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## THE LOCAL RESISTANCE IN PLASTIC PIPE FITTINGS

The results of studies of linear and local hydraulic resistance in polypropylene socket pipes, segmental elbows and bends and in systems of two or more such pipe fittings connected in series are analyzed and interpreted. The approximation of the linear resistance coefficients of the studied pipelines yielded results which were found to be in quite good agreement with the ones calculated from the Prandtl-Karman formula. Qualitative and quantitative formulas for the coefficient of local resistance in pipelines built from elbows and bends connected conformingly or alternatingly are presented.

## DENOTATIONS

$$
\begin{array}{ll}
d & \text { - the inside diameter of a pipeline or a pipe fitting, } \mathrm{m}, \\
g & \text { - the acceleration of gravity, } \mathrm{m} / \mathrm{s}^{2}, \\
\Delta h_{l} & \text { - a linear hydraulic loss head, } \mathrm{m}, \\
\Delta h_{\mathrm{m}} \text { - a local hydraulic loss head, } \mathrm{m}, \\
k & \text { - the absolute roughness of a pipeline, } \mathrm{m}, \\
l & \text { - the length of a pipeline, } \mathrm{m}, \\
l_{o} & \text { - the axial length of an elbow or a bend (or a system of such pipe fittings), } \mathrm{m}, \\
l_{e} & \text { - the equivalent length of a rectiaxial pipeline, } \mathrm{m}, \\
l_{k} & \text { - the length of the straight socket section in a segmental elbow or bend, } \mathrm{m}, \\
n & \text { - the number of elbows or bends in a system, } \\
R & \text { - the radius of curvature of a segmental elbow or bend, } \mathrm{m}, \\
R e & \text { - the Reynolds number }(R e=v d / v), \\
v & \text { - the average flow rate of liquid, } \mathrm{m} / \mathrm{s}, \\
z & \text { - the number of socket joints in a segmental elbow or bend (or in a system), } \\
\alpha & \text { - the central angle of a single elbow or bend, }{ }^{\circ}, \\
\alpha_{i} & \text { - the central angle of a segmental elbow or bend, }{ }^{\circ}, \\
\alpha_{s} & \text { - the sum of central angles in a system of segmental elbows or bends, }{ }^{\circ}, \\
\varepsilon & \text { - relative roughness }(\varepsilon=k / d), \\
v & \text { - the kinematic viscosity of a liquid, } \mathrm{m}^{2} / \mathrm{s},
\end{array}
$$

[^0]$\lambda$ - a nondimensional coefficient of linear resistance,
$\zeta$ - a nondimensional coefficient of local resistance.

## SUBSCRIPTS

$e$ - an equivalent index,
o - an axial index.

## 1. INTRODUCTION

Pipes made of plastics such as polyvinyl chloride, polyethylene or polypropylene significantly differ in the smoothness of the inside surfaces of their walls, and so in their hydraulic liquid flow conditions, in comparison with pipes made of traditional materials such as concrete, ceramics, cast iron or steel. If improper formulas are used to calculate linear losses, design errors may arise. The local resistance coefficients for elbows or bends found in the literature are mainly for steel or cast iron pipe fittings and they considerably differ from experimental results for single plastic pipe fittings and especially for two consecutive elbows or bends connected conformingly or alternatingly. As yet local resistance in systems made up of several to a dozen or so elbows or bends connected in series has not been studied. The axial length of such systems is many times shorter than the length of an equivalent (in hydraulic resistance) conventional throttling pipe. Therefore research into this subject seems to be worthwhile.

## 2. RESEARCH METHOD

As a real liquid flows in pressure conduits resistance to motion, referred to as hydraulic loss, arises due to the viscosity of the liquid. Linear and local hydraulic losses are distinguished. The hydraulic loss head $\left(\Delta h_{l}\right)$ in a rectiaxial pipeline (with length $l$ and diameter $d$ ) is expressed by the Darcy-Weisbach formula:

$$
\begin{equation*}
\Delta h_{l}=\lambda \frac{l}{d} \frac{v^{2}}{2 g} . \tag{1}
\end{equation*}
$$

