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PHYSICAL AND MATHEMATICAL MODEL FOR TRADITIONAL STORAGE RESERVOIR

The control of sewage flow in the system affects considerably the sizing and rational utilization of sewage system cross-sections. The most commonly known means of controlling flow intensity and pollutant concentration in sewage are such reservoirs that relieve hydraulic conditions in the sewage and its components at various stages of the formation and movement of wastewater. There is a growing interest in sewage storage reservoirs, and they are now an integral element of modern sewage systems. Storage reservoirs are very costly investments and the capital expenditure on their construction is proportional to their capacity. Therefore, any research aimed at minimizing this capacity, while maintaining the same level of performance, deserves special attention due to the scale of the problem and the size of the possible cost savings for the national economy.

DENOTATIONS

Am – standard cross-sectional areas of a traditional reservoir, m^2;
AK – horizontal cross-sectional area of a traditional reservoir operating in a conventional system, m^2;
AP – horizontal cross-sectional area of a flow chamber in a multi-chamber reservoir, m^2;
B – width of a conventional single-chamber reservoir, m;
F – drainage basin area, ha;
F_{zr} – reduced urban drainage basin area, ha;
Fo – cross-section area of outflow channel, m^2;
Fc – active surface of channel, m^2;
h – sewage fill height in a traditional reservoir calculated from the outflow channel axis, m;
hm – standard fill height of a traditional reservoir, m;
H – sewage height in reservoir storage chamber measured from outflow channel axis, m;
H_{rz} – difference in inflow and outflow bottom position ordinates in reservoir cross-section, m;
Hu – average fill height of reservoir storage chamber and $Hu = 0.5\ hi + hpr$, m;
l – length of discharge channel between reservoir and sewage system, m;
L – length of sewer trunk channel, m;
V – sewage flow rate in channel, dm^3/s;

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q – unitary stormwater runoff to sewage system, $dm^3/s \cdot ha$;
QA – sewage inflow to reservoir, dm^3/s;
QA_{max} – maximum sewage inflow to reservoir, dm^3/s;
QA_{aw} – inflow triggering emergency overflow at $QA_{aw} > QA_{max}$, dm^3/s;
$QA(TM)$ – maximum sewage inflow to reservoir from design storm for sizing a conventional ZK reservoir at $Td = TM$, dm^3/s;
$QA(t)$ – instantaneous sewage flow in channel at time t, dm^3/s;
QAi – discrete sequence of stormwater flow intensity of duration longer or equal to inflow time and $Tdi \geq Tp$, dm^3/s;
QAj – discrete sequence of stormwater flow intensity of duration shorter than inflow time and $Tdj < Tp$, dm^3/s;
QKo – hydraulic efficiency of trunk sewer located behind outflow channel opening, dm^3/s;
QO – sewage runoff from reservoir, variable over time; dm^3/s;
t – time, s;
$t_1, t_2, t_3, ..., t_n$ – time intervals considered during examination of sewage storage in reservoir, s;
tk – time of ground saturation for stormwater runoff, min;
tp – stormwater runoff from a given basin to channel cross-section in question, min;
tr – storage time of stormwater runoff channel, min;
T – duration of rainfall of a given frequency, min;
Td – rainfall duration, taking into account the times of ground saturation and channel storage and $Td = tp + tk + tr$, min;
Tdi – discrete sequence of rainfalls with duration longer than or equal to inflow time and $Tdi \geq Tp$, min;
Tdj – discrete sequence of rainfalls with duration shorter than inflow time and $Tdj < Tp$, min;
Te – time after which reservoir is completely filled, corresponding to maximal sewage storage in reservoir, s;
To – sewage storage time until the reservoir is completely emptied, s;
Tp – rainfall duration equal to inflow from the furthest basin point used to size the system by the method of maximal intensity, min;
TM – design storm duration for sizing a traditional single-chamber reservoir, min;
V – required capacity of reservoir relieving hydraulic conditions determined by a given method, m^3;
Vi – capacity of a given reservoir chamber for an analytically determined filling height Hi, m^3;
VI – capacity of a lower part of the reservoir or of the reservoir chamber up to the highest position of the bottom, corresponding to filling height hi, m^3;
VK – capacity of traditional single-chamber reservoir, m^3;
ZK – traditional single-chamber reservoir with conventional arrangement;
ZO – treatment reservoir acting as settling tank for treating stormwater runoff and industrial waste;
c – frequency of design storm for the purpose of sizing the sewage system, years;
c_H – coefficient for Chezy's formula, $m^{0.5} \cdot s^{-1}$;
c_Z – frequency of design storm for sizing storage reservoir, years;
C, D, E, K, u_1 and u_2 – constants of differential equations for describing sewage storage in traditional single-chamber reservoirs;
$C1$, $C2$, $C6$, $C8$ – constants of differential equations for sewage storage in traditional reservoirs;
Dd – inflow channel diameter, m;
Do – outflow channel diameter, m;
g – acceleration of gravity, $m \cdot s^{-2}$;

