# Course of optical bistability in a ring cavity in presence of an external magnetic field 

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#### Abstract

The results concerning the course of dispersive optical bistability in an isotropic, nonlinear ring cavity are presented. It is shown that in the presence of an external magnetic field, linearly polarized light transmitted through a cavity changes its polarization to the elliptical one, and reveals bistability of the intensity and polarization state. The direction of rotation of its electric vector can also vary noncontinuously.


## 1. Introduction

Since 1980 several authors have described optical and hybrid systems in which bistability of the state of light polarization can appear [1]- [7]. Polarization bistability caused by external factors has been presented in the works [8] and [9]. The authors of [8] have considered polarization bistable switching in media composed of resonantly excited two-level atoms and placed in a weak magnetic field transversal to the direction of light propagation. In the paper [9] it has been shown that externally forced anisotropy in isotropic media is able to cause bistable changes of the polarization state of light transmitted through a nonlinear Fabry-Perot cavity.

This paper deals with the behaviour of a plane, monochromatic, linearly polarized light wave transmitted through a nonlinear, isotropic ring cavity with an external magnetic field applied parallelly to the direction of propagation of the wave.

The external magnetic field splits a plane wave into right- and left-circularly polarized waves. These waves have different constants of propagation and, therefore, different resonance conditions in the ring cavity. Nonlinearity of the medium implies the interaction between the two waves. The phase of each wave depends not only on its own amplitude but also on the amplitude of the wave of the opposite circular polarization. It leads to interdependence of the intensities of the circularly polarized waves (that type of interdependence has been already presented in the paper [7]), and to bistability of the transmitted wave intesity. The transmitted wave appears to be elliptically polarized and the parameters of the polarization ellipse also reveal bistable changes. It is even possible to select geometrical parameters of the cavity in the way allowing noncontinuous changes of the direction of the electric vector rotations.

## 2. Formulation of the problem and simplifying assumptions

Ring resonator studied in this paper (see Fig. 1) is composed of four mirrors and a nonlinear crystal. The mirrors 1 and 2 have the following intensity reflectivities: $R_{\|}\left(T_{\|}+R_{\|}=1\right)$ for incident waves having the electric vector parallel to the plane of incidence, and $R_{\perp}\left(T_{\perp}+R_{\perp}=1\right)$ for incident waves having the electric vector transversal to the plane of incidence. The mirrors 3 and 4 have the intensity reflectivities equal to 1 . The crystal is made of a medium which appears to be isotropic in absence of the external magnetic field and reveals nonlinear third-order electric susceptibility. The external magnetic field is parallel to the $0 z$-axis $\left(B_{0} e_{z}\right)$ and to preserve the symmetry of the tensors of nonlinear susceptibility.


Fig. 1. Nonlinear ring resonator (1-4 mirrors)

The linear electric susceptibility tensor has the following from [10]
$\varepsilon_{j k}=\varepsilon_{r} \delta_{j k}+\alpha_{1} B_{0}^{2} \delta_{z k}+i \alpha B_{0} e_{j z k}, \quad j, k=x, y, z$
where:
$\varepsilon_{r} \quad$-relative electric permittivity of the medium (in general $\varepsilon_{r}$ is a complex number: $\varepsilon_{r}=\varepsilon_{\mathrm{T}}^{\prime}-i \varepsilon_{\mathrm{r}}^{\prime \prime}$ ),
$\alpha, \alpha_{1}$-constants characterizing the magnetooptical properties of the medium,
$\delta_{j k}$-Kronecker's tensor,
$e_{j k l}$-absolutely antisymmetric third-rank pseudotensor.
The linear magnetic susceptibility tensor is of the form
$\mu_{j k}=1, \quad j, k=x, y, z$.
The field inside the nonlinear medium is described by the equation

$$
\begin{equation*}
\operatorname{curl} \operatorname{curl} \mathscr{E}=-\mu_{0} \frac{\partial^{2} \mathscr{D}}{\partial t^{2}} \tag{2}
\end{equation*}
$$

and by the nonlinear material equation

$$
\begin{equation*}
\mathscr{D}_{j}=\varepsilon_{0} \varepsilon_{j k} \varepsilon_{k}+\mathscr{P}_{j}^{\mathrm{NL}}, \quad j, k=x, y, z \tag{3}
\end{equation*}
$$