In the literature, one can find many formulas for the linear resistance coefficient $(\lambda)$ for turbulent flows. In the case of hydraulically smooth pipelines, the PrandtlKarman formula (Polish Standard [1]):

$$
\begin{equation*}
\frac{1}{\sqrt{\lambda}}=2 \log \operatorname{Re} \sqrt{\lambda}-0.8 \tag{2}
\end{equation*}
$$

is usually applied. Sometimes (at $\operatorname{Re}>10^{5}$ ) the implicit Nikuradse formula:

$$
\begin{equation*}
\lambda=0.0032+0.221 R e^{-0.237} \tag{3}
\end{equation*}
$$

is used.
Local losses are caused by local obstacles as a result of disturbances in the fully developed velocity distribution in a long rectiaxial pipeline. The hydraulic loss head due to local resistance $\left(\Delta h_{m}\right)$ is expressed by this formula:

$$
\begin{equation*}
\Delta h_{m}=\zeta \frac{v^{2}}{2 g} \tag{4}
\end{equation*}
$$

Numerical values of coefficient $\zeta$ for single local obstructions such as elbows ( $R / d$ $\leq 1)$ or bends $(R / d>1)$ can be found in the literature dating from the 1950s - mainly for smooth or rough steel or cast iron pipe fittings - and in more recent publications (NOWAKOWSKI [2]; BrydAK-JEŻOWIECKA [3], SIWIEC [4]) - for plastic pipe fittings.

Experimental local resistance coefficient values are usually given for the fully developed turbulent flow of liquid in the 25-40 $d$ long section of the pipeline before the obstruction, taking into account the losses (due to the velocity and pressure distribution disturbances caused by the obstruction) along the $40-70 d$ long section after the obstruction. In practice, different local resistances are often closely spaced so that the flow of liquid cannot fully redevelop. Then the overall resistance coefficient for a system of pipe fittings is not equal to the direct sum of the individual resistance coefficients and it should be experimentally determined.

The aim of this research was to explore:

- the linear resistance in socket-jointed polypropylene pipes;
- the local resistance in single segmental polypropylene elbows and bends;
- the local resistance in systems of two or more bends or elbows conformingly connected in series to form, among others, spiral loops;
- the local resistance in systems of two or more bends or elbows alternatingly connected in series to form, among others, sinusoidal waves.

During the flow of a liquid with density $\rho$ and kinematic viscosity $v$ at average velocity $v$ energy (pressure) loss $\Delta p$ per a unit of length $l$ of a pipe with diameter $d$ and roughness $k$ is a function of five-dimensional variables:

$$
\begin{equation*}
\frac{\Delta p}{l}=\Pi(\rho, v, v, d, k) . \tag{5}
\end{equation*}
$$

If dimensional analysis is applied, the following equation is obtained:

$$
\begin{equation*}
\Delta p=\lambda(R e, \varepsilon) \frac{l}{d} \frac{v^{2}}{2} \rho . \tag{6}
\end{equation*}
$$

For the local resistance in a pipe fitting (a segmental elbow or bend) with diameter $d$, radius of curvature $R$, central angle $\alpha_{i}$, axial length $l_{o}$, straight socket section length $l_{k}$ and number $z$ of socket joints the following equation can be written:

$$
\begin{equation*}
\Delta p=\Pi\left(\rho, v, v, d, k, R, \alpha_{i}, l_{o}, l_{k}, z, \lambda\right), \tag{7}
\end{equation*}
$$

and applying dimensional analysis one arrives at:

$$
\begin{equation*}
\Delta p=\zeta\left(R e, \varepsilon, \frac{R}{d}, \alpha_{i}, \frac{l_{o}}{d}, \frac{l_{k}}{d}, z, \lambda\right) \frac{v^{2}}{2} \rho . \tag{8}
\end{equation*}
$$

If one opts for model studies of the flow of a liquid in polypropylene pipes and pipe fittings using scaled down actual objects (with given diameter $d$ ) and water as the medium, the experiments should be conducted for the possibly widest range of critical Reynolds number values, since the equality of numbers $R e$ as well as geometric and kinematic similarity between the model and the actual object are a necessary condition for dynamic similarity between the corresponding phenomena being studied.