H – mean annual rainfall, mm;
i – bottom slope of channel, %;
i_z – bottom slope of traditional single-chamber reservoir, %;
K – rainfall parameter covering function $K = 6.67\ H^{0.67}c^{0.33}$;
Kd – parameter characterizing geographic position, extent of basin and adopted level of sewage system reliability, called inflow parameter, $m^3 \cdot s^{1-n}$;
Ko – parameter characterizing outflow channel hydraulic efficiency, called outflow parameter, $m^{2.5} \cdot s^{-1}$;
n – exponent for formulae used to calculate design storm runoff to sewage system;
n_o – roughness coefficient of outflow channel;
n_r – coefficient of dilution by stormwater runoff on stormwater overflow;
n_{ro} – coefficient of dilution by stormwater runoff on last stormwater overflow;
β – sewage flow reduction factor in reservoir and $\beta = QO_{\max} \cdot QA(Tp)^{-1}$;
β_M – flow reduction factor for design storm and $\beta_M = QO_{\max} \cdot QA(TM)^{-1}$ or $\beta_M = QO_{\max} \cdot QA(TMW)^{-1}$;
μ – factor for sewage runoff to outflow channel.

1. INTRODUCTORY REMARKS

Traditional reservoirs that relieve hydraulic conditions in a conventional sewage system and its components consist of an inlet channel, a reservoir for storing peak overflows, and a channel to carry away sewage in amounts that depend on its hydraulic flow capacity and reservoir filling height. The release of sewage into a traditional reservoir occurs in the upper part of a single-chamber reservoir or, in the case of a double-chamber reservoir, before its partition (figure 1).

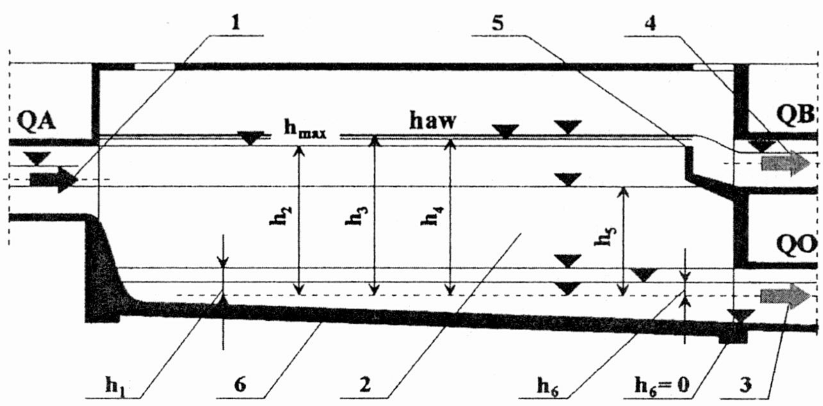

Fig. 1. Hydraulic model of a traditional single-chamber reservoir

Reservoirs of this type have many disadvantages. Among the most important are: (1) insufficient utilization of storage capacity for a given volume occupied by the reservoir, (2) inadequate utilization of the hydraulic capacity of the outflow channel during the reservoir's storage cycle, and (3) contamination of the bottom of the reservoir when the sewage inflow into the reservoir is several times lower than the maximum outflow out of the reservoir QO_{max}. Hence it is necessary to design reservoirs of considerable capacity that can be flushed frequently when they are in use.

2. MODEL OF THE TRADITIONAL STORAGE RESERVOIR

A storage reservoir in a conventional system works as follows: sewage enters the reservoir at its highest part, passes through the reservoir, and leaves it at the level of the lowest point of the reservoir bottom.