$i_{n}$ and $\mu_{0}$ denote the electric and, respectively, magnetic permittivities of vacuum, and $\varepsilon_{i k}$ is defined by (la). The components of the nonlinear polarization vector $\mathscr{P}^{\mathrm{NL}}$ are as follows [11]:

$$
\begin{equation*}
\mathscr{P}_{j}^{\mathrm{NL}}=\chi\left(\mathscr{E}_{x}^{2}+\mathscr{E}_{y}^{2}+\mathscr{E}_{z}^{2}\right) \mathscr{E}_{j} \quad j=x, y, z \tag{4}
\end{equation*}
$$

where $\chi$ is one of the components of the electric susceptibility fourth-rank tensor $\left(\chi:=\chi_{x x x x}=\chi_{y y y y}=\chi_{z z z z}\right)$.

The field inside the nonlinear medium is described by Eqs. (2)-(4), the following simplifying assumptions being used:

1. The wave field propagates in the medium parallelly to the $0 z$-axis and depends (as a function) solely on the variable $z$.
2. The wave field is a monochromatic wave field of the frequency $\omega$ (components of the field having frequencies equal to integer multiples of $\omega$ are neglected) far from the resonance frequencies of the medium.
3. The wave field incident upon the resonator is a linearly polarized, monochromatic, plane wave

$$
\begin{equation*}
\mathscr{E}^{(i)}:=\frac{1}{2} e_{x} E_{0}\left\{\exp \left[i\left(\omega t-k_{0} z\right)\right]+c . c .\right\} \tag{5}
\end{equation*}
$$

where $k_{0}=\omega / c(c$-speed of light in vacuum).
4. The wave is transmitted through the planes separating the medium and vacuum ( $z=0$ and $z=L$ ) without reflection.

The assumptions 1 and 2 imply

$$
\begin{equation*}
\mathscr{E}_{z}=\mathscr{P}_{z}^{\mathrm{NL}}=0 . \tag{6}
\end{equation*}
$$

It mens that the wave field inside the cavity is transversal to the direction of propagation.

The real field vectors $\mathscr{E}, \mathscr{P}^{\mathrm{NL}}$ can be described as sums of complex functions $\boldsymbol{E}=\left[E_{x} E_{y} E_{z}\right]^{\mathrm{T}}, \quad \boldsymbol{P}^{\mathrm{NL}}=\left[P_{x}^{\mathrm{NL}} P_{y}^{\mathrm{NL}} P_{z}^{\mathrm{NL}}\right]^{\mathrm{T}}$ and their complex conjugations:

$$
\begin{equation*}
\mathscr{E}=\frac{1}{2}\left(\boldsymbol{E}+\boldsymbol{E}^{*}\right), \quad \mathscr{P}^{\mathrm{NL}}=\frac{1}{2}\left[\boldsymbol{P}^{\mathrm{NL}}+\left(\boldsymbol{P}^{\mathrm{NL}}\right)^{*}\right] . \tag{7}
\end{equation*}
$$

The components of the complex nonlinear polarization vector $\boldsymbol{P}^{\mathrm{NL}}$ proportional to $\exp (i \omega t)$ are:

$$
\begin{align*}
& P_{x}^{\mathrm{NL}}=\frac{1}{4} \chi\left[\left(3\left|E_{x}\right|^{2}+2\left|E_{y}\right|^{2}\right) E_{x}+E_{y}^{2} E_{x}^{*}\right], \\
& P_{y}^{\mathrm{NL}}=\frac{1}{4} \chi\left[\left(3\left|E_{y}\right|^{2}+2\left|E_{x}\right|^{2}\right) E_{y}+E_{x}^{2} E_{y}^{*}\right] . \tag{8}
\end{align*}
$$