Fig. 1. Experimental setup: 1 - lower tank, 2 - pump, 3 - upper tank, 4 - expansion chamber, 5 - pipeline investigated, 6 - local resistance investigated, 7 - circuit pressure taping chamber, 8 - piezometer, 9 - control valve, 10 - measuring overfall

Model local resistance studies were carried out on forty eight geometrical arrangements of segmental polypropylene elbows ( $R / d \leq 1$ ) and bends ( $R / d>1$ ) consisting of pipe fittings with central angles $\alpha \in\left\{15^{\circ}, 30^{\circ}, 45^{\circ}\right.$ and $\left.90^{\circ}\right\}$, relative radii of curvature $R / d \in\{0.55,1.0,1.75,2.25,4.25\}$ and number of socket joints $z \in<1.48>$ at relative straight socket section length $l_{k} / d=0.70$. A diagram of the experimental setup is shown in figure 1 . The measuring lengths ( $0-4$ ) were built from straight socket sections with assembly length $L_{m}=1.0 \mathrm{~m}\left(L_{m} / d=14.1\right)$. The
inside diameter of the polypropylene pipelines and pipe fittings was determined (by the volumetric method) to be $d=71.0 \pm 0.1 \mathrm{~mm}$ (KOTOWSKI, CISOWSKA and CIEŻAK [5]).

## 3. ANALYSIS AND INTERPRETATION OF EXPERIMENTAL RESULTS

The processing of the measurements of the linear resistance coefficient $(\lambda=f(R e))$ in hydraulically smooth pipes was based on the assumption that the velocity distribution in the pipe's cross-section (including the laminar sublayer) is described by the Prandtl theory. If such conditions are fulfilled, the coefficient of linear resistance can be described by the following qualitative relation (SIWON [6], Kotowski, Cisowska [7]):

$$
\begin{equation*}
\frac{1}{\sqrt{\lambda}}=a \log R e+b \tag{9}
\end{equation*}
$$

adopted by Altszul ( $a=1.82, b=-1.64$ ), Colebrook ( $a=1.80, b=-1.52$ ), Siwoń ( $a=1.78, b=-1.45$ ) and others.

The quantitative assessment of $\lambda$ in the model was based on this equation (figure 1 ):

$$
\begin{equation*}
\lambda=\frac{g \Delta h_{2-3}}{30 v^{2}} . \tag{10}
\end{equation*}
$$

The linear approximation of 917 results of measurements and calculations of the linear resistance coefficient in the propylene pipeline ( $60 d$ long section $2-3$ in the model) yielded the following relationship:

$$
\begin{equation*}
\frac{1}{\sqrt{\lambda}}=1.808 \log R e-1.573 \tag{11}
\end{equation*}
$$

for the Reynolds number range of $R e \in\langle 47000,248000\rangle$ at correlation coefficient $R=$ 0.675 (correlation at a confidence level of 0.99 ). Ultimately, the following generalized form of the formula for the linear resistance coefficient of the pipes:

$$
\begin{equation*}
\frac{1}{\sqrt{\lambda}}=-2 \log \frac{6.11}{R e^{0.904}} \tag{12}
\end{equation*}
$$

was proposed.
Formula (12) takes into account the local resistance caused by defects in the pipe diameter and in the pipeline's socket joints spaced (assembly spacing) at every $L_{m}=$ 14.1 d . If the geometric, kinematic and dynamic similarity is taken into account, it becomes apparent that the experimentally determined values of $\lambda$ are for actual pipes
with assembly lengths $L_{m} \in\{1,2,3,4,5,6\} \mathrm{m}$ and corresponding inside diameters $d \in\{0.07,0.14,0.21,0.28,0.35,0.42\} \mathrm{m}$ for the proper ranges of $R e$.


Fig. 2. Comparison of coefficients $\lambda$ approximated by model (12) and formulas (2) and (3) for water flow in hydraulically smooth pipes

Figure 2 shows relation (12) which for the test range of $R e$ runs between the upper straight line representing equation (3) according to Nikuradse (at $R e>10^{5}$ ) and the lower straight line representing equation (2) according to Prandtl-Karman.

The quantitative assessment of local resistance coefficient $\zeta$ was based on the solution of a system of Bernoulli equations for cross-sections 1 and 2 and 1 and 3 (figure 1) of a horizontal pipeline with diameter $d$ and with a local obstacle:

$$
\begin{equation*}
\zeta=\frac{2 g}{v^{2}}\left(\Delta h_{1-3}-\frac{13}{6} \Delta h_{2-3}\right) \tag{13}
\end{equation*}
$$