Depending on the sewage level and the rate of sewage storage in a traditional reservoir (figure 1), three basic phases of reservoir operation can be distinguished: flow-through, filling, and evacuation.

Based on a detailed description of the operation of a single-chamber reservoir [1], a hydraulic model of the functioning of a flow-through reservoir during its filling and evacuation phases has been formulated (figure 1). This model allows consideration of a typical hydrogram of sewage flow in a storm drainage system and in a combined sewer system, and is the basis for the development of a mathematical model of a storage reservoir for a storm sewer system [1], [2].

Phases in filling a traditional reservoir:

1. Filling the traditional reservoir with gravity-driven sewage outflow

$$Do/2 \geq h_1 \geq 0, \quad QA > 0, \quad QA > QO, \quad QB = 0\,.$$

2. Filling the traditional reservoir under pressurized sewage outflow

$$h_{max} \geq h_2 > Do/2, \quad QA \geq QO, \quad QO \leq QO_{max}, \quad Qb = 0\,.$$

3. Filling the traditional reservoir in emergency overflow conditions

$$haw \geq h_3 > h_{max}, \quad QOaw \geq QO > QO_{max}, \quad QA \leq QAaw, \quad QB > 0\,.$$

Phases in emptying a traditional reservoir:

$$haw > h_4 \geq h_{max}, \quad QOaw > QA, \quad QO \geq QOaw, \quad QB \geq 0\,,$$

$$h_{max} > h_5 > Do/2, \quad QO_{max} > QA, \quad QO > QO(h = Do/2), \quad QB = 0\,,$$

$$Do/2 \geq h_6 \geq 0, \quad QO(h = Do/2) > QA \geq 0, \quad QB = 0\,.$$

Depending on the sewage system, the traditional reservoir reaches the capacity level h_6 during the final phase of reservoir evacuation, which corresponds to the level of sewage removed during dry weather. This occurs (1) when the reservoir is a part of a combined sewage system or (2) when a reservoir operating as a part of a stormwater sewer system is completely emptied (figure 1).

3. A METHOD FOR DESCRIBING AN ARBITRARY HYDROGRAM

Each of the established methods of determining the theoretical or empirical shape of a hydrogram for sewage overflow has certain inadequacies because of random variation in atmospheric phenomena in the basin which requires the introduction of certain simplifications in the calculation. Furthermore, climatic changes have in recent years been observed in the European area that have led to different patterns of sewage runoff; in addition, time has brought a change in the degree of urbanization in a basin and its permeability.

The validity of a method for determining the needed capacity of reservoirs is based mainly on an analysis of all the available inflow hydrograms for a given level of operational reliability of the sewer system, which makes it possible to establish a standard hydrogram by accurately simulating the sewage storage in a reservoir for a given flow reduction factor [1], [2]. In practice, it is incorrect to adopt a priori one hydrogram as standard.

Since the extreme intensity method is widely used for calculating the design storm overflow of storm sewers in Poland [3], [4], [5], as well as in Russia [6], [7], the U.S.A. [8], [9], [10], [11], and in other countries by means of more or less radical modifications, this method was thoroughly analyzed in determining the hydraulic relationships that describe the course of sewage storage in traditional and multi-chambered reservoirs. In performing this analysis, we have retained the possibility of applying any shape of inflow hydrogram or mathematical formulas after they have been transformed into a series of linear functions. For that reason, in this work the course of rainfall intensity corresponds to "Pattern II".

For "Pattern II", the variation in rainfall intensity is described by equation (1) for a hydrogram in the shape of a trapezoid or triangle

$$q = A \cdot F \cdot Td^{-n} \cdot L^{-1} = \text{const}. \tag{1}$$

Depending upon the shape of the hydrogram, rainfall intensity is described by the equations for $t < Td$

$$QA = QA\max \cdot t \cdot Td^{-1}, \tag{2}$$

while for $t > Td$

$$QA = QA\max\left[2 - t \cdot Td^{-1} + Fc^{-1}\int_{Td}^{t} q(t) \cdot dt\right]. \tag{3}$$

If after the time Td, the rain inflow stops, then $q(t) = 0$ and the decrease in overflow through the calculated cross section will correspond to the case of evacuating the trunk sewer main.

