# On striae investigation in optical glass by means of polarization shearing method* 

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## 1. Introductory remark

In this letter a method of azimuthal compensation in the heterochromatic light is described as applied to the measurement of wavefront deformation caused by weak striae being visualized by the method of linear polarization shearing. Although the results of the measurement seem to confirm the advantages offered by the method, the complete mathematical proof of its correctness has not been carried out so far.

## 2. Some remarks about the possibility of using the linear polarization shearing to investigate the optical glass blocks

Some contemporary precision optical instruments require highly uniform glass. The degree of admissible heterogeneity of optical elements may be calculated from the theory of material aberrations developed in the course of the last years. When restricting our attention to the striae it is possible to calculate the admissible relative area of the exit pupil to be occupied by the striae as a function of the wavefront deformations caused by them. The shadow methods (Schlieren or Dvořak methods) used commonly to glass examination allow us to detect the striae, but, obviously, not be measure the resulting wavefront deformations. The latter can be measured by interferometric methods, which, however, are not suitable for mass control of glass, while their accuracy does not exceed, in general $0.1 \lambda$. Thus, in order to examine the straie in glass blocks it is advisable to use rather the method of transversal polarization shearing which, in this application. was first mentioned by Hofmann [1] (without reporting on the technical solution). At our Institute of Physics the method of transversal polarization shearing has been implemented by combining the ocular of the observing telescope of a striascope (Schlierengerät) with a Savart polariscope (fig. 1). The


Fig. 1. The observing telescope of the striascope with the Savart polariscope and the azimuthal compensating plate: $S L$ - plane of the exit slit of the striascope, Pol polarizer, $P$ - phase plate acting as the azimuthal compensator, $\boldsymbol{S P}$ Savart plate, $\boldsymbol{A}$ - analyzer, $\boldsymbol{L}$ lens creating the image of the stria in the object plane of the ocular, $E$ - eyepiece

[^0]image observed is then similar to that seen in the interference-polarization microscope, but the field of view is incomparably larger, amounting to about 100 mm in diameter. Obviously, this dimension does not restrict the size of the examined glass block, as the latter may be shifted in the measurement domain across the plane perpendicular to the optical axis. The analogy with the interference-polarization microscope consists also in the offered possibility of observation in either fringed or uniform field of view. In the both cases the viewer sees the double images of striae. In the case of uniform field the optical path difference $R_{b}$ between the ordinary and extraordinary wavefronts may be varied by inclining the Savart plate, and changing thereby the colour of the viewfield background. One of the images of straie on this background has the colour coresponding to the less optical path difference $R_{l}$ (fig. 2) the colour of the other being controlled by the greater path difference $R_{r}$. The purpose


Fig. 2. The splitted image of the stria seen in the observing telescope of the striascope with the Savart polariscope: $R_{l}$ optical path difference of the ordinary and extraordinary rays creating the left hand image of the stria, $R_{r}$ - optical path difference of the rays creating right hand image of the stria, $R_{b}$ - optical path difference of the rays creating the background of the stria images, $R_{l}, R_{r}$ and $R_{b}$ refer to the observations under conditions of excluded compensator
of the measurement is to estimate the absolute difference in optical paths occurring within the stria as related to the background: $\left|R_{l}-R_{b}\right|$ and $\left|R_{r}-R_{b}\right|$. It should be noticed that the estimation of the path difference with the help of pollariscope interference colours is much inadequate, while the very image structure does not allow to apply the half-shadow methods. In this situation it is reasonable to consider several compensating methods reducing the stria colour to that of the background. At the first sight the inclination of a Savart plate seems to provide the right solution of the problem. By increasing the path difference of both the wavefronts by $R_{b}-R_{l}$ we cause that the colour of the background $R_{b}$ (fig. 2) comes over to the left hand stria image, while by decreasing it by $R_{r}-R_{b}$ the colour passes to the right hand stria image. The measurement is apparently simple, but since during the transition of the colour to one of the stria images the background colour itself varies too, the reference colour disappears causing that there exists no quarantee that the original background colour has been reconstructed exactly in the stria image. For this reason we used the azimuthal compensator covering only a part of the field of view as it shown in fig. 3. This method is
similar to that of Brace [2] with the essential difference that it is applied in heteromonochromatic rather than monochromatic light, and additionally the "half-shadow" plate


Fig. 3. Rotation of the phase plate by an angle $a$ causes the transfer of the background colour on one image of the striae. The background colour in the region uncovered by the compensating phase plate $P$ plays the part of the reference colour. I - direction of the oscillation of the electric field strength vector of the faster wave in the plate
serves to change the colour by changing the polarization state of the light in the separated region of the field of view. Such an application of the azymuthal compensation in not fully justified from the theoretical viewpoint, but the control measurements carried out for weak striae suggest that it is sufficiently accurate and correct. The proof of this thesis will be the subject of the author's interest in the nearest future. Hereafter, only a short introduction to the problem will be presented, while the problems to be proved will be only indicated.