It follows from a dimensional analysis of the local resistance phenomenon and the scope of this research that coefficient $\zeta$ of a single elbow or bend $(\alpha)$ or their systems ( $\alpha_{i}$ ) can be a function of the following quantities (similarity numbers):

$$
\begin{equation*}
\zeta=\zeta\left(\frac{R}{d}, \alpha_{i}, \frac{l_{o}}{d}, z\right) \tag{14}
\end{equation*}
$$

after eliminating from general relation (8) the following constant or slightly variable quantities:

- $R e$ as it was shown that above boundary value $R e_{\text {bdry }} \approx 150000$ coefficient $\zeta$ does not depend on $\operatorname{Re}$ (figure 3);
- $\varepsilon=k / d$ - the polypropylene pipes and pipe fittings are hydraulically smooth which is tantamount to the assumption that $k / d=0$;
- $l_{k} / d$ since this parameter does not change in the model $\left(l_{k} / d=0.7=\right.$ idem $)$;
- $\lambda$ since this coefficient hardly varies in the model above $R e_{\text {bdry }}$, i.e. $\lambda \in\langle 0.0165$, $0.0150\rangle$ (figure 3), in comparison with the considerable variability of coefficient $\zeta$ in the model ( $\zeta \in\langle 0.15,8.0\rangle$ ).


Fig. 3. Selected measurement results for coefficients $\lambda$ and $\zeta$ versus $\operatorname{Re}$ (at $R / d=1.75$ ) for:
(a) segmental bend $\alpha_{i}=90^{\circ}$ (consisting of two single bends $\alpha=45^{\circ}$ ); (b) two alternatingly connected bends $\alpha_{s}=2 \times 90^{\circ}$ (consisting of four bends $\alpha=45^{\circ}$ ); (c) a loop consisting of four conformingly connected bends $\alpha_{s}=4 \times 90^{\circ}$ (consisting of eight bends $\alpha=45^{\circ}$ )

It was shown (Kotowski [5]) that in the test range of Reynolds number (Re $\in$ $\langle 47000,248000\rangle$ ) coefficient $\zeta$ is only slightly dependent on $R e$, i.e. for the forty eight local resistances under investigation the relative percentage error of coefficient $\zeta$ defined as

$$
\begin{equation*}
\delta=\frac{\zeta_{\mathrm{av}}-\zeta_{\text {const }}}{\zeta_{\mathrm{av}}} \cdot 100 \% \tag{15}
\end{equation*}
$$

was $\delta \in\langle+5.3,-1.9\rangle \%$ (on average $+1.15 \%$ ). In equation (15)
$\zeta_{\mathrm{av}}-$ the arithmetic average of the coefficient in the whole measuring range of $R e$,
$\zeta_{\text {const. }}$ - the arithmetic average of the coefficient above $R e_{\text {bdry }}=150000$.
The above proves that the coefficient $\zeta$ is only slightly variable and can be regarded as constant above $R e_{\text {bdry }}=150000$ (figure 3). WERSZKO [8] showed that $R e_{\text {bdry }}$ $\approx 100000$ in the case of rough elbows. This may be due to larger boundary layer disturbances in rough elbows than in the smooth elbows and bends investigated.

Experimentally determined coefficient $\zeta$ of a single bend with $\alpha=90^{\circ}$ and $R / d=$ 0.55 is 1.18 (KOTOWSKI [5]). For a similar PE bend with $\alpha=90^{\circ}$ and $R / d \approx 0.6, \zeta_{\left(90^{\circ}\right)}$ $=1.08$ (SIWIEC [4]). According to the Standard [1], $\zeta_{\left(2 \times 90^{\circ}\right)}=2 \zeta_{\left(90^{\circ}\right)}$ for sequences of conformingly connected double elbows $\alpha_{s}=2 \times 90^{\circ}=180^{\circ}$, i.e., it is recommended to double the value of $\zeta_{i}$ of the individual elbows, whereas for alternatingly connected double elbows the coefficient is $\zeta_{\left(2 \times 90^{\circ}\right)}^{\prime}=4 \zeta_{\left(90^{\circ}\right)}$. In neither case has this been corroborated by the results of the studies. The above recommendations result in overestimated local losses in the pipe fittings under investigation.
a)

b)


Fig. 4. Coefficient $\zeta$ versus $l_{o} / d$ at $R / d=1.75, \alpha=45^{\circ}$ and $\alpha_{i}=90^{\circ}$ in systems of:
(a) alternatingly connected pipe fittings (forming, among others, sinusoidal waves); (b) conformingly connected pipe fittings (forming, among others, spiral loops)

