Optica Applicata, Vol. XIV, No. 4, 1984

Computer modelling of three-dimensional Fresnel-diffraction pattern at circular, rectangular and square apertures

Y. P. KATHURIA*

Institute of Applied Physics, Schlossgarten Str. 7, 61 TH Darmstadt, West Germany.

1. Introduction

Fresnel diffraction of various apertures has been a subject of interest in laser physics, in optics and in the antenna theory. Circular, rectangular, square apertures and Fresnel number play an important role in these related phenomena. This aspect has been dealt by many authors with different parameters of interest. But only few authors have tried to study the effect of diffraction dependence on Fresnel number. In the past few years Fresnel number played an important role in defining the region of the diffraction field. For example, three regions can be defined as follows (Fig. 1):

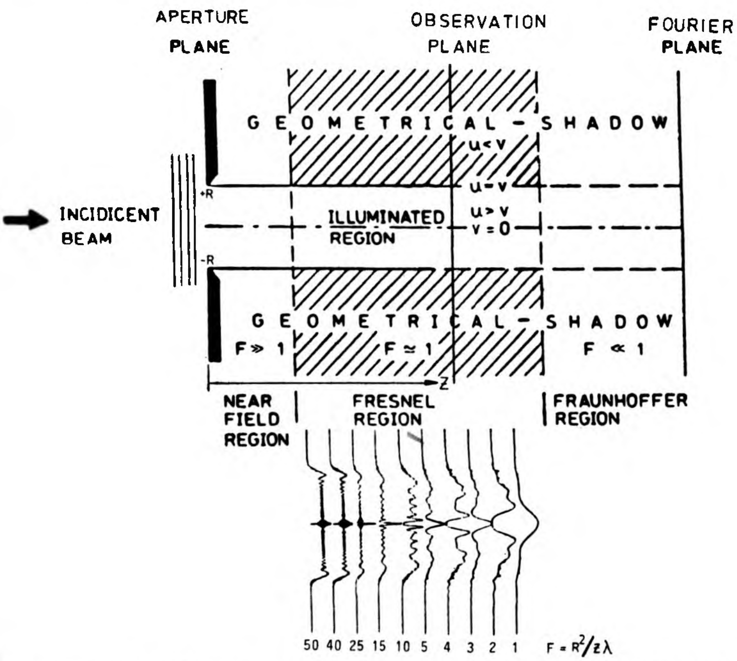

Fig. 1. Block diagram indicating the different regions and the boundaries of diffracted field for circular aperture

* The author is on long leave at the following address: Ashoo Industries Ltd., Faridabad–121001, India.

i) Near field region: region with the Fresnel number much greater than unity, i.e., $F \gg 1$.

ii) Fresnel region: this is the region where the Fresnel number is of the order of unity, i.e., $F \simeq 1$.

iii) Fraunhoffer region: in this region the Fresnel number is much smaller than unity, i.e., $F \ll 1$.

Therefore it is of great importance to know the structure of the Fresnel diffraction produced by these apertures, the Fresnel number being a parameter. This communication describes analytically the generation of such 3D-intensity patterns. The algorithm used here involves the exact solution of the Fresnel-Kirchhoff diffraction integral in terms of the Fresnel-integrals and Lommel functions and is much faster and more accurate than the direct numerical solution of the wave equation [1].

2. Analysis

Considering the first order approximation, the field diffracted by an aperture in the Fresnel region can be approximated by [2, 3]

$$U_p = -\frac{j}{\lambda z}\exp(jkz)\int\int_{-\infty}^{+\infty} U_A \exp\left\{\frac{jk}{2z}\{(x_1-x_0)^2+(y_1-y_0)^2\}\right\}dx_1 dy_1 \tag{1}$$

where: $U_A = U_A(x_1, y_1)$ is the field in the aperture plane,
$z = |z_1 - z_0|$ is the distance between the two planes,
$(x_1, y_1); (x_0, y_0)$ represent the coordinate system in the plane of the aperture and in the plane of observation, respectively,
$k = 2\pi/\lambda$ (λ – the wavelength of radiation).

