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Geodesic lens of rotational symmetry with axis parallel to light propagation direction

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In this paper a new type of a geodesic lens, of which axis of rotational symmetry is parallel to the planar thin film waveguide has been proposed. In particular, the method of evaluation of a parallel beam of rays passing through the particular type of a geodesic lens has been proposed and the results of numerical calculations have been presented. This lens is created by rotating the curve (generating curve) of the form $z = f(v) = a(1-v)^{1/2}$, where z — axis of symmetry (axis of revolution), v — normalized distance of surface point with the axis, a — constant factor.

1. Introduction

In the papers [1–6] the theoretical principles and practical realizations of geodesic lenses with axis perpendicular to the plane of waveguide have been examined. For this type of lenses the change of the beam width is not possible. This follows from the principal formulae presented, for example, in [6].

Let a rotational surface be given by parametric equation $\mathbf{r}(u, v)$ (see Fig. 1):

$$\mathbf{r}\begin{cases} v\cos u \\ v\sin u \\ f(v) \end{cases} \tag{1}$$

where v — distance of surface point with the z-axis lying at the plane of waveguide, u — angle of rotation measured from positive direction of x-axis, $f(v)$ — function of v-parameter (the so-called generating curve [1]).

The trajectories of rays falling onto this surface and passing throught it satisfy the system of differential equations

$$\begin{cases} \dfrac{d^2u}{ds^2} + \dfrac{2}{v}\dfrac{du}{ds}\dfrac{dv}{ds} = 0, \\ \dfrac{d^2v}{ds^2} - \dfrac{v}{1+f'^2}\left(\dfrac{du}{ds}\right)^2 + \dfrac{f'f''}{1+f'^2}\left(\dfrac{dv}{ds}\right)^2 = 0, \end{cases} \tag{2}$$

where $f'(v)$ and $f''(v)$ are the first and second derivatives of $f(v)$, respectively.

The first square form of surface (1) (if both index of refraction and thickness of guided film are constant) is written as follows:

$$ds^2 = v^2 du^2 + (1+f'^2) dv^2, \tag{3}$$

where s — arc length. For the sake of simplicity, the index of refraction of guided film has been assumed to be unity.

Moreover, according to the well known Clairaut's theorem we have

$$v \sin a = C, \tag{4}$$

and from the first equation in (2), we obtain

$$v^2 \frac{du}{ds} = C, \tag{5}$$

where a — angle between the meridian line (line u = const) and the geodesic line (the ray), C — constant.

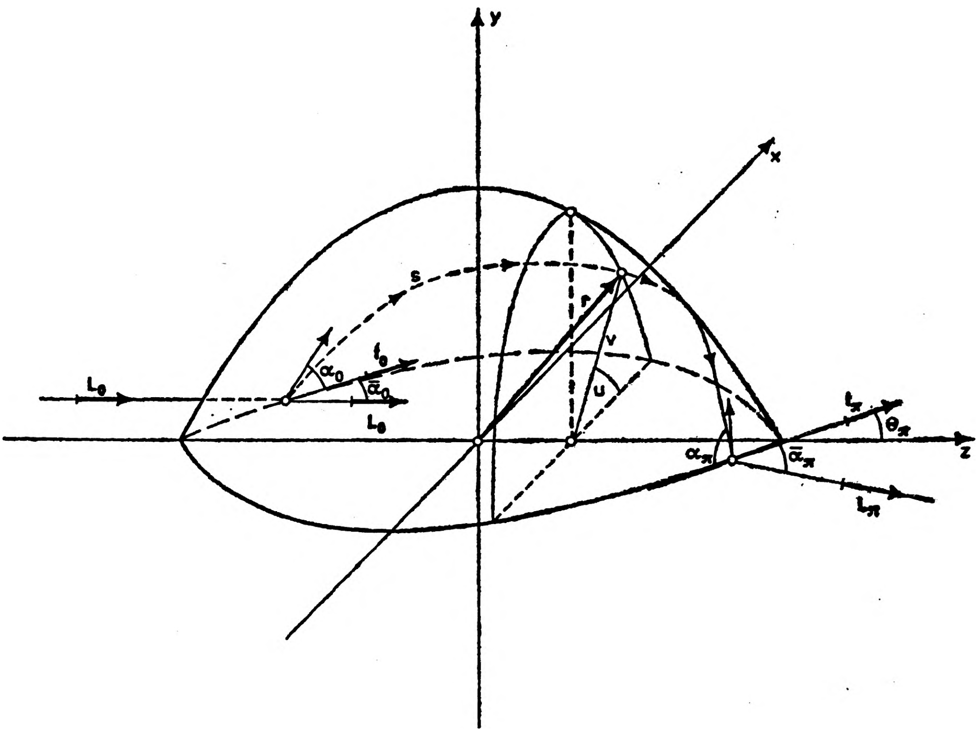

Fig. 1. Geodesic lens: z–axis of symmetry, x, z–plane of waveguide, curve s–light ray

The light ray parallel to the z-axis (Fig. 1) falls on the geodesic lens at the point $u = 0$, $v = v_0$, and leaves that lens at the point $u = \pi$, $v = v_\pi$. The function $f(v)$ can be chosen so that v_π/v_0 be equal to the prescribed value and that the exit ray be parallel to the z-axis.