In the case of systems of a few to a dozen or so alternatingly connected elbows or bends (figure 4a) forming sinusoidal waves, the total resistance coefficient is approximately a direct sum ( $n$ ) of coefficients $\zeta_{i}$ of the individual segmental elbows ( $R / d$ $\leq 1)$ or bends $(R / d>1)$, as opposed to systems of conformingly connected pipe fittings forming spiral loops where the total resistance of a loop is much lower than the direct sum of the resistance coefficients of the individual elbows or bends (figure 4b). Generally, for the same $R / d$ ratio, total numerical axial length $l_{o} / d$ of elbows or bends,
sum of central angles $\alpha_{s}$ and number of socket joints $z$, the hydraulic resistance of spiral-loop systems is much lower than that of sinusoidal-wave systems.


Fig. 5. Model (Werszko [8]) of $\zeta$ versus $R / d$ for single elbows and bends ( $90^{\circ}$ ) with the measurements marked for double elbows and bends $\left(2 \times 90^{\circ}\right)$ in conformingly or alternatingly connected systems, including sinusoidal wave ( $4 \times 90^{\circ}$ ) and spiral loop ( $4 \times 90^{\circ}$ )
(• - according to SiwIEC [4] for $90^{\circ}$ PE elbow)
Coefficient $\zeta$ of the studied segmental polypropylene elbows and bends is mainly determined by the $R / d$ ratio (figure 5). The influences of the other parameters of function (16) on $\zeta$ for given $R / d$ are comparable, as shown in table 1.

Table 1
Calculated linear regression of local resistance coefficient $\zeta$ depending on dimensionless similarity numbers for systems of alternatingly connected pipe fittings at $R / d=1.75$

| No. | Dimensionless similarity number $x_{i}$ | Form of formula $\hat{y}=a_{0}+a x$, where: $\hat{y}=\hat{\zeta} \text { and } a_{0}=\zeta_{0}$ | Range of similarity numbers $x_{i}$ | Correlation coefficient $R$ | $\begin{gathered} \text { Critical } \\ \text { value } \\ R_{\mathrm{cr}}(0.01) \end{gathered}$ | Sum of squares of deviations $\sum(\zeta-\hat{\zeta})^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $l_{0} / d$ | $\hat{\zeta}=0.132 l_{o} / d+0.0915$ | $2.75 \leq l_{o} / d \leq 21.99$ | 0.9997 | 0.9587 | 0.00258 |
| 2 | $\alpha_{s}$ | $\zeta=0.00403 \alpha_{s}+0.0915$ | $90^{\circ} \leq \alpha_{s} \leq 720^{\circ}$ | 0.9997 | 0.9587 | 0.00258 |
| 3 | $z$ | $\hat{\zeta}=0.181 z+0.0915$ | $2 \leq z \leq 16$ | 0.9997 | 0.9587 | 0.00258 |
| 4 | $\alpha_{s} / \alpha_{i}$ | $\hat{\zeta}=0.181 \alpha_{s} / \alpha_{i}+0.0915$ | $2 \leq \alpha_{s} / \alpha_{i} \leq 16$ | 0.9997 | 0.9587 | 0.00258 |

Table 1 shows that the value of coefficient $R$ for relations $\zeta=\zeta\left(l_{o} / d\right)$, $\zeta=\zeta\left(\alpha_{s}\right), \zeta=\zeta(z)$ and $\zeta=\zeta\left(\alpha_{s} / \alpha_{i}\right)$ for a given $R / d$ is the same. Thus in this case, parameters $l_{o} / d, \alpha_{s}, z$ and dimensionless similarity number $\alpha_{s} / \alpha_{i}$ have equivalent significance (a significance level of 0.01 ).

To generalize the experimental results for coefficient $\zeta$ of segmental polypropylene elbows and bends the following model without the minimum of function $\zeta=f(R / d)$ (see PIGOTT [9]):

$$
\begin{equation*}
\zeta=\zeta_{0}+\frac{a}{(R / d)^{3}} \tag{16}
\end{equation*}
$$

was adopted. The choice was dictated by the absence of measuring points in area $R / d$ $\in(2.25,4.25)$ and the fact that ratio of $R / d \in\langle 0.5,2.25\rangle$ is the most common for elbows and bends made of plastics. The relationship between $\zeta$ and $R / d$ for systems made up of four alternatingly connected segmental elbows or bends with central angles $\alpha_{i}=90^{\circ}\left(\alpha_{s}=360^{\circ}\right)$ is shown in figure 6.


