_{C}}{R_{C}^{5}}\right. \\
& \left.-\frac{x_{O^{\prime}} y_{O^{\prime}} z_{O^{\prime}}}{R_{O^{\prime}}^{5}} \pm \mu\left(\frac{x_{O} y_{O} z_{O}}{R_{O}^{5}}-\frac{x_{R} y_{R} z_{R}}{R_{R}^{5}}\right)\right],  \tag{17}\\
& G=\frac{x_{C}^{3}}{R_{C}^{5}}-\frac{x_{O^{\prime}}^{3}}{R_{O^{\prime}}^{5}} \pm \mu\left(\frac{x_{O}^{3}}{R_{O}^{5}}-\frac{x_{R}^{3}}{R_{R}^{5}}\right),  \tag{18}\\
& H=\frac{x_{C} y_{C}^{2}}{R_{C}^{5}}-\frac{x_{O^{\prime}} y_{O^{\prime}}^{2}}{R_{O^{\prime}}^{5}} \pm \mu\left(\frac{x_{o} y_{O}^{2}}{R_{O}^{5}}-\frac{x_{R} y_{R}^{2}}{R_{R}^{5}}\right),  \tag{19}\\
& I=\frac{x_{C}^{2} y_{C}}{R_{C}^{5}}-\frac{x_{O^{\prime}}^{2} y_{O^{\prime}}}{R_{O}^{5}} \pm \mu\left(\frac{x_{o}^{2} y_{O}}{R_{O}^{5}}-\frac{x_{R}^{2} y_{R}}{R_{R}}\right),  \tag{20}\\
& J=\frac{y_{C}^{3}}{R_{C}^{5}}-\frac{y_{O^{\prime}}^{3}}{R_{O^{\prime}}^{5}} \pm \mu\left(\frac{y_{O}^{3}}{R_{O}^{5}}-\frac{y_{R}^{3}}{R_{R}^{5}}\right) . \tag{21}
\end{align*}
$$

It may be seen that the four last coefficients are independent of the surface shape, but the others change with the curvature radius of the hologram.

If in the formulae (12) -(21) we substitute $\varrho \rightarrow \infty$ (plane surface) then we obtain:

$$
\begin{aligned}
A & =\frac{1}{R_{C}^{5}}-\frac{1}{R_{O^{\prime}}^{5}} \pm \mu\left(\frac{1}{R_{O}^{5}}-\frac{1}{R_{R}^{5}}\right) \\
B & =\frac{x_{C}}{R_{C}^{5}}-\frac{x_{O^{\prime}}}{R_{O^{\prime}}^{5}} \pm \mu\left(\frac{x_{O}}{R_{O}^{5}}-\frac{x_{R}}{R_{R}^{5}}\right) \\
C & =\frac{y_{C}}{R_{C}^{5}}-\frac{y_{O^{\prime}}}{R_{O^{\prime}}^{5}} \pm \mu\left(\frac{y_{O}}{R_{O}^{5}}-\frac{y_{R}}{R_{R}^{5}}\right) \\
D & =\frac{x_{C}^{2}}{R_{C}^{5}}-\frac{x_{O^{\prime}}^{2}}{R_{O^{\prime}}^{5}} \pm \mu\left(\frac{x_{O}^{2}}{R_{O}^{5}}-\frac{x_{R}^{2}}{R_{R}^{5}}\right)
\end{aligned}
$$

$$
\begin{aligned}
E & =\frac{y_{C}^{2}}{R_{C}^{5}}-\frac{y_{O^{\prime}}^{2}}{R_{O^{\prime}}^{5}} \pm \mu\left(\frac{y_{O}^{2}}{R_{O}^{5}}-\frac{y_{R}^{2}}{R_{R}^{5}}\right) \\
F & =\frac{x_{C} y_{C}}{R_{C}^{5}}-\frac{x_{O^{\prime}} y_{O^{\prime}}}{R_{O^{\prime}}^{5}} \pm \mu\left(\frac{x_{o} y_{o}}{R_{O}^{5}}-\frac{x_{R} y_{R}}{R_{R}^{5}}\right)
\end{aligned}
$$

The coefficients $G, H, I, J$ remain unchanged. The obtained formulae are the same as those given by Latta for the plane holograms [8].

