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Theoretical and experimental investigations on coupling-out mirrors with hole for a cw CO_2 laser

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In this paper investigations on output mirror transmittance and output power of a cw CO_2 laser as a function of coupling-out hole diameter are described. While considering the maximum output laser power the optimum value of mirror transmittance and the optimum diameter of coupling-out hole have been theoretically calculated. The results obtained were verified experimentally.

1. Introduction

The laser optical cavities equipped with the output totally reflecting mirror with coupling-out hole in its centre are of importance if the lasers operate in the infra-red region. Such a structure of the laser cavity permits an easy regulation of output mirror transmittance through the choice of coupling-out hole diameter. It is especially convenient in the high power laser structures, such as continuous wave CO_2 lasers. These inexpensive mirrors are moreover characterized by a high thermal resistance.

In this paper theoretical and experimental investigations on the properties of concave output mirror with coupling-out hole are presented. Theoretical results were experimentally verified using a low power cw CO_2 laser.

2. Transmittance of output mirror with hole

Assuming the Gaussian intensity distribution of the laser beam at the surface of the output mirror (see Fig. 1) the transmittance t can be obtained as a function of coupling-out hole diameter $2r$ [1]. The output power P of the laser

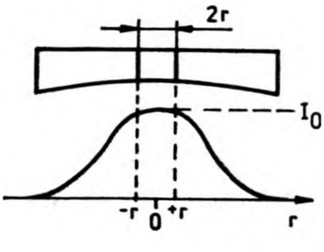

Fig. 1. Ouptut mirror with hole and distribution of laser beam intensity I at mirror surface, vs. the distance r from the beam axis

radiation is

$$P = tP_0 \tag{1}$$

where P_0 is the total power inside the laser cavity. The intensity $I(r)$ of the laser beam inside the laser cavity may be described by the equation

$$I(r) = I_0 \exp[-2r^2/w^2] \tag{2}$$

where I_0 is the intensity of the laser beam at its axis, r — the distance from the beam axis, $2w$ — the diameter of the laser beam inside resonator.

Integrating Eq. (2) over the radius r from 0 to ∞ we get the total power of the radiation inside the laser cavity

$$P_0 = \pi w^2 I_0/2. \tag{3}$$

The output power contained in a hole area of radius r is

$$P = \int_0^r I(r) 2\pi r \, dr = \frac{1}{2} w^2 I_0 [1 - \exp(-2r^2/w^2)]. \tag{4}$$

Combining Eqs. (1), (3) and (4) we get the expression which describes the transmittance t of output mirror in term of coupling-out hole diameter $2r$

$$t = 1 - \exp[-2r^2/w^2]. \tag{5}$$

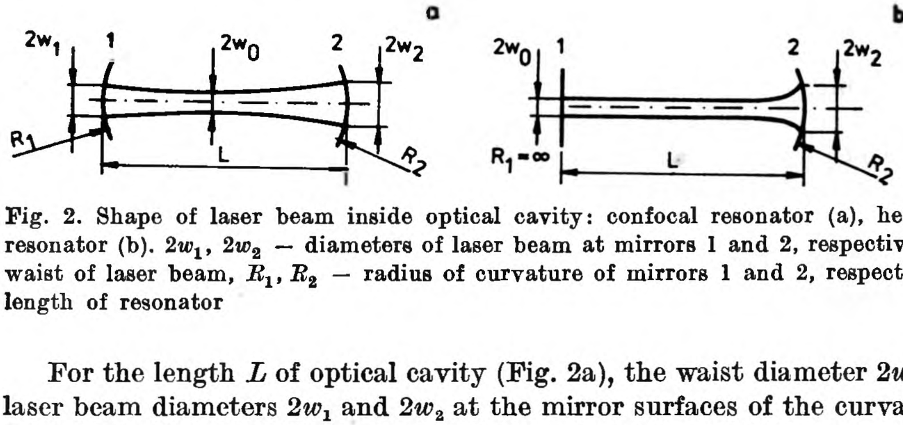

Fig. 2. Shape of laser beam inside optical cavity: confocal resonator (a), hemispherical resonator (b). $2w_1$, $2w_2$ — diameters of laser beam at mirrors 1 and 2, respectively, $2w_0$ — waist of laser beam, R_1, R_2 — radius of curvature of mirrors 1 and 2, respectively, L — length of resonator

For the length L of optical cavity (Fig. 2a), the waist diameter $2w_0$ and the laser beam diameters $2w_1$ and $2w_2$ at the mirror surfaces of the curvature radii R_1 and R_2, respectively, in a general case, are given by [2]:

$$w_0^4 = (\pi/\lambda)^2 \frac{L(R_1-L)(R_2-L)(R_1+R_2-L)}{(R_1+R_2-2L)^2}, \tag{6}$$

$$w_1^4 = (\lambda R_1/\pi)^2 \frac{L(R_2-L)}{(R_1-L)(R_1+R_2-L)}, \tag{7}$$

$$w_2^4 = (\lambda R_2/\pi)^2 \frac{L(R_1-L)}{(R_2-L)(R_1+R_2-L)} \tag{8}$$

where λ is the wavelength of the laser radiation.

