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TM waves in magnetooptic-nonlinear waveguide

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The TM waves guided by the linear thin film sandwiched between magnetooptic semi-infinite substrate and nonlinear Kerr-type semi-infinite cladding have been investigated theoretically and numerically. Both the dispersion relation and the guided power flow have been derived. The power flow dependence of the nonreciprocal behaviour of these waves is discussed. Introducing a nonlinear material into magnetooptic waveguide structure leads to an interesting application for the new class of integrated devices which combine magnetooptic and nonlinear effects.

1. Introduction

The magnetooptical waveguides have been receiving a great deal of attention because they have the application as basic elements for the nonreciprocal devices realized in planar form [1]–[3]. The nonreciprocal devices, such as isolators and circulators, are needed to stabilize the semiconductor–laser operation to avoid self-oscillation of the laser due to reflection in the communication system. Most proposals for waveguide type isolators are based on nonreciprocal TE-TM mode conversion [4]. In order to achieve efficient TE-TM conversion, the precise phase matching between the interacting modes is necessary by orienting the optic axis of the materials precisely and by controlling the thickness of the film accurately. These requirements are difficult to fulfil in practice. The other concept for an optical isolator is based on the nonreciprocal phase shift for TM modes in magnetooptical waveguide in the so-called Voigt (or equatorial) configuration [5], [6]. It was shown that propagation constant of the TM mode is different for the propagation in the opposite direction in the presence of the dc magnetic field applied perpendicular to the direction of propagation. This concept is advantageous as it involves one polarization only, namely TM, and therefore, there is no need of phase matching.

The purpose of this paper is to present a new configuration of the magnetooptic layered structure in which one medium (*e.g.*, top layer) is nonlinear. Nonlinear optical waveguides exhibit a number of interesting properties that have been studied extensively in recent years (see, for example, [7] and references in review articles [8]–[10]), because of their potential applications in all-optical signal processing. To date, closed form solution has been obtained only for TE-polarized waves. For the TM waves, the presence of two electric field component, one parallel and one perpendicular to the interfaces makes the analysis considerably complicated. To obtain analytical description of the TM-case a number of approximations have been employed [11], [12].

The introduction of the nonlinear material into magnetooptic waveguide gives an additional "degree of freedom", namely power flow down the guide [13], [14]. The power dependence of the propagation constant allows good control of the nonreciprocal behaviour of the counterpropagating TM modes. This shows great potential for application of the magnetooptic-nonlinear waveguide in the design and the realization of a new class of integrated devices the functions of which combine the magnetooptic and nonlinear effects. As an example such device can be nonreciprocal limiter of the guided power flow. Another possible device is power controlled integrated isolator based on the magnetooptic directional coupler. In such a structure, the nonlinearity acts on propagation constants of the counterpropagating TM modes modifying the coupling condition, and therefore the transfer efficiency which can be optimized, say in backward direction and reduced for the forward propagation.

2. Dispersion equation for TM modes

The guiding structure to be considered consists of three layers – the magnetooptic substrate, the linear nonmagnetic film and the nonlinear top layer. The external dc magnetic field H_0 is applied parallel to film plane and perpendicular to the direction of propagation which is along the z-axis (Voigt configuration). Neglecting losses, the dielectric tensor of magnetooptic (gyrotropic) material has the form

$$\varepsilon^{\mathrm{MO}} = \begin{pmatrix} \varepsilon_x & 0 & i\delta \\ 0 & \varepsilon_y & 0 \\ -i\delta & 0 & \varepsilon_z \end{pmatrix} \tag{1}$$

where diagonal elements are even powers of the applied magnetic field $|H|$ and the off-diagonal element is an odd power of $|H|$ [15].

It is well known that the modes in the Voigt configuration can be separated into $\mathrm{TE}[E_y, H_x, H_z]$ and $\mathrm{TM}[H_y, E_x, E_z]$ polarizations. We consider only TM modes as these are nonreciprocal in Voigt configuration, while the nature of the TE modes is preserved even in the presence of the externally applied magnetic field. For simplicity, the magnetooptic medium is assumed to be isotropic $\varepsilon_x = \varepsilon_y = \varepsilon_z = \varepsilon_r$. We suppose the nonlinear cladding to be Kerr-type and isotropic, so that the nonlinear dielectric subtensor for the TM waves reads as

$$\varepsilon^{\mathrm{NL}} = \begin{pmatrix} \varepsilon_{xx} & 0 \\ 0 & \varepsilon_{zz} \end{pmatrix} \tag{2}$$

where elements ε_{xx} and ε_{zz} depend on the local intensity and we assume

$$\varepsilon_{xx} = \varepsilon_{zz} = \varepsilon^{\mathrm{NL}} = \varepsilon_c + \alpha(|E_x|^2 + |E_z|^2) \tag{3}$$

(electrostrictive or thermal nonlinearity), where ε_c is linear part of the dielectric permittivity and α is the nonlinear coefficient.