2. Detailed formulae

We have examined, in particular, the geodesic lens created by rotation of the generating curve $f(v)$ of the forms

$$f_1(v) = a_1(1-v)^{1/2}, \text{ for } z < 0, \qquad (6)$$
$$f_2(v) = a_2(1-v)^{1/2}, \text{ for } z > 0,$$

where a_1 and a_2 are constant factors and the values of parameter v satisfy the inequality $0 \leqslant v \leqslant 1$.

In view of these assumptions the eqs. (2) take the following form

$$\frac{d^2u}{ds^2} + \frac{2}{v}\frac{du}{ds}\frac{dv}{ds} = 0.$$
$$\frac{d^2v}{ds^2} - \frac{4v(1-v)}{4(1-v)+a_i^2}\left(\frac{du}{ds^2}\right)^2 + \frac{a_i^2}{2(1-v)[4(1-v)+a_i^2]}\left(\frac{dv}{ds}\right)^2 = 0, \qquad (7)$$

for $i = 1$, and $i = 2$.

In order to examine the course of light rays by solving the eqs. (7), the values of du/ds and dv/ds at the point $s = 0$ ($u = 0$, $v = v_0$) should be calculated. This can be done as follows: From (4) and (5) we obtain (for $s = 0$)

$$\left(\frac{du}{ds}\right)_0 = \frac{C}{v_0^2} = \frac{\sin a_0}{v_0}. \qquad (8)$$

Assuming that the tangential component of a ray direction vector, with respect to the lens boundary, preserves its value after refraction on line $u = 0$ (lens boundary) we can write (see Fig. 1 and [6]):

$$a_0 = \bar{a}_0. \qquad (9)$$

Cosine of angle $\bar{a}_0$ may be evaluated as the inner product of the ray direction vector $\mathbf{L}_0\{0, 0, 1\}$ and tangential vector to the lens boundary

$$t_0 = \frac{1}{\sqrt{1+f_0'^2}}\{1, 0 f_0'\}. \qquad (10)$$

This yields

$$\sin \overline{a_0} = \frac{1}{\sqrt{1+f_1'^2}} = \sin a_0. \qquad (11)$$

By combining (8) and (11) we obtain the first initial condition

$$\left(\frac{du}{ds}\right)_0 = \frac{1}{v_0\sqrt{1+f_0'^2}}, \qquad (12)$$

therefore, in particular with respect to (6), we have

$$\left(\frac{du}{ds}\right)_0 = \frac{2}{v_0}\sqrt{\frac{1-v_0}{4(1-v_0)+a_1^2}}. \tag{13}$$

The second initial condition can be obtained from (3) and (13):

$$\left(\frac{dv}{ds}\right)_0 = \frac{f'_0}{1+f_0'^2}, \tag{14}$$

in our case we have

$$\left(\frac{dv}{ds}\right)_0 = \frac{2a_1\sqrt{1-v_0}}{4(1-v_0)+a_1^2}. \tag{15}$$

In order to determine the direction of a ray leaving a lens at the point $u = 0$, $v = v_\pi$ we can use the relations similar to (8) and (9), namely:

$$\bar{a}_\pi = a_\pi \tag{16}$$

and

$$\left(\frac{du}{ds}\right)_\pi = \frac{C}{v_\pi^2} = \frac{\sin a_\pi}{v_\pi}, \tag{17}$$

where a_π and $\overline{a_\pi}$ are the angles between the ray and the lens boundary (line $u = \pi$) before and after the refraction of the ray on line $u = \pi$, respectively, (Fig. 1).

The unit vector $\boldsymbol{t}_\pi$ tangential to the line $u = \pi$ can be expressed by the formula similar to that for the vector $\boldsymbol{t}_0$ (1), i.e.

$$\boldsymbol{t}_\pi = \frac{1}{\sqrt{1+f'^2(v_\pi)}}\left\{1,\ 0, -f'(v_\pi)\right\},$$

and, in our case we have

$$\boldsymbol{t}_\pi = \frac{2\sqrt{1-v}}{\sqrt{4(1-v_\pi)+a_2^2}}\left\{1,\ 0, \frac{a_2}{2\sqrt{1-v_\pi}}\right\}, \tag{18}$$

x and z components of the vector $\boldsymbol{t}_\pi$ are $\sin\theta$ and $\cos\theta$, respectively, where the angle θ means the angle between vector $\boldsymbol{t}_\pi$ an z-axis (Fig. 1).