The total reflecting mirror was replaced by the plane diffraction grating in the considered optical cavity. Treating the grating as a plane mirror with the curvature of radius $R_1 = \infty$ (Fig. 2b) we obtain

$$w_0^4 = w_1^4 = (\lambda/\pi)^2(R_2-L)L, \tag{9}$$

$$w_2^4 = (\lambda/\pi)^2R_2^2L/(R_2-L). \tag{10}$$

Since in the considered optical cavity the laser power is coupled-out through the mirror with the radius of curvature $R_2 = R$, we may replace w in Eq. (5) by w_2 to obtain finally

$$t = 1-\exp\left\{-\frac{2r^2[L(R-L)]^{1/2}}{(\lambda/\pi)LR}\right\}. \tag{11}$$

Figure 3 shows the variations of the output mirror transmittance t in term of the coupling-out hole (radius r), where the radius R is a parameter. These theoretical curves have been calculated from Eq. (11) for optical cavity 1.07 m long.

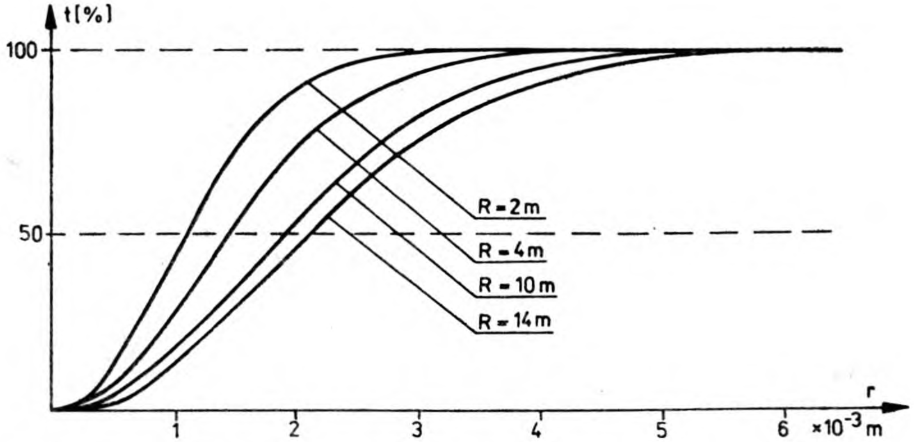

Fig. 3. Theoretical plots of output mirror transmittance t vs. the coupling-out hole of radius r (R – radius of mirror curvature)

3. Output power of laser

In this Section we consider the laser output as a function of the output mirror transmittance and the active medium parameters.

Let us consider the active medium enclosed within two mirrors: the transmitting mirror 1 with the effective reflectance $r_1 = 1-a-t$ (where a is the net intracavity losses) and the total reflecting mirror 2 with the effective reflectance

$r_2 = 1$ separated by the distance L. The laser beam of the initial relative intensity $\beta_1 = I_1/I_s$ (I_1 and I_s being the intracavity radiation intensity and the saturation intensity, respectively) that starts from the mirror 1 reaches the value $\beta_2 = I_2/I_s$ and so on (Fig. 4). If the output laser beam intensity coupled-out through the mirror 1 is I, then we can write

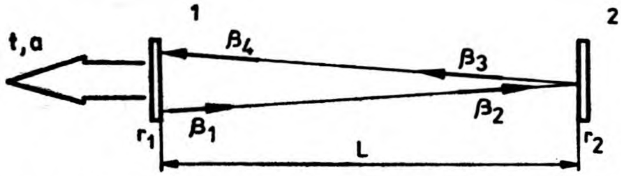

Fig. 4. Growth of laser beam intensity during each transit inside the optical cavity of the length L: t — transmittance of output mirror, a — all losses of mirror except for the transmission ones, r_1 and r_2 — reflectance of output mirrors 1 and 2, respectively, β_1, β_2, β_3 and β_4 — relative intensities of laser beam

$$I/I_s = \beta_3 t, \tag{12}$$

or, for the output power P,

$$P/P_s = \beta_3 t \tag{13}$$

where P_s is the saturation power.