The electric and magnetic field vectors for TM waves propagating along z-axis with angular frequency ω and propagation constant β are written as

$$E(x, z) = [E_x(x)i_x + E_z(x)i_z]\exp[i(\beta z - \omega t)],$$

$$H(x,z) = H_y(x) i_y \exp[i(\beta z - \omega t)].$$

For the magnetooptic medium ($x < 0$), the magnetic field amplitude $H_y(x) \equiv H(x)$ is solution of the wave equation

$$\frac{d^2H}{dx^2} + k_0\left[\frac{\varepsilon_s^2 - \delta^2}{\varepsilon_s} - \frac{\beta^2}{k_0^2}\right]H = 0 \tag{5}$$

where k_0 is the free-space wave number. In the nonlinear cladding ($x > 0$) the field amplitude H obeys the equation

$$\frac{d}{dx}\left[\frac{1}{\varepsilon^{\mathrm{NL}}}\frac{dH}{dx}\right] + k_0\left[\varepsilon^{\mathrm{NL}} - \frac{\beta^2}{k_0^2}\right]\frac{H}{\varepsilon^{\mathrm{NL}}} = 0. \tag{6}$$

arbitrary form of the intensity-dependent dielectric function $\varepsilon(I)$. The corresponding electric field $\boldsymbol{E}$ can be calculated from Maxwell equations. Assuming decaying field in the magnetooptic substrate (and nonlinear cover) and taking into account the conservation law in nonlinear medium [16] and applying the boundary condition at the film-substrate and film-cover interfaces one obtains the relationship

$$k_0 d\sqrt{n_f^2 - N^2} = \operatorname{arctg}\left[\frac{n_f}{n_s}\right]^2 \frac{\sqrt{N^2 - n_s^2} - N\delta/\varepsilon_s}{\sqrt{n_f^2 - N^2}} - \operatorname{arctg}\left[\frac{\varepsilon_f}{\varepsilon_c \eta}\sqrt{\frac{(3\eta - 1)(N^2 - \eta^{(\mathrm{NL})^2})}{(\eta + 1)(n_f^2 - N^2)}}\right] \tag{7}$$

where d is the film thickness, $N = \beta/k_0$ is the effective index of refraction (mode index), $n_s^2 = \dfrac{\varepsilon_s^2 - \delta^2}{\varepsilon_s}$, $n_f^2 = \varepsilon_f$, $\eta = 1 + \dfrac{\alpha}{\varepsilon_c}(|E_{xc}|^2 + |E_{zc}|^2)$ – boundary value normalized dielectric permittivity of nonlinear medium, $\eta^{(\mathrm{NL})^2} = \dfrac{2n_c^2\eta^2}{3\eta_c - 1}$, $n_c^2 = \varepsilon_c$.

Equation (7) is the dispersion relation of TM modes which determines the propagation constant β. The nonreciprocity of TM modes is evident from (7). Due to the off-diagonal elements in dielectric tensor (1), term linear in β enters this equation. Thus, the value of the propagation constant is different for propagation in opposite direction, depending on the sign of β. We note that changing the sign of δ (reverse the direction of dc magnetic field) has the same effect as changing the sign of β. It follows also from dispersion relation that propagation constant depends on the guided power flow (via power density at the film-cover interface). Note that the limit $\varepsilon^{\mathrm{NL}} \to \varepsilon_c$ in Eq. (7) yields the dispersion relation for linear magnetooptic waveguide [5].