This fact and relations (16) and (17) allow to determine the coordinates of the unit direction vector L_π in x, y, z system:

$$\begin{aligned} L_{\pi x} &= \cos\overline{a_\pi}\sin\theta - \sin\overline{a_\pi}\cos\theta, \\ L_{\pi y} &= 0, \\ L_{\pi z} &= \cos\overline{a_\pi}\cos\theta + \sin\overline{a_\pi}\sin\theta. \end{aligned} \tag{19}$$

3. Numerical results

Numerical evaluations of the ray trajectories have been made for several values of the coefficients a_1 and a_2. For $a_1 = -1/2\sqrt{3}$ the results have been illustrated both graphically (Fig. 2) and numerically (Table 1). The results of the computations for $a_1 = -\sqrt{3}$, $a_2 = 1/2\sqrt{3}$ are presented in Fig. 3 and in Table 2.

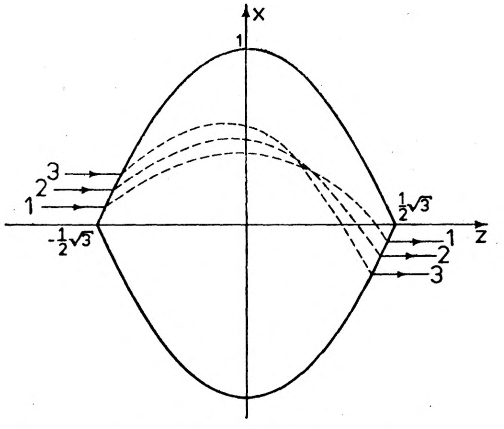

Fig. 2. Top view of ray trajectories for lens with generating curve $f(v) = \pm\frac{1}{2}\sqrt{3}(1-v)^{1/2}$; $1 - v_0 = 0.1$, $2 - v_0 = 0.2$, $3 - v_0 = 0.3$

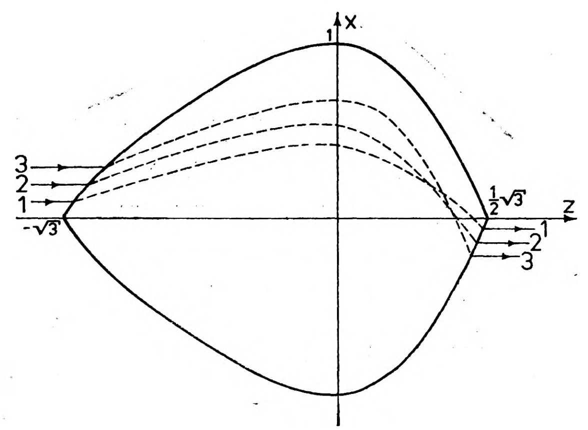

Fig. 3. Top view of ray trajectories for lens with generating curve $f_1(v) = -\sqrt{3}(1-v)^{1/2}$ (for $z < 0$), and $f_2 = \frac{1}{2}\sqrt{3}(1-v)^{1/2}$ (for $z > 0$); $1 - v_0 = 0.1$, $2 - v_0 = 0.2$, $3 - v_0 = 0.3$

4. Conclusions

A new type of geodesic lens has been presented. Collimated beam preserves the collimation after passing through these lenses. Moreover, in second example the beam changes its width. The beams leaving these lenses are uncorrected.

Table 1. Results of computations for lens shown in Fig. 2

Number of the ray in Fig. 2	v_0	v_π	$L_{\pi z}$
1	0.1	0.10331	0.999837
2	0.2	0.19271	0.999946
3	0.3	0.30181	0.999521

Table 2. Results of computations for lens shown in Fig. 3

Number of the ray in Fig. 3	v_0	v_π	$\frac{v_\pi}{v_0}$	$L_{\pi z}$
1	0.1	0.07762	0.77620	0.993866
2	0.2	0.15545	0.77727	0.998814
3	0.3	0.23584	0.78612	0.999639

In order to obtain the perfect geodesic lens with prescribed change of beam width a suitable continuous change of the coefficient a_i in the function $f(v)$ should be introduced. An alternative way is to inspect another form of the function $f(v)$. These problems will be considered in the next papers.

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Геодезическая линза с осью вращения симметрии, параллельной направлению распространения света

Предложен новый тип геодезической линзы, ось вращения симметрии которой параллельна плоскости волновода. В частности, разработан метод пересчёта параллельного пучка лучей через специального типа линзу, а также приведены результаты расчётов. Эта линза создаётся посредством вращения кривой вида: $z = f(v) = a(1-v)^{1/2}$, где z — ось вращения, v — нормализованное расстояние точки кривой от оси, a — постоянный коэффициент.