# To be or not to be: whether a point-source is on the optical axis or not. A contribution to the consideration of the optical aberrations of a hologram 

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The paper entitled Numerical investigations of the imaging by curved holographic lens [1] by J. Nowak and M. Zając has been published in Optica Applicata, Vol. 18 (1988), 51. The results of the authors' investigations on imaging quality for some particular cases are presented. The investigations are devoted to both plane and spherical holographic lenses and take account of the position of the entrance pupil of the system. Besides an extensive consideration concerning the imaging by the plane holographic lenses and their aberrations (Section 2), the authors present in Section 3.1 the aberrations of the imaging occurring in spherical holographic lenses. Both from the considerations and the papers cited it follows that the examinations are based on the theory of aberration according to Champagne [2] used also in Mustafin's work [3]; the latter being devoted to the imaging quality of spherical holograms.

After a broad discussion on the physical model of the examined optical system, the approach and formulation of the problem and the aberration of the spherical holographic lens which I had with the authors. I feel obliged to express my view in the columns of your highly estimated quarterly. In my opinion, the interpretation of the aberrations formulated in the said paper [1] arouses some doubts and is difficult to accept. As it is commonly known, the theory of third order aberrations formulated by Champagne for holograms refers to imaging of the single off-axis object point and therefore is perfectly suitable for determining the imaging errors of point sources taking part in the hologram creation. Consequently, opposed to the Meier theory [4], here, there appear only three monochromatic aberrations: spherical aberration, coma and astigmatism. In the particular case of imaging of the point source located on the axis we have to do with the spherical aberration. The characteristic magnitude in the Champagne concept is the distance $R$ of the point source from the hologram centre (or from the vertex of the spherical hologram). This magnitude (beside the Cartesian coordinates) occurs in all the relations and formulas determining the direction of the principal rays, the image locations and the third order aberration coefficients. Therefore, the spherical aberration for the off-axis image is defined along the direction of image principal ray and not along the optical axis of the system to which we are generally accustomed. If the optical axis of the system is identical with the $z$ axis of the Cartesian coordinate system (Fig. 1), then the coefficient of third

order spherical aberration for the spherical hologram of the curvature $\varrho$ takes the form

$$
\begin{align*}
S= & \frac{1}{R_{C}^{3}}-\frac{1}{R_{3}^{3}} \pm \mu\left(\frac{1}{R_{1}^{3}}-\frac{1}{R_{R}^{3}}\right)+\frac{2}{\varrho}\left[\frac{z_{C}}{R_{C}^{3}}-\frac{z_{3}}{R_{3}^{3}} \pm \mu\left(\frac{z_{1}}{R_{1}^{3}}-\frac{z_{R}}{R_{R}^{3}}\right)\right] \\
& +\frac{1}{\varrho^{2}}\left[\frac{z_{C}^{2}}{R_{C}^{3}}-\frac{z_{3}^{2}}{R_{3}^{3}} \pm \mu\left(\frac{z_{1}^{2}}{R_{1}^{3}}-\frac{z_{R}^{2}}{R_{R}^{3}}\right)\right] \tag{1}
\end{align*}
$$

where $z_{1}, z_{R}, z_{c}, z_{3}$ are coordinates of the object, reference, reconstructed image points, respectively, taken in the direction of optical axis of the system. In this case, the spherical aberration is determined along the principal ray of the image point produced out of the optical axis of the system. However, in general, the position of the image with respect to the hologram vertex depends on the hologram curvature $R_{3}=R_{3}(\varrho)$. This means that for the fixed recording and reconstruction systems the position of the image changes with the change of the hologram curvature. But for the holographic lens in the case of axial imaging the reconstructing point source falls on the optical axis of the system $x_{c}=0$, (Fig. 2). Then we have

$$
\lim _{x_{c} \rightarrow 0} R_{c}\left(x_{c}\right)=z_{c},
$$

and the image point position defined by $R_{C}$ takes the simple form independent of the hologram curvature

$$
\begin{equation*}
\frac{1}{z_{3}}=\frac{1}{z_{c}} \pm \mu\left(\frac{1}{z_{1}}-\frac{1}{z_{R}}\right) . \tag{2}
\end{equation*}
$$

This means that the third term of the expression (1) vanishes and the spherical aberration coefficient takes the form, given in work [4], determining the spherical aberration of the axial point

$$
\begin{equation*}
S=\frac{1}{z_{C}^{3}}-\frac{1}{z_{3}^{3}} \pm \mu\left(\frac{1}{z_{1}^{3}}-\frac{1}{z_{R}^{3}}\right)+\frac{2}{\varrho}\left[\frac{1}{z_{C}^{z}}-\frac{1}{z_{3}^{2}} \pm \mu\left(\frac{1}{z_{1}^{2}}-\frac{1}{z_{R}^{2}}\right)\right] \tag{3}
\end{equation*}
$$

Under these circumstances, the coefficients of the field aberrations: coma and astigmatism occuring in the expression (3) of the above mentioned work [1] has no raison d'être since the point source is on the axis. Besides this statement follows


Fig. 2. Reconstruction of an off-axis $P_{3}$ and an axis-image $P_{3}^{0}$ by the holographic curved lens
immediately from the considerations by Mustafin given in paper [3]. The given point source may be located either on the axis or off-axis. One should choose: To be or not to be the optical axis. The formula for the coefficient of spherical aberration occuring in expression (3) of the discussed paper [1] refers to the axial point, while the formulae for coma and astigmatism - to the off-axis point.

Summing up, it should be unquestionably stated that the considerations published in paper [1], based on the Champagne theory and concerning the aberrations of the spherical hologram imaging, contain some uncertainty due to their inconsequences since Champagne defines distinctly the aberrations given point source and this should be respected. The formulation of spherical aberration in the form defined by expression (3) in paper [1] shifts the point source on the optical axis of the system, where no coma or astigmatism occur. If, in spite of this, we still speak of these aberrations then the spherical aberration due to Champagne should be used, i.e., along the principal ray independent of what value it takes. Otherwise. we perturb the physical model of the system we base on and the Champagne model has no reason of existence.

An alternative approach to the description of the aberration of the holographic lens on a spherical substrate based of Meier concept [4] free from the said uncertainties has been given in [5].

## References

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[2] Champagne E. B., J. Opt. Soc. Am. 57 (1967), 51.
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[4] Meier R. W., J. Opt. Soc. Am. 55 (1965), 987.
[5] Jagoszewski E., Optik 69 (1985), 85.

