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# Chromatic aberration of a system of holographic lenses produced on non-plane surfaces

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In this work the dependence determining the chromatic aberrations of a system of arbitrary number of holographic lenses produced on the rotationally symmetric surfaces of second degree has been derived. A method of designing an achromatic system of holographic lenses is presented.

## 1. Introduction

The aberrations are one of the main factors restricting the quality of holographic imaging. The formulae for third order aberration for plane holograms were given in the papers [1, 2], while those for the holograms produced on surfaces of rotational symmetry may be found in papers [3, 4]. Since the aberrational expressions depend to a significant degree upon the light wavelength used during the reconstruction, the construction of an achromatic system composed of holographic optical elements is more difficult than that of a classical optical system. For holograms chromatic dispersion is so great that it may be compensated only by other holographic elements. This is caused by the diffraction nature of holograms. The purpose of this work is to derive the formulae determining the chromatic aberration of the chromatic system of arbitrary number of holographic lenses produced on the non-plane surfaces. Also, a method of designing an achromatic system of holographic lenses is suggested.

## 2. Chromatic aberration of a system of holographic lenses

In order to examine the hologram aberrations produced on the non-plane surfaces a system of $p$ holographic lenses, as shown in Figure, is considered. The equation determining the position of the image produced by the first $i$ holograms is the following:

$$\frac{1}{R_{O'_i}} = -2K_i + \frac{1}{R_{C_i}} \pm \frac{\lambda'}{\lambda_i}\left(\frac{1}{R_{O_i}} - \frac{1}{R_{R_i}}\right), \tag{1}$$

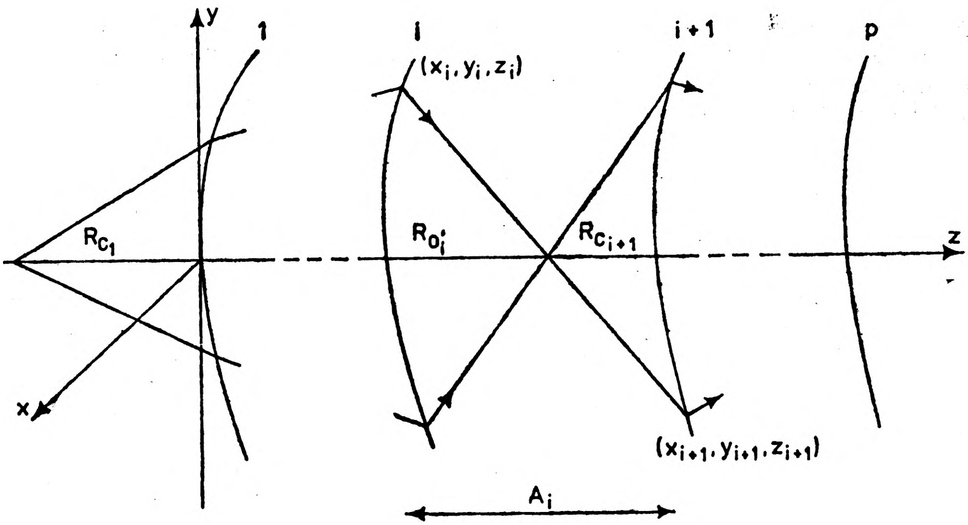
where $\lambda_i$ — wavelength of the light used for recording the $i$-th hologram,
$\lambda'$ — wavelength of the light used for reconstruction.
$K_i = (1+\varepsilon_i)/\varrho_i$, $\varepsilon$ and $\varrho$ denote the same as in preceeding paper [4]

of the author. The power of $i$-th hologram, i.e., the reciprocity of its focal length is determined

$$D_i = \frac{1}{R_{O'_i}} - \frac{1}{R_{C_i}} = F_i, \tag{2}$$

where

$$F_i = \pm\mu\left(\frac{1}{R_{O'_i}} - \frac{1}{R_{R_i}}\right) - 2K_i. \tag{3}$$




System of holographic lenses

The chromatic difference of the value $D$, for reconstructing wavelengths $\lambda'_1$ and $\lambda'_2$ denoted by $\Delta D$, amounts to

$$D_i = D_{i\lambda_1} - D_{i\lambda_2} = -\frac{\Delta R_{O'_i}}{R^2_{O'_i}} + \frac{\Delta R_{C_i}}{R^2_{C_i}}. \tag{4}$$

For an arbitrary hologram it may be written

$$\Delta R_{C_{i+1}} = R_{O'_i} - A_i, \tag{5}$$

where $A_i$ – distance between the $i$-th and $(i+1)$-th holograms. Hence, it may be seen that

$$R_{C_{i+1}} = R_{O'_i}. \tag{6}$$

Therefore, the eq. (4) may be written in the form

$$-\frac{\Delta R_{O'_i}}{R^2_{O'_i}} = \Delta D_i - \frac{\Delta R_{C_i}}{R^2_{C_i}} = \Delta D_i - \frac{\Delta R_{O'_{i-i}}}{R^2_{C_i}}. \tag{7}$$

Since we deal with paraxial considerations it may be seen from Figure that the following dependence is true

$$\frac{y_i}{y_{i+1}} = \frac{R_{O'_1}}{R_{C_{i+1}}}. \tag{8}$$

The eq. (7) takes now the form

$$-\frac{\Delta R_{O'_i}}{R^2_{O'_i}} = \Delta D_i - \left(\frac{y_{i-1}}{y_i}\right)^2 \frac{\Delta R_{O_{i-1}}}{R^2_{O'_{i-1}}}. \tag{9}$$

Thus, a recurrence formula has been obtained which allows to calculate the longitudinal chromatism after $i$-th hologram if its value after $(i-1)$-th hologram is known.