For the common case of point object hologram which is recorded and reconstructed with plane waves in one plane only the coefficients of fifth order aberrations take the form:

$$
\begin{gather*}
A=-\frac{1}{R_{O^{\prime}}^{5}} \pm \mu \frac{1}{R_{O}^{5}}+\frac{3}{\varrho}\left(\frac{z_{O^{\prime}}}{R_{O^{\prime}}^{5}} \pm \mu \frac{z_{O}}{R_{O}^{5}}\right)+ \\
-\frac{3}{\varrho^{2}}\left(-\frac{z_{O^{\prime}}^{2}}{R_{O^{\prime}}^{5}} \pm \frac{z_{O}^{2}}{R_{O}^{5}}\right)  \tag{12a}\\
\\
-\frac{1}{2 \varrho^{3}}\left(-\frac{z_{O^{\prime}}^{3}}{R_{O^{\prime}}^{5}} \pm \mu \frac{z_{O}^{3}}{R_{O}^{5}}\right)  \tag{13a}\\
B=-\frac{x_{O^{\prime}}}{R_{O^{\prime}}^{5}} \pm u \frac{x_{O}}{R_{O}^{5}}+\frac{1}{\varrho}\left(-\frac{x_{O^{\prime}} z_{O^{\prime}}}{R_{O^{\prime}}^{5}} \pm \mu \frac{x_{O} z_{O}}{R_{O}^{5}}\right)  \tag{15a}\\
 \tag{18a}\\
-\frac{1}{\varrho^{2}}\left(-\frac{x_{O^{\prime}} z_{O^{\prime}}^{2}}{R_{O^{\prime}}^{5}} \pm \mu \frac{x_{O} z_{O}^{2}}{R_{O}^{5}}\right) \\
D=-\frac{x_{O^{\prime}}^{2}}{R_{O^{\prime}}^{5}} \pm \mu \frac{x_{O}^{2}}{R_{O}^{5}}-\frac{1}{\varrho}\left(-\frac{x_{O^{\prime}}^{2} z_{O^{\prime}}}{R_{O^{\prime}}^{5}} \pm \mu \frac{x_{O^{2}}^{2} z_{O}}{R_{O}^{5}}\right) \\
G=-\frac{x_{O^{\prime}}^{3}}{R_{O^{\prime}}^{5}} \pm \mu \frac{x_{O}^{3}}{R_{O}^{5}}
\end{gather*}
$$

In all the formulae given above no magnification factor appears, because in the most common case of optical holography it is assumed to be equal to 1 . If, additionally, the hologram is recorded and reconstructed with the same wavelength $(\mu=1)$, then it follows from the formulae that the third order and fifth order disappear in two most simple cases, i.e. when the reference and reconstructed wave are plane ( $R_{R}=R_{C}=\infty$ ) and when the reference wave source is the symmetric (with respect to the $z$ axis) image of the object.

## Conclusions

The goal of the paper presented was to derive the dependences describing the aberration of the hologram recorded on the surfaces of rotational symmetry. The advantage of such a hologram is that there appear two additional parameters: the curvature radius and the coefficient of asphericity, which may be used for correction of imaging. The problem of correction of these holograms will be considered in the next paper.

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## Аберрации 3-го и 5-го ряда голограмм, образуемых на вращательной поверхности 2-й степени

На основании классического метода Зайделя выведены формулы на третьерядные аберрации голограмм на асферических поверхностях, а также формулы на аберрации 5-го ряда голограмм на сферических поверхностях. Рассуждения касались исключительно монохроматического случая.


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