3. Power flow

The time averaged guided wave power in watts per meter in the y-direction is given in terms of the Poynting vector

$$P = \frac{1}{2}\mathrm{Re}\int\limits_a^b E_x H_y dx.$$

The results in the magnetooptic substrate P_s and in the linear film are:

$$P_s = \frac{\varepsilon_s\beta-\delta\gamma_s}{4\varepsilon_0\gamma_s(\varepsilon_s^2-\delta^2)\omega}B^2, \quad P_f = \frac{\beta}{4\varepsilon_0\varepsilon_f\gamma_f\omega}\left(\frac{\sin2(\gamma_f d+\Phi)-\sin2\Phi}{2}+\gamma_f d\right)A^2 \tag{8}$$

where:

$$\gamma_s = k_0\sqrt{N^2-n_s^2}, \quad \gamma_f = k_0\sqrt{\varepsilon_f-N^2}, \quad \Phi = \mathrm{arctg}\left(\frac{\beta\delta-\varepsilon_s\gamma_s}{(\varepsilon_s^2-\delta^2)\gamma_f}\varepsilon_f\right).$$

A and B are amplitudes of the magnetic field in the film and in the substrate, respectively. The power flow in nonlinear cladding can be evaluated without integration over the field distribution. This means that it is not necessary to know the analytical form of the nonlinear field profile across the nonlinear top layer. The basic integration required is

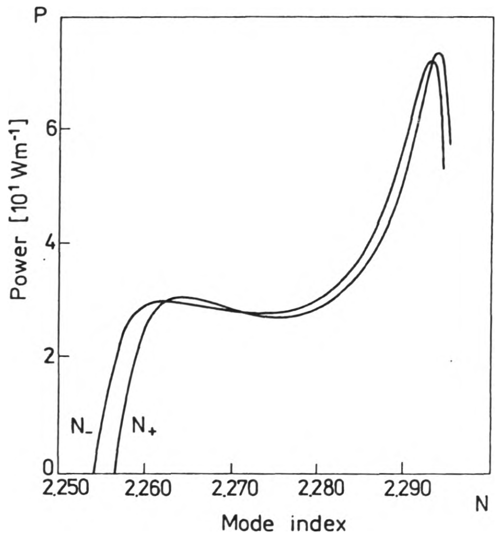
$$P_c = \frac{\omega\varepsilon_0\varepsilon_c}{2\beta}\int\limits_d^\infty \eta\rho_x^2 dx = \frac{\omega\varepsilon_0\varepsilon_c^2}{\beta^2}\int\limits_{\eta_m}^{\varepsilon_c}\eta\rho_x^2\frac{d(\beta x)}{\delta\eta}d\eta$$

$$= \frac{\omega\varepsilon_0\varepsilon_c^2}{2\beta^2\alpha}\int\limits_{\eta_m}^{\varepsilon_c}\frac{1+3\eta^2-a\eta^3}{2\eta(2-a\eta)^2}\sqrt{\frac{\eta+1}{-2a\eta^2+3\eta-1}}d\eta \tag{9}$$

where η_m is the maximal value of normalized permittivity of the nonlinear medium. The integral (9) has to be evaluated numerically. The total power flow down the guide is $P = P_s+P_f+P_c$.

4. Results

For the integrated nonreciprocal devices an important parameter is difference $\Delta N = N_+ - N_-$ between effective index of forward and backward propagating waves, respectively. This difference describes the nonreciprocal phase shift between counterpropagating modes. The total power flow as a function of both effective indices N_+ and N_- is shown in the Figure for the case when the nonlinear cladding is focusing ($\alpha > 0$). The value of the magnetooptic parameter δ is two order larger than typical value. This is done to accentuate the effect of δ on the nonreciprocity. For nonreciprocal device applications it is important to achieve a large difference $N_+ - N_-$ ($\beta_+ - \beta_-$). As shown in the Figure, the difference ΔN depends on guided power flow. Note that $N_+ - N_-$ vanishes for particular values of power flow. Therefore, in designing the nonreciprocal devices (phase shifter, isolators, gyrators) comprising in the structure a nonlinear cladding it is important to keep the power flow which maximizes the nonreciprocity.



Total power flow as a function of the TM mode index for forward (N_+) and backward (N_-) propagation ($\varepsilon_f = 5.198$, $\varepsilon_s = 4.796$, $\varepsilon_s = 5.033$, $\alpha = 6.4 \times 10^{-12}$ m^2V^{-2}, $\delta = 0.19$)

5. Conclusions

We have presented a new configuration of the planar waveguide composed of a linear thin film sandwiched between magnetooptic substrate and Kerr-type nonlinear cladding. A theoretical and numerical analysis of the nonreciprocal behaviour of the TM modes propagating in opposite direction has been carried out. The nonreciprocal phase shift of the counterpropagating TM waves in such structure is sensitive to the power flow along the guide. The proposed magnetooptic-nonlinear waveguiding structure can be applicable to production of a new type of integrated devices the operations of which combine the magnetooptic and nonlinear effects.

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