According to RIGROD [3], we have finally

$$P = P_s t\beta_3 = P_s t\,\frac{2g_0L+\ln(1-a-t)}{2(a+t)} \tag{14}$$

where g_0 is the unsaturated gain coefficient.

This expression is obtained at the assumption that an emission laser line is broadening homogeneously. The broadening of the emission line for the investigated cw CO_2 laser will be determined.

4. Broadening of the CO_2 laser emission line

In order to determine the broadening of a CO_2 laser emission line the gas temperature T in discharge tube should be estimated. According to [4], the temperature of a CO_2 laser gas mixture can be approximately calculated from the following equation:

$$T^2 = T_0^2+0.255\ A^{-1}I_dE \tag{15}$$

where T_0 is the temperature of the discharge tube wall, I_d is the current intensity, E — the electric field intensity inside the laser tube, $A = \Delta\lambda_\Sigma/\Delta T$ — the directional coefficient of the thermal conductivity λ_Σ vs. the temperature T

For the three-component gas mixture of the CO_2 laser, like in our experiment the summary coefficient of thermal conductivity may be calculated from [5]:

$$\lambda_\Sigma = \frac{\lambda_{CO_2}}{1+0.81(p_{N_2}/p_{CO_2})+0.23(p_{He}/p_{CO_2})} + \frac{\lambda_{N_2}}{1+1.4(p_{CO_2}/p_{N_2})+0.34(p_{He}/p_{N_2}} + \frac{\lambda_{He}}{1+0.34(p_{CO_2}/p_{He})+2.7(p_{N_2}/p_{He})} \tag{16}$$

where λ_{CO_2}, λ_{N_2} and λ_{He} are the thermal conductivity coefficients for CO_2, N_2 and He molecules, respectively; p_{CO_2}, p_{N_2} and p_{He} are the partial pressures of the CO_2 laser gas mixture components. In our case $p_{CO_2}:p_{N_2}:p_{He} = 1.7:1.7:6.6$. Taking the values of λ_{CO_2}, λ_{N_2} and λ_{He} at different temperatures T_1 and T_2 from [6] we obtain

$$A = \frac{\Delta\lambda_\Sigma(T)}{\Delta T} = \frac{\lambda_\Sigma(T_2)-\lambda_\Sigma(T_1)}{T_2-T_1} \approx 2.3\cdot 10^{-6}\,\mathrm{W/cm\ K}. \tag{17}$$

The electric field intensity E inside the discharge tube, at the discharge length $l_e = 0.75$ m, the voltage drop across the tube $U_{ac} = 7750$ V and the voltage cathode drop $U_c = 450$ V [7], is

$$E = \frac{U_{ac}-U_c}{l_e} \approx 100\ \mathrm{V/cm}. \tag{18}$$

According to Eqs. (15)–(18) at the laser tube wall temperature $T_0 \approx 288$ K and at the optimal discharge current $I_d = 20$ mA, the temperature of the gas mixture of investigated CO_2 laser is $T \approx 550$ K.

The spectral width of an inhomogeneous broadening emission line (Doppler broadening) $\Delta\nu_D$ may be calculated from [8]:

$$\Delta\nu_D = 2\nu_0\left(\frac{2kT\ln 2}{Mc^2}\right)^{1/2} \tag{19}$$

where k is the Boltzmann constant ($k = 1.35805\cdot 10^{-23}$ J/K), c — the light velocity ($c = 3\cdot 10^8$ m/s), ν_0 — the frequency of the emission line centre (ν_0(P20) $\approx 2.83\cdot 10^{13}$ Hz [9]).

For the CO_2 molecules at $T \approx 550$ K we get

$$\Delta\nu_D(CO_2) \approx 70\ \mathrm{MHz}. \tag{20}$$

It is well known that in a homogeneous broadening, $\Delta\nu_L(CO_2)$, of CO_2 laser emission line the CO_2-CO_2, CO_2-He and CO_2-N_2 collisions play the most important role [10]. The homogeneous broadening $\Delta\nu_L(CO_2)$, depends on the pressure p and temperature T of the CO_2 laser gas mixture and may be estimated by means of expression [11]:

$$\Delta\nu_L = 7.58(\gamma_{CO_2} + 0.73\ \gamma_{N_2} + 0.64\ \gamma_{He})\ p(300/T)^{1/2} \tag{21}$$

where:

$$\gamma_{CO_2} = \frac{N_{CO_2}}{N_{CO_2}+N_{N_2}+N_{He}},$$
$$\gamma_{N_2} = \frac{N_{N_2}}{N_{CO_2}+N_{N_2}+N_{He}}, \tag{22}$$
$$\gamma_{He} = \frac{N_{He}}{N_{CO_2}+N_{N_2}+N_{He}}.$$

For the optimal gas mixture composition $N_{CO_2}:N_{N_2}:N_{He} = 1:1:4$, the optimal total pressure $p = 1330$ Pa (10 Torr) and the gas mixture temperature $T \approx 550$ K we get

$$\Delta\nu_L(CO_2) \approx 40\ \text{MHz}. \tag{23}$$

According to [12] the broadening of the emission line of the CO_2 laser active medium can be treated as being homogeneous, because the values of $\Delta\nu_D(CO_2)$ and $\Delta\nu_L(CO_2)$ have the same order of magnitude. Hence, for calculation of the CO_2 laser output power the expression (14) can be used.