Stormwater sewage plays the dominant role in the process of sewage retention in a wastewater system. Therefore the flow balance equation considers the specific characteristics of stormwater runoff, as well as variation in the inflow of any type of sewage, by using an appropriate procedure to describe the inflow hydrograph in the following form:

- equations of discrete linear functions for separate areas of variation in QA over time,
- equations that describe any flow QA over time by means of continuous curvilinear functions that can be converted into a sequence of elementary values of the linear variability QA_i,
- graphic description of the variation in QA over time by means of coordinates of the points of intersection of straight lines of different slopes,
- a table of the variation in QA over time, most often determined by means of direct measurements over a period of time.

In summary, each of the above analytical or graphic methods for describing the variation of flow intensity for any type of sewage QA over time can be presented as a set of elementary linear functions having three unique characteristics: (1) they increase for $QA2 > QA1$ (figure 2a); (2) they decrease for $QA2 < QA1$ (figure 2b); (3) for constant flow when $QA2 = QA1 = \text{const}$ (figure 2c) for known coordinates $M(ti, QA_i)$ and $N(tj, QA_j)$. For a specified time t, the general equation (4) determines the value of the flow $QA(t)$.

It is possible to substitute an inflow hydrogram of any shape, even that described by a complicated mathematical curve, for an equivalent hydrogram illustrating the variability of wastewater inflow in the form of a specified series of linear functions. The proposed system makes it possible to maintain the shape of a hydrogram similar to the assumed one and, even more importantly, permits a considerable simplification in the mathematical description. Thus, each hydrogram can be expressed in the form of an ordered set of three kinds of overflow variation:

- segments with a linear increase in overflow over time, in which for $t_2 > t_1$, $QA2 > QA1$ (figure 2a),
- segments with constant overflow over time, in which for $t \langle t_1;\ t_2\rangle$ $QA1 = QA2$ (figure 2c),
- segments with a linear decrease in overflow over time, in which for $t_{i+1} > t_i$, $Qa_{i+1} < QA_i$ (figure 2b).

Keeping this in mind, a general relationship has been established which makes it possible to determine sewage overflow at any time for known coordinates of characteristic points of an inflow hydrogram, dividing it into segments of variation QA in time (figure 2), using the equation of a straight line defined by two points:

