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Two-dimensional phase decoding from bounded fringe patterns by using the Fourier-transform method

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An efficient and accurate modification of the Fourier transform method for phase reconstruction from a given interferogram is described. In particular, a simple technique of fringe supplementing in the cases when the analysed area is neither square nor rectangle is proposed. Typical numerical experiments show that the phase retrieval process can be significantly improved, even with small microcomputer systems.

1. Introduction

Among the known methods of fringe pattern analysis, the Fourier transform method seems to be the best one for the patterns recorded photographically [1]. All we need when employing the above technique is an optical projection system with distortion properly corrected, and a microcomputer which communicates a scanning photodetector system (i.e., a CCD camera nowadays). Such equipment is sufficient for decoding the phase distribution from a single interferogram, provided that the fringe intensity profiles are nearly sinusoidal, and that the Fizeau reference fringes are introduced with proper tilt of interfering wavefronts.

However, the Fourier method has also some substantial disadvantages. Firstly, in many cases a great number (such as 1024 or even greater) of intensity samples along a single line is required to achieve good accuracy of decoded phase values, and this makes the processing procedure surprisingly slow, even when both the FFT algorithm written in machine code and fast microhardware is applied. In consequence, the method is not always useful for real-time processing and, what is also important, large amount of memory is needed for two-dimensional (2–D) analysis. Secondly, the inherent property of 2–D FFT algorithm is that it uses square, or rectangular areas of integration, whereas typical optical fringe patterns are bounded with circle; as a result, significant errors in decoded phase values can be created.

In this work, we present a remedy to the above difficulties. Corresponding theoretical considerations are given, and experimental results that confirm their usefullness are demonstrated.

2. Algorithm for 2–D analysis of interferograms

To minimize the processing time as well as the memory required, the 2–D analysis of interferograms is carried out with the aid of 1–D FFT algorithm. This simple trick enables one to analyse only a selected part of the pattern (in fact, the number of lines to be processed may be arbitrary). The analysis of intensity distribution is therefore performed sequentially (line-by-line), and the results can either be reduced or transferred successively to an external memory. That is why such an algorithm can be easily implemented even in small microcomputer systems. The entire procedure consists of the following consecutive steps:

A. Data acquisition.
B. Aperture contouring.
C. Fringes supplementing.
D. 1–D phase decoding.
E. Correction of both constant and line components for each line.
F. Parametrization of the decoded phase distribution.
G. Graphical display of the phase surface.

At first the intensity samples are transferred to the microprocessor unit; alternatively, in the case when the amount of data is too large to accomodate the microcomputer's RAM, appropriate transfers to the floppy disc are carried out. Next, the accepted data are analysed in order to establish the shape and dimensions of the effective area of the pattern under study. One can either restrict the area in advance, or choose a piece of it for further processing. As it was mentioned earlier, a part of the processed area is free from fringes, and this usually indroduces numerical errors in the restored phase distribution. To minimize these errors, a procedure of fringe supplementing with appropriate sinusoid is applied [2]. (A method of effective fitting of the sinusoid will be described in detail in the next Section of the present paper).

The next of the whole numerical procedure is the 1–D phase decoding from the fringe pattern. Consecutive scanning lines that consist of 2^N intensity samples (256 samples at least) are processed indepedently, line-by-line, by using the TAKEDA'S method [3]. After the sequential processing, both the piston and tilt terms in the phase function computed for successive lines are properly corrected (note that the phase function may have contained different constant components for different lines). The next step is to compose the 2–D phase surface from its successive cross-sections (i.e., from successive scanned lines). After the composition, the phase distribution is parametrized: in particular, such useful parameters like peek-to-valley deviation and RMS deviation of the wavefront are determined. Finally, the 3–D plot of the phase distribution is displayed on a graphical monitor.

3. Fringe supplementing technique

As is well-known, the fringe analysis technique based on the Fourier transform method described by TAKEDA et al. [3] gives the possibility to reconstruct the phase coded in a pattern cross-section by using 2^N intensity samples of interferogram.

However, when some of the samples have non-zero values, the corresponding phase error increases rapidly as the number of samples containing useful information decreases. Typical interferograms are bounded with circular apertures. To avoid errors in the retrieved phase, a square aperture inscribed in the circular one is taken into computations. In such a case certain amount of information is lost.

To analyse the whole circular area (or another shape), a new algorithm is proposed. Line intervals with absence of fringes are supplemented with a properly matched sinusoid. Let us assume that the intensity distribution at a given cross-section of the interferogram under study is of the form

$$I(n) = a(n)+b(n)\cos\left[w_0 n+f(n)\right], \quad n_l < n < n_r \tag{1}$$

where $a(n)$, $b(n)$ represent the background intensity and the fringe visibility, respectively; $f(n)$ is the phase function containing information of interest; w_0 is the spatial carrier frequency: $a(n)$, $b(n)$, $f(n)$ vary slowly in comparison with the variations introduced by w_0; $n = 1,2,\ldots, 256$; n_l, n_r denote the ordering numbers of samples which describe the range with non-zero values.