After multiplying both sides of the eq. (3) by $y_i/y_1$, expending and substituting the expression for $i = p, p-1, \ldots, 1$ as well as exploiting the condition $\Delta R_{C_1} = 0$ it may be obtained that

$$-\left(\frac{y_p}{y_1}\right)^2 \frac{\Delta R_{O'_p}}{R^2_{O'_p}} = \sum_1^p \left(\frac{y_i}{y_1}\right)^2 \Delta D_i. \tag{10}$$

By exploiting the eq. (2) it may be written

$$\Delta D_i = \frac{F_i}{\lambda_i} \Delta \lambda. \tag{11}$$

where $\Delta\lambda' = \lambda'_1 - \lambda'_2$. The eq. (10) takes now the form

$$-\left(\frac{y_p}{y_1}\right)^2 \frac{\Delta R_{O'_p}}{R^2_{O'_p}} = \sum_1^p \left(\frac{y_i}{y_1}\right)^2 \frac{\Delta\lambda'}{\lambda'} F_i. \tag{12}$$

The condition of chromatism correction is $\Delta R_{O'_p} = 0$.

As it should be expected the configuration and the power of the lens as well as the difference of the wavelength used during reconstruction have some influence on the chromatism value. The eq. (12) is valid also for holographic lenses produced on plane substrates.

After introducing the notion of the first chromatic sums

$$S_{I_{\mathrm{chrom}}} = \sum_{1}^{p} y_i^2 F_i, \tag{13}$$

the chromatic aberration of position may be written as follows

$$\Delta R_{O_p'} = \left(\frac{R_{O_p'}}{y_p}\right)^2 \frac{\Delta\lambda'}{\lambda'} S_{I_{\mathrm{chrom}}}. \tag{14}$$

## 3. Design of chromatic system of holographic lenses

As an example, the system of two contacting holographic lenses has been examined. Then we have $p = 2$, $y_1 = y_2$, $S_{I_{\mathrm{chrom}}} = y_1^2 F_1 + y_2^2 F_2$. The condition of chromatism correction $S_{I_{\mathrm{chrom}}} = 0$ is reduced to the form $F_1 + F_2 = 0$. Since in this case the total power of such a doublet is $F = F_1 + F_2$, then the achromatic condition denotes the zero power. An analogical result has been obtained by applying the procedure proposed by Benett [5] for plane holograms.

Now, the holographic lenses are shifted aside from each other to the distance $A$. Then $p = 2$, $y_1 \neq y_2$.

The condition of chromatic correction is the following

$$\frac{R^2_{O_1'}}{(R_{O_1'} - A_1)^2} F_1 + F_2 = 0. \tag{15}$$

Hence, the powers of the particular components must have opposite signs. For $A = 0$, this condition is reduced to that for the contacting elements.

For a three-components system $p = 3$, $A_1 \neq A_2$, $y_1 \neq y_3$. The condition for chromatism may be written in the form

$$\frac{R^2_{O_1'} R^2_{O_2'}}{(R_{O_1'} - A_1)^2 (R_{O_2'} - A_2)^2} F_1 + \frac{R^2_{O_2'}}{(R_{O_2'} - A_2)^2} F_2 + F_3 = 0. \tag{16}$$

This dependence does not impose any constructional restrictions on the system of these holographic lenses, except for the fact that it must contain elements of both positive and negative powers. The results obtained are analogous to those obtained for plane holograms [5].

As a numerical example of applying the obtained dependences to design, an achromatic system, of holograms, an achromatic doublet of holographic lenses produced on non-plane holograms was examined. After taking account of the dependence describing the focussing properties of the lenses, the eq. (15) takes the form

$$-A^2 R_{C_1}^2 F_1^3 + \left(A^2 R_{C_1}(F R_{C_1} - 1)\right) F_1^2 + (2AFR_{C_1} - 2FR_{C_1} + 2AR_{C_1} - A) F_1 + F(R_{C_1} - A)^2 = 0. \tag{17}$$

In an achromatic doublet of interlens spacing, say $A = 10$ mm, working as a microscope objective of 10 $D$ power, two roots of the eq. (17) of third degree are of practical importance.

*The solution 1*

power of the first lens $F_1 = 45.4$ $D$,
power of the second lens $F_2 = -65.13$ $D$.

*The solution 2*

power of the first lens $F_1 = -40.5$ $D$,
power of the second lens $F_2 = 92.66$ $D$.
If we want to obtain the objective of 5 $D$ power the same procedure gives:

*The solution 1*

power of the first lens $F_1 = 8.05$ $D$,
power of the second lens $F_2 = -3.31$ $D$.

*The solution 2*

power of the first lens $F_1 = -14$ $D$,
power of the second lens $F_2 = 22.09$ $D$.
The third root in both these cases leads to high powers of opposite signs and is of no practical use.

## 4. Final remarks

The problem of chromatism is here treated in general terms. The derived formulae concern arbitrary number of system elements. They are valid also for holograms of arbitrary shape. In some cases for two- or three-element combination of plane holograms the formulae come over to the formulae obtained by using other methods for these particular cases, which are shown in this work. In practice only the simplest combinations of two or three elements is possible, due to diffraction efficiency, aberration value and the presence of the beams diffracted in the higher diffraction orders. These factors cause that the restricted spectra are used and classical optical elements are usually applied both in the recording and reconstruction processes. Because of this restriction the shown example concerned the design of the achromatic doublet of holographic lenses. It may have also a practical application in designing the holographic microscopic objectives and other holographic optical elements.

## References

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## Хроматические аберрации системы голографических линз, создаваемых на неплоских поверхностях

Выведена зависимость, определяющая хроматическую аберрацию системы любого числа голографических линз, создаваемых на поверхностях вращения второго порядка. Показан также метод конструкции ахроматической системы голографических линз.