$$QA(t)=(QA_{i+1}-QA_i)(t-t_i)(t_{i+1}-t_i)^{-1}+QA_i\,. \tag{4}$$

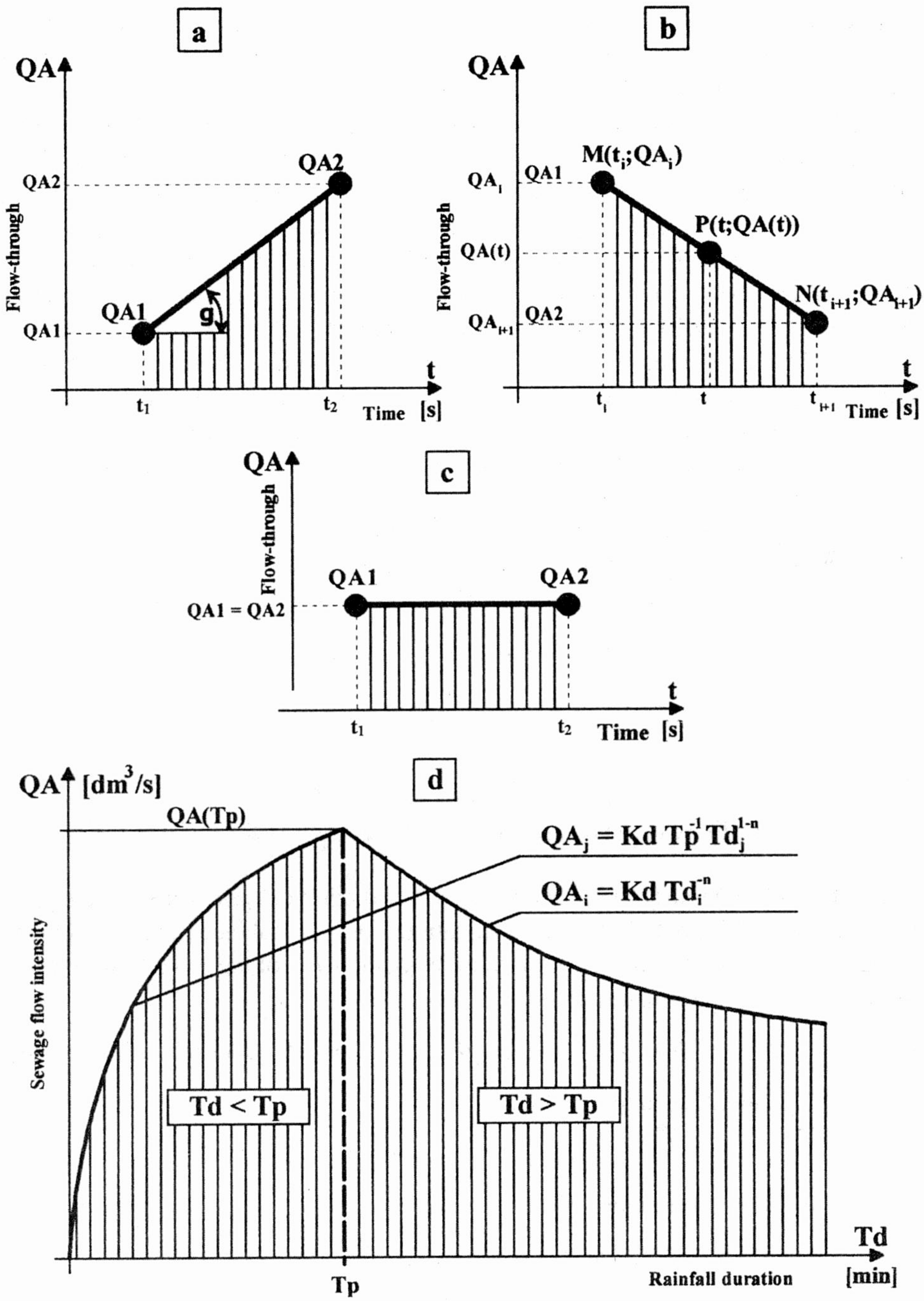

Fig. 2. Graphic description of general equation of inflow variation (4) characteristic of limit stress method

This dependence makes it possible to define the value of wastewater overflow at any time if the slope angle of the straight line or the values of the coordinates for the boundaries of the segments of overflow variation $Q = f(t)$ are known.

Taking into account equation (4), when establishing the differential equations for sewage flow balance in reservoirs of any type, it is possible to define unambiguously the filling and evacuation processes over time in a traditional reservoir, an overflow chamber, and the remaining storage chambers of multi-chamber reservoirs, as well as to define the necessary capacity of the reservoir by using the specified equations [12].

The proposed method can be applied independently of the particular procedure used to determine overflow variation or independently of the mathematical model used to describe stormwater runoff overflow in sewage mains, both of which are the result of the hydraulic analysis of the overflow obtained from the transformation of effective precipitation into surface runoff, until the main is filled and overflow appears in the main, as well as the result of the rate of increase over time in overflow observed in the cross-section and along the length of the main.

Depending upon the characteristic of rainfall duration Td in relation to the inflow time Tp, different procedures have been determined for calculating the value of sewage inflow QA in order to size reliably the channel cross-section (figure 2d):

$$QA_i = Kd \cdot Td_i^{-n} \quad \text{for} \quad Td \geq Tp\,, \tag{5}$$

$$QA_j = Kd \cdot Tp^{--} \cdot Td_j^{1-n} \quad \text{for} \quad Td < Tp\,. \tag{6}$$

4. MATHEMATICAL MODEL OF THE TRADITIONAL STORAGE RESERVOIR

On the basis of previously published work on the theoretical principles of sizing traditional reservoirs [1], [13] only basic model equations for flow balance have been given for different areas of inflow variability with the following division of the model parameters:

- Input dependent parameters WZ:

$$WZ[AK, c, c_z, Do, Fzr, Fo, H, Kd, Ko, q, QKo, QA(Tp), QA_i, QA_j, Td_i, Td_j, Tp, \beta, \mu]\,.$$

- Input independent parameters WN:

$$WN\,[B, Hu, Hrz, i_z, l, t_k, t_r, n, n_o, n_r, n_{ro}, \zeta, \chi]\,.$$

- Output resultant parameters WW:

$$WW\,[Am, hm, QA(TM), Te, To, TM, VK, \beta_M]\,.$$

- Dependent variable $ZZ\,[h]$ and independent variable $ZN\,[t]$.