The function $I(n)$ has non-zero values within the range bounded from the left side by the n_l-th sample, and from the right – by the n_r-th sample. To obtain 2^N non-zero samples we add to both sides two periodic functions with parameters w_0, $a(n)$, $b(n)$ similar to proper parameters of the function $I(n)$. When viewed from the left side, the function has the form

$$I_l(n) = a_l(n)+b_l\cos(n\,w_0+f_l) \tag{2}$$

where a_l, b_l fit the functions $I_l(n)$ in scope of contrast and modulation of intensity; $f_l(n)$ expresses the phase correction, whereas w_0 stands for the approximate carrier frequency obtained from the analysed intensity distribution. The frequency w_0 is given by the formula

$$w_0 = (L_{max}+L_{min}+1)/(n_l-n_r) \tag{3}$$

where L_{max}, L_{min} stand for the ordering number of maximum and minimum intensity of the fringe pattern, respectively. The functions $a_l(n)$ and $b_l(n)$ are replaced with constants in such a manner that intensities $I_l(n)$ and $I(n)$ are equal in the common point: namely,

$$a_l = (I_{max}+I_{min})/2, \quad \text{and} \quad b_l = (I_{max}-I_{min})/2 \tag{4}$$

where I_{max}, I_{min} are the values of the intensity in the first maximum and first minimum of the fringe pattern, respectively. The phase correction in this point is assumed to be

$$\cos f_l = \left[I(n_l)-a_l\right]/b_l. \tag{5}$$

According to the above considerations, Eq. (2) becomes

$$I_l(n) = a_l+b_l\cos(n\,w_0)\cos f_l+\sin\left[n\,w_0(1-\cos^2 f_l)^{1/2}\right]. \tag{6}$$

In the case when the function $I(n)$ increases at the common point n_l, we choose the "$-$" sign in the above relationship; if this is not the case, the "$+$" sign is selected.

Analogous relation is obtained for the function $I_r(n)$, which should be added from the right side of the intensity distribution $I(n)$.

In order to estimate the errors of the method, the numerical verification has been performed on the base of a theoretical distribution. Figure 1**a** shows the analysed intensity distribution ($n = 256$) and its Fourier transform. The phase reconstructed from the pattern (see Fig. 1**b**) is spherical and the errors (Fig. 1**c**) obtained by

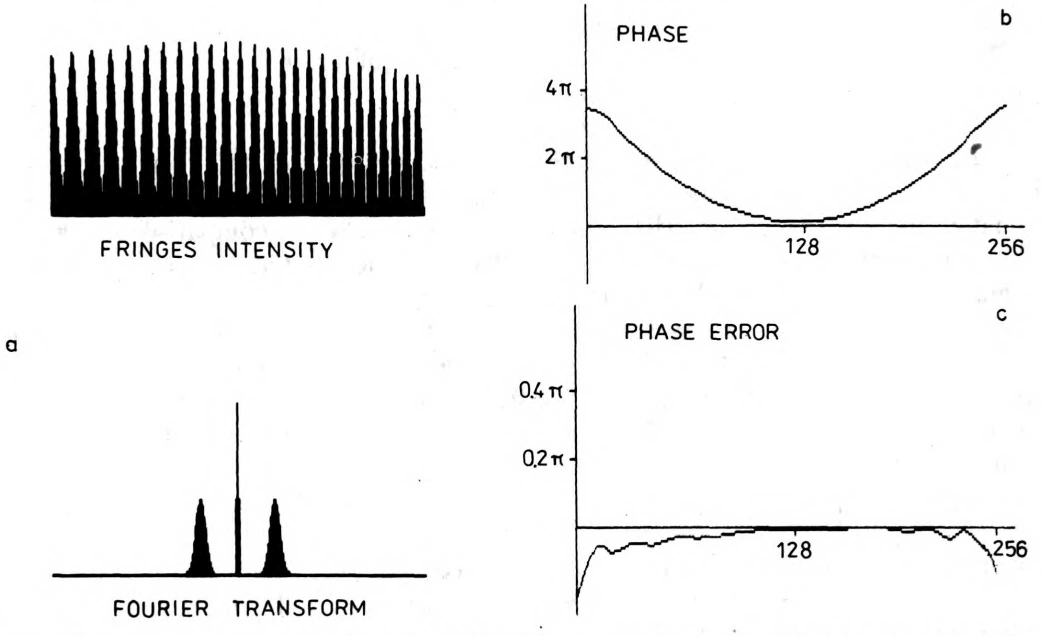


Fig. 1. Fringe pattern cross-section given by 256 intensity samples: its spectrum (**a**), the retrieved phase (**b**) and the distribution of the phase errors (**c**)

