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Waves guided by nonlinear interface

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Guided TM waves by an interface between two nonlinear media with Kerr-type nonlinearity are analyzed. The possibility of the existence of the waves of the lateral type for the above case is presented. The dispersion relation has also been investigated.

The electromagnetic field propagating at a boundary between different media exhibits a number of interesting properties which are the subject of many studies in literature [1]–[6].

In the case of linear media, this field can be represented by different types of waves, such as lateral, surface or leaky ones [2]–[4].

In this connection the following questions arise:

– what properties has the electromagnetic field propagating along an interface between nonlinear media,

– can there exist, in these circumstances, some counterparts of waves characteristic of linear media,

– if so, then what conditions and relations should be satisfied for the fields to exist?

It is widely known that in this case there can excists a class of nonlinear surface waves [7]–[11]. The properties of these waves and the conditions for their existence are now well recognized [9]. The problem of the existence of other types of electromagnetic waves at the boundary between two nonlinear media, however, has not been satisfactorily investigated yet.

The objective of this paper is to study the possibility of the existence of the waves type: the lateral and/or leaky ones for the case mentioned above. Only TM modes will be considered.

Two semi-infinite regions, $x > 0$ and $x < 0$, separated by a plane interface $x = 0$ will be considered, respectively. Each of the regions contains a uinaxial nonlinear medium of the Kerr type described by a tensor of dielectric permeability ${}^j\hat{\varepsilon}$ ($j = 1, 2$)

$$ {}^j\hat{\varepsilon} = \begin{pmatrix} {}^j\hat{\varepsilon}_{11} & 0 & 0 \\ 0 & {}^j\varepsilon_{22} & 0 \\ 0 & 0 & {}^j\varepsilon_{33} \end{pmatrix}, \tag{1} $$

where:

$$ {}^j\varepsilon_{11} = {}^j\varepsilon, \tag{2} $$

$$ {}^j\varepsilon_{22} = {}^j\varepsilon_{33} = {}^j\varepsilon_1 + \alpha_j |{}^jE_z|^2. \tag{3} $$

In the latter expressions, ${}^{j}\varepsilon$ and ${}^{j}\varepsilon_1$ denote the linear permeabilities, α_j is the nonlinear coefficient of $x > 0$ (${}^{1}\varepsilon$, ${}^{1}\varepsilon_1$, α_1) and $x < 0$ (${}^{2}\varepsilon$, ${}^{2}\varepsilon_1$, α_2) regions, respectively. E_z is the longitudinal component of the electric field wave.

For TM-polarized waves Maxwell equations describing the propagation of the electromagnetic field can be written as:

$$\frac{\partial^j H}{\partial z} = i\omega\varepsilon_0{}^{j}\varepsilon_{11}{}^{j}E_x = ik_0 h^j H, \tag{4}$$

$$\frac{\partial^j H}{\partial x} = -i\omega\varepsilon_0{}^{j}\varepsilon_{33}{}^{j}E_z, \tag{5}$$

$$k_0 h^{2j}E_x + ih\frac{\partial^j E_z}{\partial x} = k_0{}^{j}\varepsilon_{11}{}^{j}E_x. \tag{6}$$

Estimating ${}^{j}E_x$ and ${}^{j}H$ from the set of Eqs. (4)–(6), one obtains the wave equations for the j-th region

$$\frac{d^{2j}E_z}{dx^2} - \frac{{}^{j}k^2}{{}^{j}\varepsilon}({}^{j}\varepsilon_1 + \alpha_j|{}^{j}E_z|^2){}^{j}E_z = 0 \tag{7}$$

where: k_0 is the vacuum wave number, h is the propagation constant of the wave in the direction of the z-axis and ${}^{j}k^2 = k_0^2(h^2 - {}^{j}\varepsilon)$.