The rain sewage volume variation in traditional reservoir dV (figure 3a) in the given time interval dt can be described by equation:

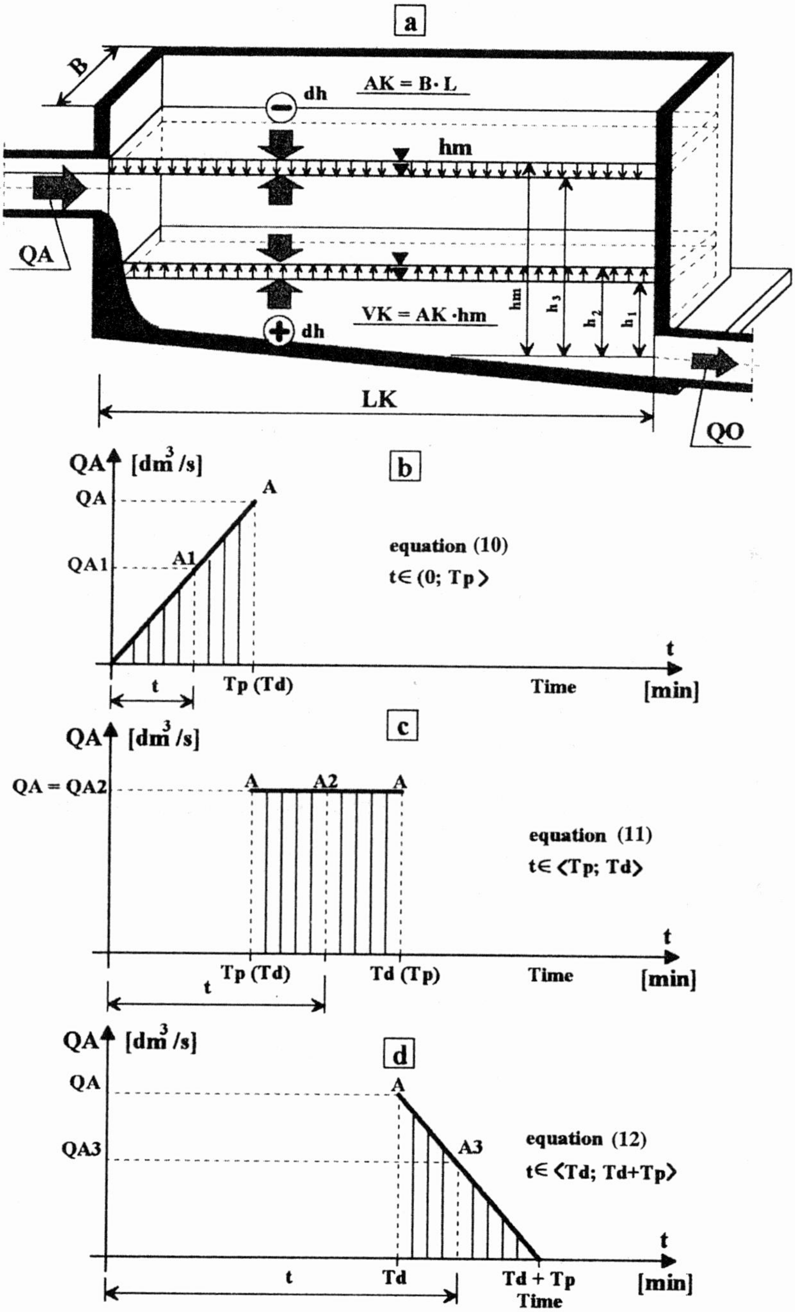

Fig. 3. Model of storage and diagrams of variations of sewage inflow to traditional reservoir for characteristic rainfall durations

$$dV = AK \cdot dh = QA(t)dt - QO(t)dt\ , \tag{7}$$

where

$$QO(t) = 2^{0.5} \cdot g^{0.5} \cdot \mu \cdot F_o \cdot h(t)^{0.5}, \tag{8}$$

$$\mu = \left(8 \cdot g \cdot l \cdot c_H^{-2} \cdot Do^{-1} + \sum \zeta + 1\right)^{-0.5}. \tag{9}$$