comparing the theoretical phase distribution with the reconstructed one has its maximum values at the edge of the interval. This is the well-known effect occuring due to the limited length of the processed signal. Figure 2 shows the analysis of truncated fringe patterns ($n_1 < n_2 < 256$) for Fig. 2**b** and 2**a**, respectively. It may be noticed that the errors of the retrieved phase do not exceed the errors obtained when analysing the entire ($n = 256$) signal (see Fig. 2**c**). The method of supplementing the fringe pattern with a sinusoidal-type signal gives positive results even for a few samples. The limit of the sample number results from the necessity to determine the carrier frequency w_0. However, if a common carrier w_0 is assumed for the 2–D analysis, the phase may be reconstructed even from a few samples.

After line-by-line processing, the pattern is composed by using the algorithm mentioned above: as a result we obtain the 2–D phase distribution that corresponds to the circular aperture of interest (see, also, Fig. 3, and note that the phase sign is changed there to make the plot more legible). The computational errors over the whole circular area do not exceed the values that have been achieved in the case of rectangular one.

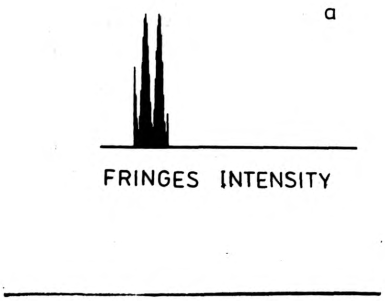
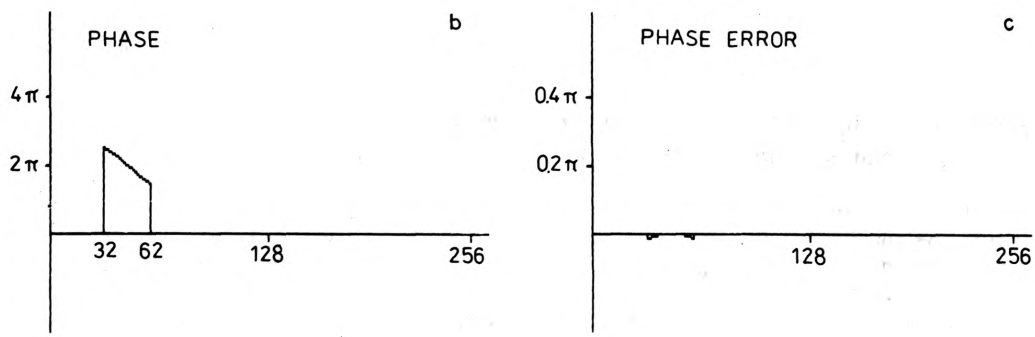


Fig. 2. The same as in Fig. 1, except that the number of samples is reduced to 30



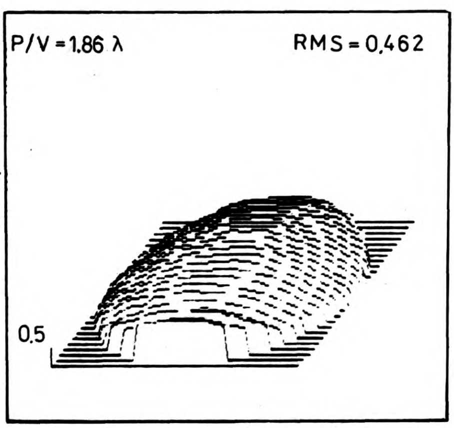


Fig. 3. Example of 2–D phase distribution decoded from a fringe pattern bounded with circular aperture

4. Conclusions

The numerical procedure described here is useful for phase decoding from fringe patterns that are associated with apertures having sharply defined edges. In comparison with the commonly employed 2–D Fourier analysis, the technique based on line-by-line application of the 1–D discrete Fourier transform has some advantages: both running time and memory required for processing can be shortened, and a selected piece of the pattern can be only processed, if necessary. The sinusoidal supplementing applied over the processed areas that are beyond the aperture of the fringe pattern, significantly reduces the numerical errors inside the aperture.

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Двуфазовая процедура декодирования фазы из ограниченных полосатых образов, используюшая метод изображения функции Фурье

Описан способ модификации метода изображения функции Фурье к эффективной и точной реконструкции фазовой информации, содержимой в интерферограмме. Особенно описана техника дополнения полосок в случае, когда область интерферограммы не ограничена квадратом или прямоугольником. Помещены численные результаты, из которых вытекает, что процесс декодирования фазы может быть более совершенным, даже в случае применения на небольших системах микро-ЭВМ.