Let us assume that a phase plate of optical path difference $\Delta R$ is inserted in the image plane of the observing telescope, covering only a part of the total field of view. The eigenvectors of the phase plate are directed initially along the directions of transmission of both the polarizer and analyzer of the Savart polariscope. It may be proved that although such positioning of the phase plate changes the states of polarization of the light transmitted through this plate, neverthelles the colour of any fragment of the image remains unchanged. Both the parts of the field of view: the free one and that covered by the phase plate have still the same colour in the background and the striae. The proof may be, most simply, carried out by using the Poincare sphere (fig. 4). The intervals of polarization states in the


Fig. 4. Illustration of the transformation of the polarization states of the light waves creating the spectrum $R_{l}$ of the left hand image of the stria under the zeroth azimuth ( $\alpha=0$ ) of the phase plate. The angular distance $2 \beta$ of the representation of the polarization state for an arbitrary wavelength $\lambda$ before and after the transformation from the point $A$ representing the azimuth of the analyzer remaining constant
visible range of three spectra are marked on the zero meridian of the Poincare sphere; these are the light spectra creating: the left hand stria image $R_{l}$, the right hand stria image $R_{r}$, and the background $R_{b}$. Each state of polarization passes through the Savart analyzer with the intensity:

$$
\begin{equation*}
I(\lambda)=I_{0}\left(t_{0}-t_{90}\right) \cos ^{2} \beta(\lambda)+t_{90}, \tag{1}
\end{equation*}
$$

where $t_{0} \quad$ - transmission coefficient of the analyser for the light vector oscillating along the transmission direction,
$t_{90}$ - transmission coefficient of the analyzer for the light vector oscillating perpendicularly to the transmission direction,
$\beta(\lambda)$ - half angular distance of the analyzer representation to that of the first vector of the phase plate on the Poincaré sphere,
$I_{0} \quad$ - light intensity in front of the analyzer.
The distribution of $I$, as an implicite function of the wavelength appearing in the formula (1), decides about the colour of both the background and the double images of striae. In figure 4 it may be seen that, all the wavelengths of each spectrum are polarized elliptically in the horizontal (or vertical) direction and differ by helicity and ellipticity. When inserting a phase plate in the course of the light beam under the azimuth $0^{\circ}$ the state of polarization is changed depending upon the wavelength and the intitial ellipticity. The first vector of the phase plate $P$ corresponds to the coordinates $(0,0)$ on the Poincare sphere. The coordinates corresponding to the position of analyzer $A$ of the Savart polariscope are positioned at the other end of the diameter passing through the point $P$. In order to find a new state of polarization each point on the shpere corresponding to any wavelength of the spectrum should be rotated on this sphere arround the axis $P A$ in the clockwise direction by an angle

$$
\begin{equation*}
\delta_{p}=2 \pi \frac{\Delta R}{\lambda} \tag{2}
\end{equation*}
$$

Since, however, the rotation was carried out arround the axis $P A$ the angular distance $2 \beta$ of a representation of a given point of the spectrum from the analyzer $A$ is constant for any given wavelength, independently of the length of the are travelled during the rotation. Hence, the distribution of the value of $I(\lambda)$ in the formula ( 1 ) is not changed.

It is not much obvious what happens during the rotation of the compensating phase plate (which is the necessary condition to transfer the colour from the background to any of both stria images). By rotating the plate it is possible to extinguish, at certain angle (fig. 5), the same wavelength of the spectrum as that which is extinguished in the background spectrum. Strictly speaking, this is possible only approximately. Now, by rotating each point


Fig. 5. Illustration of the transformation of the polarization states of the light waves creating the spectrum $R_{l}$ of the left hand image of the stria for the azimuth of the phase plate $P$ different from zero
of the spectrum on the Poincare sphere arround the axis passing through the new position of the point $P$, the said wavelength may be shifted to the meridian and, if necessary, by a slight rotation of the analyzer, cause its extinction. There in no reason to suppose that the angular distance $2 \beta$ of the polarizer representation on the Poincaré sphere to the polarization states is for the given wavelength constant before and after transformation. This is so only when the rotation angle of the phase plate amounts to $a=45^{\circ}$ and the dispersions of both the phase plate birefringence and the glass examined are equal. If, during the rotation of the phase plate by an angle $a$, the stria accepts the colour of the background, then the wavefront deformation on the stria is calculated from the formula [2]:

$$
\begin{equation*}
\left|R_{r}-R_{d}\right|=\left|R_{l}-R_{b}\right|=\frac{\lambda}{2 \pi} \operatorname{arctg}\left[(\sin 2 \alpha) \operatorname{tg} 2 \pi \frac{\Delta R}{\lambda}\right] . \tag{3}
\end{equation*}
$$

The reservations cited above have, probably, no essential meaning for small values of $\Delta R$ since, in practice, we managed to transfer the background colour successively to both the stria with high fidelity. The background in the region of the field of view uncovered by the phase plate being used as a reference colour, the accuracy of the measurement was improved. The described method was also succesfully applied to realize the compensation in the linear polariscope without shearing. It is necessary to carry out some tedious calculations to estimate the influence of the said reservations on the result of measurement and, hence, to eastablish the range of applicability of the method.

## References

[1] Hofmann C., Jenaer Rundschau 22 (1977), 1.
[2] Jerrard H. G., J. Opt. Soc. Am. 38 (1948), 1.


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