For simplicity we have assumed a uniform illumination of the apertures, i.e., $U_A = 1$

2.1. Circular aperture

Writing the integral (1) for a circular aperture of radius R as

$$U_{\mathrm{circ}} = \frac{j}{\lambda z}\exp(jkz)\int\int_{\mathrm{circ}} \exp\left\{\frac{jk}{2z}\{(x_1-x_0)^2+(y_1-y_0)^2\}\right\}dx_1 dy_1, \tag{2}$$

and transforming it into polar coordinate system [3] its solution can be written in terms of normalized intensity in the following form [4]:

$$\begin{aligned} I_{\mathrm{circ}} &= U^*_{\mathrm{circ}} U_{\mathrm{circ}} \\ &= I_1|_{u<v} + I_2|_{u=v} + I_3|_{u>v} + I_4|_{v=0}\,. \end{aligned}$$

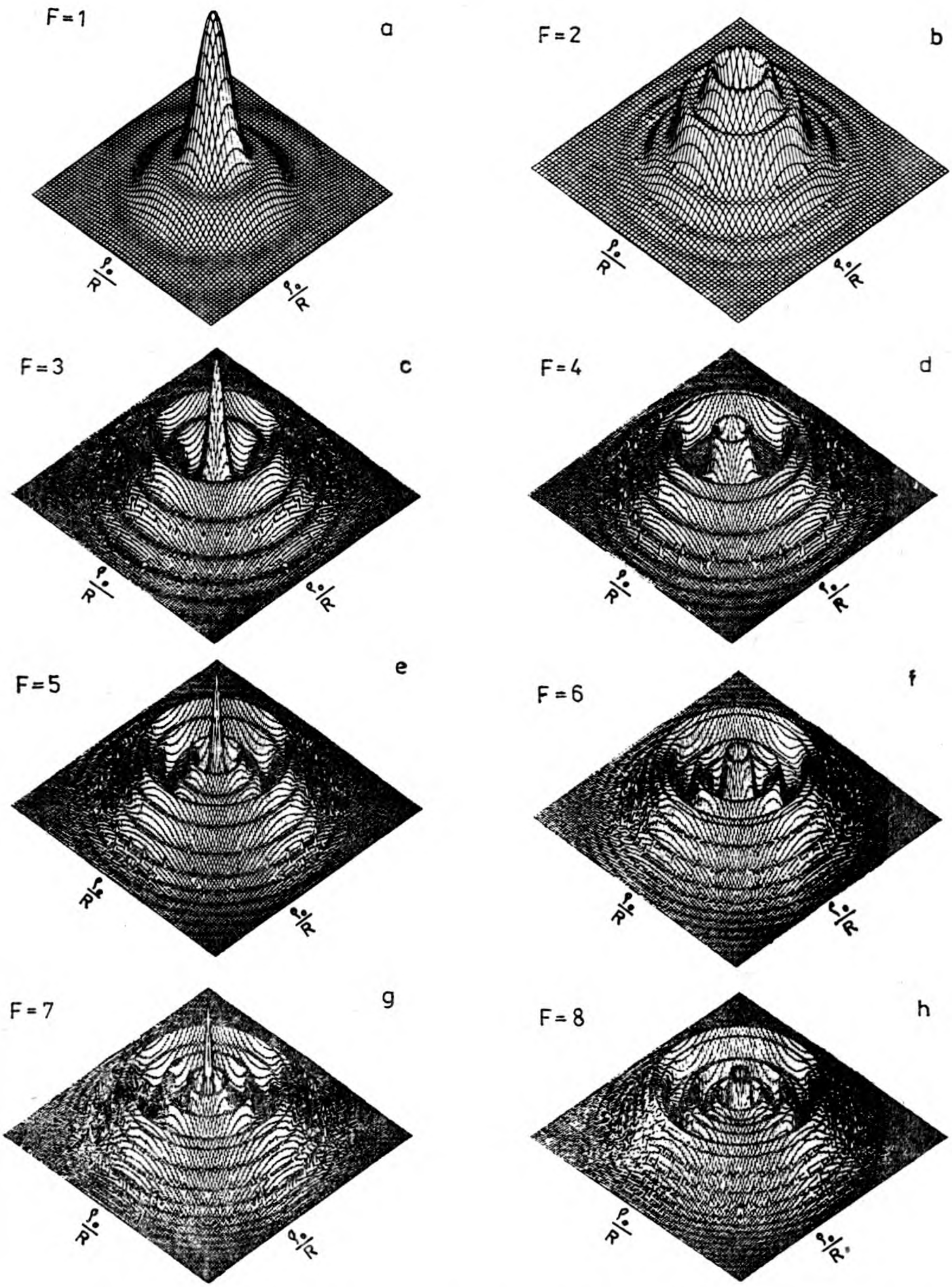

Fig. 2. Normalized intensity in arbitrary units vs. normalized coordinates (ϱ_0/R) along x- and y-axes for circular aperture with respective Fresnel numbers: $F = 1$ (a), $F = 2$ (b), $F = 3$ (3), $F = 4$ (d), $F = 5$ (e), $F = 6$ (f), $F = 7$ (g) and $F = 8$ (h)

Here I_1, I_2 and I_3, I_4 are the intensities corresponding to the four different boundaries of the diffraction field, as shown in Fig. 1:

— for geometrical shadow

$$I_1 = U_1^2(u, v) + U_2^2(u, v),$$

— for boundary of geometrical shadow and illuminated region

$$I_2 = \frac{1}{4}\,[1 - 2J_0(u)\cos(u) + J_0^2(u)],$$

— for illuminated region

$$I_3 = 1 + V_0^2(u, v) + V_1^2(u, v) - 2V_0(u, v)\cos\left(\frac{u}{2} + \frac{v^2}{2u}\right) - 2V_1(u, v)\sin\left(\frac{u}{2} + \frac{v^2}{2u}\right),$$

— for optical axis

$$I_4 = \sin^2(u/4)$$

where $u = 2\pi F$, $v = 2\pi F(\varrho_0/R)$, $\varrho_0 = \sqrt{x_0^2 + y_0^2}$ ($F = R^2/\lambda z$ – Fresnel number), U_1, U_2 and V_0, V_1 are the Lommel functions.

Figures 2a–h show the computer-generated 3D-Fresnel diffraction pattern of the normalized intensity vs. normalized coordinate (ϱ_0/R) for circular aperture with Fresnel number $F = 1, 2, 3, \ldots$, respectively. It is observed from these diagrams that the intensity becomes maximum for odd Fresnel number, and minimum for even Fresnel number. It is seen, moreover, that the number of peaks is equal to the Fresnel number. These results agree very well with the results of CAMPILLO [1], who generated the 2D-curve numerically. The latter procedure requires more computational time than that employed here which uses exact solution, consumes less computational time for generating 3-D intensity patterns and gives more information.

2.2. Rectangular aperture

Applying the above integral (1) to a rectangular aperture we get

$$U_{\mathrm{rect}} = -\frac{j}{\lambda z}\exp(jkz)\int_{-a}^{a}\int_{-b}^{b}\exp\left\{\frac{jk}{2z}\,\{x_1 - x_0)^2 + (y_1 - y_0)^2\}\right\}dx_1\,dy_1 \tag{3}$$

This integral can be solved. Its solution for the normalized intensity is given by GOODMAN [5]

$$\begin{aligned} I_{\mathrm{rect}} = U^*_{\mathrm{rect}}\,U_{\mathrm{rect}} = \frac{1}{4}\,&\{[C(\xi_2) - C(\xi_1)]^2 + [S(\xi_2) - S(\xi_1)]^2\} \\ &\times\{[C(\eta_2) - C(\eta_1)]^2 + [S(\eta_2) - S(\eta_1)]^2\} \end{aligned} \tag{4}$$