In the presently investigated problem we seek such solutions that would be oscillating and periodic in region 2 ($x < 0$) and vanishing in region 1 ($x > 0$). We have therefore for $x > 0$

$$E_z(x) = \left(\frac{a_1}{|b_1|}\right)^{1/2} \{\mathrm{ch}[a_1(x - x_{01})]\}^{-1}, \tag{8}$$

and for $x < 0$

$${}^{2}E_z(x) = \delta_2 \mathrm{cn}[\gamma_2(x - x_{02})|m] \tag{9}$$

where cn is a specific Jacobian elliptic function, x_{0j} is the integration constant, while the δ_j and γ_j parameters are given by:

$$\delta_j = \left[\frac{a_j + (a_j^2 + 4|b_j|C_1)^{1/2}}{2|b_j|}\right], \tag{10}$$

$$\gamma_j = (a_j^2 + 4|b_j|C_1)^{1/4}, \tag{11}$$

with:

$$a_j = \frac{{}^{j}k^2\,{}^{j}\varepsilon_1}{{}^{j}\varepsilon}, \tag{12}$$

$$b_j = \frac{\alpha_j}{2}\frac{{}^{j}k^2}{{}^{j}\varepsilon}, \tag{13}$$

C_1 is now the constant of integration.

The continuity conditions for the tangential component at the interface $x = 0$ give the dispersion relations

$$\frac{^2\varepsilon\gamma_2}{k_2^2}\operatorname{sn}(\gamma_2 x_{02})\operatorname{dn}(\gamma_2 x_{02}) = \frac{^1\varepsilon a_1}{k_1^2}\operatorname{th}(a_1 x_{01}). \tag{14}$$

The dispersion equation (14) gives, after some simple transformations, an elementary form of the algebraic equation

$$h^4 k_0^2(^1\varepsilon\alpha_1 - {}^2\varepsilon\alpha_2) + h^{2\,2}\varepsilon[^1\varepsilon(\alpha_2-\alpha_1)k_0^2 - (^1\varepsilon\alpha_1 - {}^2\varepsilon\alpha_2)(1+k_0^2)]$$
$$- {}^1\varepsilon\,{}^2\varepsilon^2(\alpha_2-\alpha_1)(1+k_0^2) = 0. \tag{15}$$

In the last equation, h is a wave number for the mode propagating along the interface, k_0 is the vacuum wave number and the remaining symbols are the so-called material parameters. The solution of this equation have the form:

$$h_1 = \pm k_0^{-1}\sqrt{^2\varepsilon C}, \tag{16}$$

$$h_2 = \pm\sqrt{-\frac{^1\varepsilon\,{}^2\varepsilon B}{A}} \tag{17}$$

where:

$$A = (^1\varepsilon\alpha_1 - {}^2\varepsilon\alpha_2), \tag{18}$$
$$B = (\alpha_2 - \alpha_1), \tag{19}$$
$$C = 1 + k_0^2. \tag{20}$$

Solution (16) is real and represents the wave number of the wave propagating along the z-axis. However, solution (17) can be either imaginary or real. When

$$\frac{B}{A} > 0, \tag{21}$$

the root of (17) is imaginary

$$h_2 = \pm i\sqrt{\frac{^1\varepsilon\,{}^2\varepsilon B}{A}}. \tag{22}$$

In the case when

$$\frac{B}{A} < 0, \tag{23}$$

the root of (17) is real

$$h_2 = \pm\sqrt{\frac{^1\varepsilon\,{}^2\varepsilon B}{A}}. \tag{24}$$

The electromagnetic field at a boundary between two nonlinear media possesses a number of interesting properties which are not known for linear media. One of such properties is represented by the solutions (8), (9), (16), (22) and (24). These solutions describe the electromagnetic field propagating along the boundary $x = 0$, vanishing in the $x > 0$ region and oscillating for $x < 0$. Such a behaviour of the electromagnetic field is known for the linear case and is characteristic of the so-called "lateral wave" [2]. For the case of the interface between nonlinear media, the problem becomes more complex because in this situation one or two waves of this type can be guided.

The wave number given by Equation (16) represents a wave propagating along the interface independently of the conditions and relations between material constants describing two media. This is the first of the waves of lateral type. It constitutes a constant component of the electromagnetic field on the boundary between nonlinear media.

The second type of "lateral" waves can appear even outside the boundary depending on the relation between the material constants. In the case when these constants fulfil relations (23), the wave number given (24) is real and on the boundary between nonlinear media appears the lateral wave of the second type. The electromagnetic field at the interface is then represented by two waves of "lateral" type propagating at different velocities along the boundary.

When the metarial constants for bordering media fulfil relation (21), the wave number of the second of these waves becomes imaginary, the field of this wave vanishes quickly and at the border only one wave dominates constituting, as said above, a stable picture of the field. This wave is given by wave number (16).

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