Fig. 6. $\zeta$ versus $R / d$ for systems of alternatingly connected pipe fittings at $\alpha_{i}=90^{\circ}$ and $\alpha_{s}=4 \times 90^{\circ}=360^{\circ}\left(\Sigma_{\text {squares of deviations }}=0.00453\right)$

In order to determine the influence of the particular dimensionless similarity numbers on $\zeta$, the analyses as the one shown in table 2 were carried out for systems of alternatingly connected pipe fittings at $\alpha_{i}=45^{\circ}$.

As table 2 shows, all the similarity numbers have a more or less significant influence on the accuracy of the formula for the local resistance coefficient of the investigated systems of segmental elbows and bends.

To generalize the experimental results for the systems of alternatingly connected pipe fittings (including ones forming sinusoidal waves) an attempt at a mathematical description of local resistance coefficient $\zeta$ as a function of the particular dimensionless similarity numbers was made. For this purpose the thirty two variants of measurements relating to the systems of alternatingly connected pipe fittings were divided into families depending on the segments' central angles $\alpha_{i} \in\left\{30^{\circ} ; 45^{\circ} ; 60^{\circ} ; 90^{\circ}\right\}$. The following formulas for the local resistance coefficient of such systems were pro-
posed; by combining the families of relations $\zeta$ for $\alpha_{i} \in\left\{30^{\circ}, 45^{\circ}, 60^{\circ}\right\}$ one obtains a sufficiently accurate mathematical expression:

$$
\begin{equation*}
\zeta=-0.0756+\frac{1.635}{(R / d)^{3}}-0.00223 \cdot \alpha_{s} / \alpha_{i}+0.780 \cdot l_{o} / d-0.823 \cdot z \tag{17}
\end{equation*}
$$

at a sum of the squares of the deviations of 0.055 and a standard deviation of 0.054 . Relation (17) holds for the following intervals (closely corresponding to the scope of the studies) of dimensionless similarity numbers:

$$
\begin{array}{cc}
R / d \in<1.75, & 4.25>;
\end{array} \quad \alpha_{s} / \alpha_{i} \in<2.12>;
$$

Table 2
Results of multiple linear regression calculations of local resistance coefficient $\zeta$ depending on dimensionless similarity numbers for systems of alternatingly connected pipe fittings at $\alpha_{i}=45^{\circ}$

| No. | Dimensionless similarity numbers | Form of formula $\hat{y}=a_{0}+a_{1} x_{1}+\ldots+a_{n} x_{n},$ <br> where: $\hat{y}=\hat{\zeta}$ and $a_{0}=\zeta_{0}$ | Correlation significance level | Sum of squares of deviations $\sum(\zeta-\hat{\zeta})^{2}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $R / d$ | $\zeta=1.140-\frac{0.115}{(R / d)^{3}}$ | <0.05 | 7.758 |
| 2 | $R / d, z$ | $\zeta=0.831-\frac{0.068}{(R / d)^{3}}+0.0342 z$ | <0.05 | 7.338 |
| 3 | $R / d, l_{o} / d$ | $\zeta=0.604-\frac{0.028}{(R / d)^{3}}+0.0491 l_{o} / d$ | <0.05 | 6.666 |
| 4 | $R / d, \alpha_{s} / \alpha_{i}$ | $\zeta=-0.139+\frac{0.0495}{(R / d)^{3}}+0.247 \alpha_{s} / \alpha_{i}$ | 0.05 | 1.375 |
| 5 | $R / d, \alpha_{s} / \alpha_{i}, l_{o} / d$ | $\zeta=0.105-\frac{0.0114}{(R / d)^{3}}+0.368 \alpha_{s} / \alpha_{i}-$ | 0.01 | 0.0314 |
| 6 | $R / d, \alpha_{s} / \alpha_{i}, l_{o} / d, z$ | $\begin{aligned} & -0.0794 l_{o} / d \\ & \zeta=0.0162+\frac{1.00813}{(R / d)^{3}}-2.169 \alpha_{s} / \alpha_{i}+ \\ & +6.320 l_{o} / d-6.270 z \end{aligned}$ | 0.01 | 0.0131 |