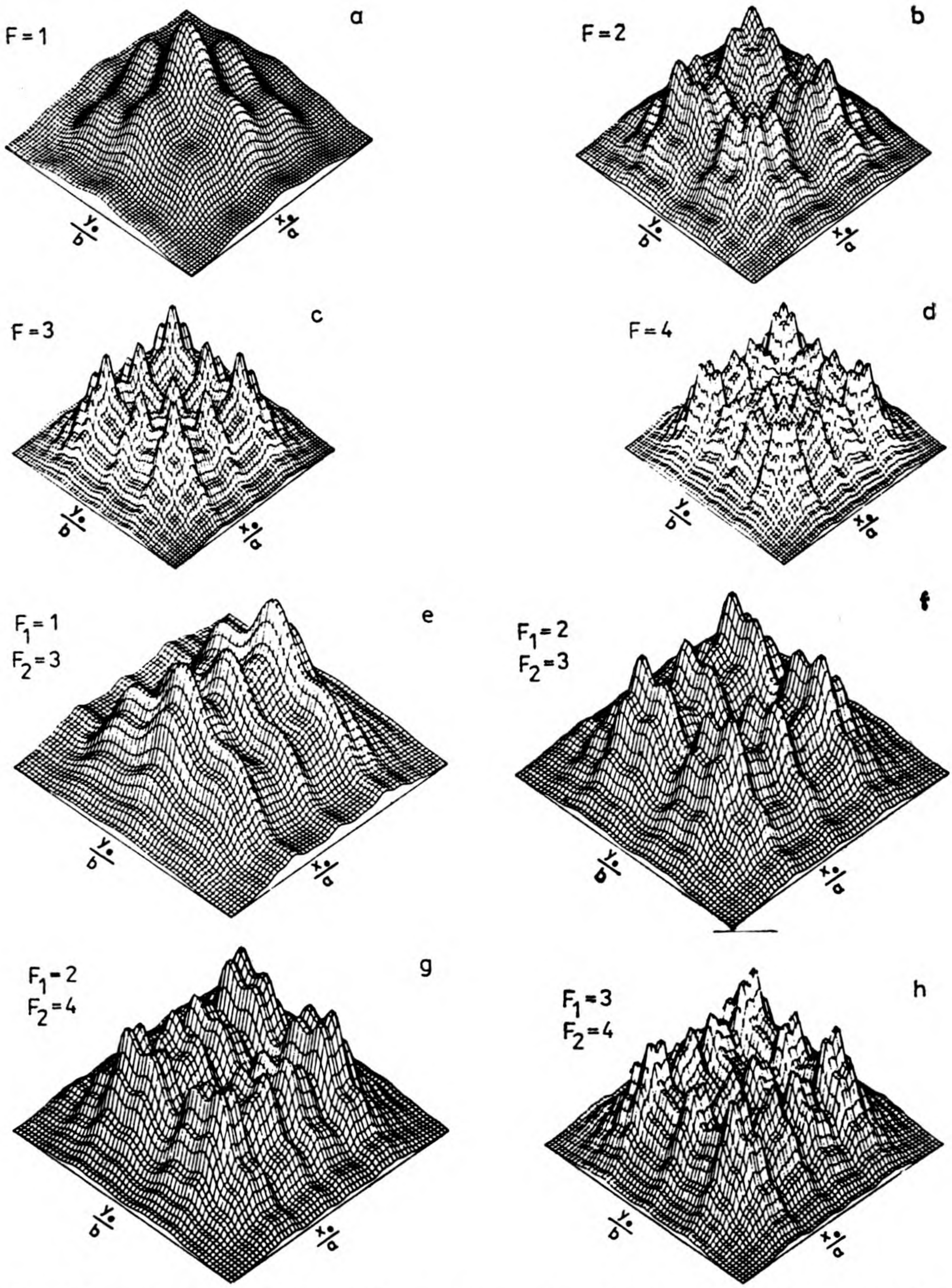

Fig. 3. Normalized intensity in arbitrary units vs. normalized coordinates (x_0/a, y_0/b) along x- and y-axes for: square aperture with respective Fresnel numbers: $F = 1$ (a), $F = 2$ (b), $F = 3$ (c), $F = 4$ (d), and for rectangular aperture with respective Fresnel numbers: $F_1 = 1$, $F_2 = 3$ (e), $F_1 = 2$, $F_2 = 3$ (f), $F_1 = 2$, $F_2 = 4$ (g), $F_1 = 3$, $F_2 = 4$ (h)

where: $\xi_1 = -\sqrt{2F_1}\left(1+\frac{x_0}{a}\right)$, $\xi_2 = \sqrt{2F_1}\left(1-\frac{x_0}{a}\right)$,

$$\eta_1 = -\sqrt{2F_2}\left(1+\frac{y_0}{b}\right),\quad \eta_2 = \sqrt{2F_2}\left(1-\frac{y_0}{b}\right),$$

C and S are the Fresnel integrals, ($F_1 = a^2/\lambda z$, $F_2 = b^2/\lambda z$ being Fresnel numbers for rectangular aperture).

This is the general relation to calculate the intensity distribution of the rectangular aperture. For $x_0 = y_0 = 0$, $b/a = 0.5$ and $F_1 = 1, 2, \ldots, 20, \ldots$ the same Eq. (4) can be used for determining the intensity distribution along the optical axis.

2.3. Square aperture

This is a special case of a rectangular aperture for $a = b$, i.e., $F_1 = F_2 = F$,

$$I_{\text{square}} = |I_{\text{rect}}| \quad \text{for } a = b.$$

Figures 3a–h show a computer-generated 3D-plots of the normalized intensity vs. the normalized coordinates (x_0/a, y_0/b) along the two axes of the square and rectangular apertures for different values of the Fresnel numbers. It is interesting to observe that the number of peaks is equal to the product of the Fresnel numbers F_1 and F_2. Furthermore, the position of the peak can be ascertained with the element (p, q), where $p = 1, 2, \ldots, F_1$ and $q = 1, 2, \ldots, F_2$, respectively. Besides, alternative minima and maxima are found in the centre of the square of rectangular pattern for an even or odd product of F_1 and F_2.

3. Conclusion

We have presented the results which demonstrate the systematic effect of the Fresnel number on the 3D-Fresnel diffraction patterns at circular, rectangular and square apertures. These results show that for circular aperture, the number of maxima is equal to the Fresnel number, whereas for rectangular or square apertures, these peaks are represented through the matrix elements of the two Fresnel numbers.

References

[1] CAMPILLO A. J. et al., Appl. Phys. Lett. **23** (1973).
[2] FEIOCK F. D., J. Opt. Soc. Am. **68** (1978).
[3] KATHURIA Y. P. et al., Proc. SPIE Vol. 288, 1981.
[4] BORN M., WOLF E., *Principles of optics*, Pergamon Press, 1959.
[5] GOODMAN J. W., *Introduction to Fourier optics*, McGraw Hill Inc., 1968.

Received April 4, 1984