5. Experimental arrangement and results

The experimental investigations were carried out using the cw CO_2 laser of the following structure:

The discharge laser tube 60 cm long and 1.4 cm in diameter was operated with nonflowing gas at the discharge current of 20 mA and the voltage drop tcross the tube of 7750 V. In order to reduce the output power fluctuations ahe discharge current stabilizer was used. The gas mixture of $CO_2:N_2:He$ gases of partial pressures given at 1:1:4, respectively, was applied at the total pressure of 10 Torr. Each end of the discharge tube was closed with a NaCl window places at Brewster's angle and covered with antihygroscopic CaF_2 layer. The laser radiation was coupled out through the hole in the centre of the gold-coated concave mirror of the radius curvature $R = 10$ m. The coupling-out hole in the glass mirror was made by ultrasonic machining method. The opposite side of the laser cavity was terminated by the diffraction grating placed at the distance $L = 1.07$ m from the output mirror. The plane diffraction grating was used to select laser lines. All the measurements were performed at P20 line of 10.4 μm band. The output laser power was measured by noncooling CdHgTe detectors [13], scaled by a bolometer. The experimental investigations were performed on coupling-out holes of the radii 0.50, 0.75, 1.00, 1.25 and 1.50 mm.

The output laser power $P(r, R)$ and the transmittance $t(r, R)$ of the concave output mirror ($R = 10$ m) with hole vs. the coupling-out hole of the radius r are plotted in Fig. 5 from the formulae (11) and (14), respectively. In this figure the experimental results are also shown [14]. The experimental data are in satisfactory agreement with the theoretical results.

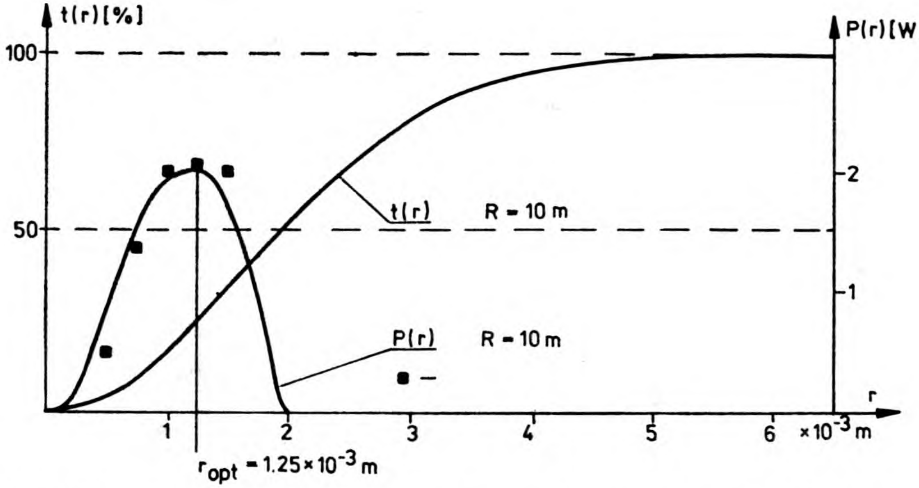

Fig. 5. Theoretical plots of transmittance $t(r, R)$ and output power $P(r, R)$ vs. the coupling-out hole radius r at the radius of the output mirror curvature $R = 10$ m. The experimental results are also marked

6. Conclusions

All the above considerations are valid only at a single-transverse-mode laser operation. At the upper order transverse modes and at noncorrect adjustment of the optical cavity the expressions (11) and (14) become not valid. Some aspects of these cases are published in another paper [15].

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Теоретические и эксперименталные исследования выводных зеркал с отверстием в CO_2 лазерах

В работе представлены исследования пропускания выходных зеркал и выходной мощности CO_2 лазера в зависимости от диаметра ввод-выводного отверствия. На основе максимальной выходной мощности лазера теоретически вычислены оптимальное пропускание зеркала и оптимальный диаметр ввод-выводного отверстия. Полученные результаты проверены экспериментально.