For systems of alternatingly connected pipe fittings at $\alpha_{i}=90^{\circ}$ the following implicit formula for $\zeta$ :

$$
\begin{equation*}
\zeta=-0.0923+\frac{2.437}{(R / d)^{3}}+0.0654 \cdot \frac{\alpha_{s}}{\alpha_{i}}+\frac{0.0648 \cdot\left(\alpha_{s} / \alpha_{i}\right)^{2}}{(R / d)^{3}}+0.210 l_{o} / d-0.205 z \tag{18}
\end{equation*}
$$

at sum of the squares of the deviations of 0.072 and a standard deviation of 0.077 in parameter ranges: $R / d \in<1.0,4.25>; \alpha_{s} / \alpha_{i} \in<2, \quad 8>; \quad l_{o} / d \in<3.14 ; 53.41>$; $z \in<2,48>$ is proposed.

Also the experimental results for ten geometric arrangements of conformingly connected elbows and bends ( $\alpha_{i}=90^{\circ}$ ), including spiral loops, have been generalized. The following formula:

$$
\begin{equation*}
\zeta=0.0891+\frac{0.968}{(R / d)^{3}}+0.0515 \cdot \alpha_{s} / \alpha_{i}+0.170 \cdot l_{o} / d-0.168 \cdot z \tag{19}
\end{equation*}
$$

at sum of the squares of the deviations 0.056 and a standard deviation of 0.075 in parameter ranges: $R / d \in<1.0,4.25>; \alpha_{s} / \alpha_{i} \in<2, \quad 8>; l_{o} / d \in<3.14,28.27>$; $z \in<2, \quad 24>$ is proposed.

## 4. CONCLUSIONS

Linear resistances and local resistances in hydraulically smooth pipelines and pipe fittings have been quantitatively studied in a wide range of Reynolds number and qualitatively described. In the literature on the subject, there is a lack of experimental results relating to the coefficient of local resistance for systems of conformingly or alternatingly connected two or more elbows $(R / d \leq 1)$ or bends $(R / d>1)$, forming, among others, spiral loops or sinusoidal waves. One of the practical functions of the systems considered is the throttling of the flow of liquid in water supply and sewerage systems.

The following conclusions can be drawn from the research:

1. The (socket-jointed) polypropylene pipes and pipe fittings belong to hydraulically smooth pipes (with imperceptible roughness). As regards the pipelines (with diameter $d=0.071 \mathrm{~m}$ ), quantitative expression (12) for the linear resistance coefficient was found to be in quite good agreement with Prandtl-Karman formula (2).
2. In the test range of Reynolds numbers, $R e \in\langle 47000,248000\rangle$, coefficient $\zeta$ depends to a slight degree on $R e$. Above Reynolds number boundary value $R e_{\text {bdry }} \approx$ 150000 coefficient $\zeta$ of polypropylene pipe fittings can be regarded as invariable.
3. $R / d$ ratio and then equally central angle $\alpha_{i}$ (or ratio of central angles $\alpha_{s} / \alpha_{i}$ ), relative axial length $l_{0} / d$ and number of socket joints $z$ have the most significant effect on the local resistance coefficient of single segmental polypropylene elbows or bends and their systems.
4. The local resistance coefficient of the studied systems which consist of ( $n$ ) segmental elbows or bends and are either conformingly connected or alternatingly connected is radically different: in the former case the values of total resistance coeffi-
cient $\zeta_{(n)}$, which is not a direct sum of resistance coefficients $\zeta_{i}$ of the component pipe fittings, are much lower than in alternatingly connected systems, where the hydraulic resistance of systems built from the same pipe fittings as above is much greater and $\zeta_{(n)}$ is approximately a direct sum of components $\zeta_{i}$.

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## OPORY MIEJSCOWE W KSZTAŁTKACH Z TWORZYW SZTUCZNYCH

Przedstawiono analizę i interpretację wyników badań hydraulicznych oporów liniowych oraz miejscowych w rurach kielichowych oraz kolanach i łukach segmentowych z polipropylenu, w tym w układach dwu i więcej kształtek połączonych szeregowo. Odnośnie do współczynnika oporów liniowych badanych rurociagów otrzymano znaczną zgodność wyników aproksymacji ze wzorem Prandtla-Karmana. Przedstawiono zależności jakościowe i wzory ilościowe dla współczynnika oporów miejscowych w układach kolan i łuków zgodnych oraz przemiennych.


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