Using model diagrams of inflow variability, differential balance equations for a traditional reservoir in the filling range $h_{\max} \geq h \geq 0$ have been determined as follows:

$$\frac{dh}{dt} = C1 \cdot t - C2 \cdot h^{0.5} \quad \text{for} \quad QA2 > QA1 \quad \text{(figure 3b)}\ , \tag{10}$$

$$\frac{dh}{dt} = C6 - C2 \cdot h^{0.5} \quad \text{for} \quad QA2 = QA1 \quad \text{(figure 3c)}, \tag{11}$$

$$\frac{dh}{dt} = C8 - C1 \cdot t - C2 \cdot h^{0.5} \quad \text{for} \quad QA2 < QA1 \quad \text{(figure 3d)}\ . \tag{12}$$

Equations (10), (11) and (12) make it possible to determine the dynamics of changes in filling $QA = f(t)$, as well as the outflow from reservoir $QO = f(h)$ in a traditional reservoir as it varies over time under conditions of variation, including the hydrographs specific to the method of extreme intensities for rain duration times Td_i longer than or equal to inflow time Tp, and Td_j shorter than the outflow time.

The constants given in general balance equations can be calculated from the following equations:

$$C1 = QA \cdot AP^{-1} \cdot Tp^{-1}\ , \tag{13}$$

$$C2 = 1.41 \cdot \mathrm{g}^{0.5} \cdot \mu \cdot Fo \cdot AP^{-1}\ , \tag{14}$$

$$C6 = QA \cdot AP^{-1}, \tag{15}$$

$$C8 = C6 + C7 = QA \cdot AP^{-1}(1 + Td \cdot Tp^{-1}). \tag{16}$$

5. CONCLUSIONS

Based on an analysis of existing studies, the basis has been developed for mathematical models of traditional storage reservoirs that relieve hydraulic and associated conditions. Using a previously developed mathematical model for a storage reservoir in a storm sewer system and using the results of numerical solutions, the essential research results are presented for a mathematical model of a traditional, single-chamber reservoir operating in any given sewage system. The scope and level of detail of this paper result from the need to present in convincing detail each of the hydraulic models of traditional reservoirs that will be proposed later. This detail is the basis for comparing variation in the process of sewage accumulation, as well as for comparing the advantages of controlling this process, in order to make possible the optimum utilization of reservoir storage space.

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PROPOZYCJA MODELU FIZYCZNEGO I MODELU MATEMATYCZNEGO KLASYCZNYCH ZBIORNIKÓW RETENCYJNYCH

Zbiorniki retencyjne należą do bardzo kosztownych inwestycji, a o nakładach finansowych związanych z ich budową decyduje przede wszystkim ich kubatura. Zatem wszelkie badania zmierzające do uzasadnionego minimalizowania pojemności, przy równoczesnym zachowaniu identycznego poziomu niezawodności ich działania, powinny zasługiwać na praktyczne wykorzystanie, jeśli mamy na uwadze skalę problemu i oszczędności inwestycyjne w gospodarce wodno-ściekowej. Opracowano uniwersalną metodę transformacji dowolnego hydrogramu dopływu w formie uporządkowanego zbioru trzech obszarów zmienności przepływu, niezależnie od przyjętej procedury wyznaczania zmienności dopływu ścieków deszczowych do zbiornika. Sformułowany model fizyczny uwzględnia charakterystyczne fazy napeł-

niania i opróżniania zbiornika klasycznego. Na jego podstawie opracowano model matematyczny funkcjonowania tego typu zbiorników w różnych systemach kanalizacji. Umożliwiają one porównanie zmian w badanym procesie akumulacji ścieków i zalet zbiorników wielokomorowych. Z uwagi na dominującą rolę ścieków deszczowych w procesie retencjonowania ścieków w systemach kanalizacji grawitacyjnej uwzględniono ich specyfikę i losową zmienność ich przepływu w kanałach w modelu funkcjonowania zbiornika klasycznego, odciążającego hydraulicznie sieć i inne elementy systemu kanalizacji.