## Imaging of one and two slits incoherent apodized optical system

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The imaging of one and two slit objects in incoherent and apodized optical system has been analysed. The apodizers of the types $1 / 2\left(1+\varrho^{2}\right), 1-\varrho^{2}$ and tche one proposed by Hamed [1] were assumed. The latter consisted of four irscles with uniform transparency located symmetrically in the aperture. For the prescribed systems the point spread function and the image intensity ditribution for one and two slits object have been calculated.

A stationary and linear optical systems with incoherent illuminator mah ea described as a convolution of intensity distribution $I(x)$ of the object wity bn intensity point spread function $S(x)$ of the optical system

$$
\begin{equation*}
I\left(x^{\prime}\right)=\int_{-\infty}^{\infty} I(x) S\left(x^{\prime}-x\right) d x \tag{1}
\end{equation*}
$$

The relation between point spread function $S(x, y)$ and pupil function $T(\xi, \eta)$ as given by the squared modulus of the two dimensional Fourier transform. This relation may be simplified to the squared modulus of Hankel transform


Fig. 1. The amplitude transmission for filters I, III, IV


Fig. 2. The filter composed of some circular apertures and external ring (filter II)
in the case of rotational symmetry of the system

$$
\begin{equation*}
S\left(r^{\prime}\right)=\left|\int_{-\infty}^{\infty} T(\varrho) J_{0}\left(\frac{k \varrho r^{\prime}}{f}\right) \varrho d \varrho\right|^{2}=|P S F|^{2} \tag{2}
\end{equation*}
$$

For the amplitude apodizers of the types (Figs. 1, 2):
the intensity point spread function takes the forms:

$$
\begin{aligned}
& S_{\mathrm{I}}\left(r^{\prime}\right)=\left[\frac{2 J_{1}\left(r^{\prime}\right)}{r^{\prime}}\right]^{2} \\
& \mathrm{~S}_{\mathrm{I}}(W)=\left\{2 \pi \varrho_{\max }^{2}\left[\frac{\Delta}{\varrho_{\max }} J_{0}(W)+\frac{1}{4} \frac{J_{1}(1 / 4 W)}{(W / 4)} \cos ^{2}\left(\frac{3 W}{8}\right)\right]\right\}^{2}
\end{aligned}
$$

$$
W=k \varrho_{\max } r^{\prime \prime} \mid f, k=2 \pi \lambda, r^{\prime \prime \prime}=x^{\prime 2}+y^{\prime 2}
$$

$$
\begin{equation*}
S_{\mathrm{III}}\left(r^{\prime}\right)=\left\{J_{1}\left(r^{\prime}\right)\left[\frac{1}{r^{\prime}}-\frac{4}{r^{\prime 3}}\right]+\frac{2}{r^{\prime 2}} J_{0}\left(r^{\prime}\right)\right\}^{2} \tag{4}
\end{equation*}
$$

$$
S_{\mathrm{ID}}\left(r^{\prime}\right)=\left[\frac{8 J_{1}\left(r^{\prime}\right)}{r^{\prime 3}}-\frac{4 J_{0}\left(r^{\prime}\right)}{r^{\prime 2}}\right]^{2}
$$

$$
\begin{align*}
& \mathrm{I} T(\varrho)= \begin{cases}1 & |\varrho| \leqslant 1 \\
0 & |\varrho|>1\end{cases} \\
& \text { II } T(\varrho)=\delta(\Delta)+\sum_{i=1}^{\dot{4}} T_{i}(\varrho), \quad[1], \Delta=\varrho-\varrho_{\max } \\
& T_{i}(\varrho)=\left\{\begin{array}{l}
1\left|\varrho-\varrho_{i}\right| \leqslant 1 / 4 \varrho_{\max } \\
0 \text { otherwise },
\end{array}\right. \\
& \varrho_{i}=x_{i}^{2}+y_{i}^{2}, \quad x_{i}=y_{i}=\left(\frac{3}{4} \sqrt{2}\right) \varrho_{\max }, \quad \varrho_{i}=\frac{3}{4} \varrho_{\max } . \\
& \text { III } T(\varrho)= \begin{cases}1 / 2\left(1+\varrho^{2}\right)|\varrho| \leqslant 1, \\
0 & |\varrho|>1,\end{cases}  \tag{3}\\
& \text { IV } T^{\prime}(\varrho)= \begin{cases}1-\varrho^{2} & |\varrho| \leqslant 1, \\
0 & |\varrho|>1 .\end{cases}
\end{align*}
$$



Fig. 3a. The point spread functions for filters I,III, IV


Fig. 3b. The point spread function for filter II, PSF, $1-J_{0}(W), 2-J_{1}(W / 4) \cos ^{2}(3 y / 8) /(W / 4)$

The corresponding point spread function has been drawn in Figs. 3a, b, the influence of the external ring for the aperture from Fig. 2 being presented in Fig. 3c. The intensity distribution for one and two object are shown in Figs. $4 a-c$, and Figs. 5a-c, respectively. From the results obtained we see that, for


Fig. 3c. The influence of the width $\Delta$ of the external ring of the filter II on the corresponding point spread function


Fig. 4a




Fig. 4. The image intensity distributions for one slit object and different filters. The widths of the slit are: $2 b=1$ (a), $2 b=2$ (b), and $2 b=3$ (c)


Fig. 5a


Fig. 5. The same as in Fig. 4, but for two slit object. Distances between slits, equal to the slits, are: $2 b=1$ (a), $2 b=2$ (b) and $2 b=3$ (c)
$2 b=1$ and filters I, III, IV (Figs. 4a, 5a), the images are not recognizable. For $2 b=2$, the images of one and two slit objects (Figs. $4 b, 5$ b) are similar to those of one- and two-point objects, respectively [2]. For all the objects considered (Figs. 4,5) the best recognizability is given by the filter II.

## References

[1] Hamed A. M., Optica Applicata 13 (1983), 265.
[2] Magiera A., Pluta M., Optica Applicata 12 (1982), 363.