## 3. Nonlinear Helmholtz's equations in the nonlinear medium and their solutions

The components of the wave field proportional to $\exp (i \omega t)$ are described by the following system of scalar equations:

$$
\begin{align*}
& \frac{d^{2}}{d z^{2}} E_{x}=-k_{0}^{2} \varepsilon_{r} E_{x}+i k_{0}^{2} \alpha B_{0} E_{y}-\mu_{0} \omega^{2} P_{x}^{\mathrm{NL}} \\
& \frac{d^{2}}{d z^{2}} E_{y}=-k_{0}^{2} \varepsilon_{r} E_{y}+i k_{0}^{2} \alpha B_{0} E_{x}-\mu_{0} \omega^{2} P_{y}^{\mathrm{NL}} \tag{9}
\end{align*}
$$

The wave field is decomposed into two fields of right and left circular polarizations:

$$
\begin{equation*}
E_{1}:=\frac{1}{\sqrt{2}}\left(E_{x}+i E_{y}\right), \quad E_{2}:=\frac{1}{\sqrt{2}}\left(E_{x}-i E_{y}\right), \tag{10}
\end{equation*}
$$

hence, the system (9) may be replaced by the following system of scalar Helmholtz's nonlinear equations (equivalent to (9)):

$$
\begin{align*}
& {\left[\frac{d^{2}}{d z^{2}}+k_{0}^{2}\left(\varepsilon_{r}-\alpha B_{0}\right)\right] E_{1}=-\frac{1}{2} \mu_{0} \omega^{2} \chi\left(\left|E_{1}\right|^{2}+2\left|E_{2}\right|^{2}\right) E_{1},} \\
& {\left[\frac{d^{2}}{d z^{2}}+k_{0}^{2}\left(\varepsilon_{r}+\alpha B_{0}\right)\right] E_{2}=-\frac{1}{2} \mu_{0} \omega^{2} \chi\left(2\left|E_{1}\right|^{2}+\left|E_{2}\right|^{2}\right) E_{2} .} \tag{11}
\end{align*}
$$

The assumption 4 allows us to expect the following form of the solution of the system (11) $[12]:$

$$
\begin{equation*}
E_{j}(z, t)=\xi_{j}(z) \exp \left\{i\left[\Phi_{j}(z)-k_{j} z\right]\right\} \exp (i \omega t), \tag{12}
\end{equation*}
$$

where

$$
\begin{equation*}
k_{j}^{2}:=k_{0}^{2}\left[\varepsilon_{r}^{\prime}+(-1)^{j} x B_{0}\right] . \tag{13}
\end{equation*}
$$

The amplitudes $\zeta_{1}, \check{\zeta}_{2}$ and the phase function $\Phi_{1}, \Phi_{2}$ are real, slowly varying functions of the variable $z$ :

$$
\begin{align*}
& \left|\frac{d^{2}}{d z^{2}} \xi_{j}\right| \ll 2 k_{j}\left|\frac{d}{d z} \xi_{j}\right|, \\
& \left|\frac{d^{2}}{d z^{2}} \Phi_{j}\right| \ll 2 k_{j}\left|\frac{d}{d z} \Phi_{j}\right|,  \tag{14}\\
& \left|\frac{d}{d z} \Phi_{j}\right| \ll k_{j}, \quad j=1,2 .
\end{align*}
$$

The approximation of slowly varying functions [12] leads to a system of first-order differential equations which, after separating real and imaginary parts,
yields the following equations determining the amplitudes and the phase functions of the circularly polarized waves:

$$
\begin{align*}
& \frac{d}{d z} \xi_{j}=-\varrho_{j} \xi_{j} \\
& \frac{d}{d z} \Phi_{j}=-\gamma_{j}\left(\xi_{j}^{2}+2 \xi_{3-j}^{2}\right), \quad j=1,2 \tag{15}
\end{align*}
$$

where the constants denote:

$$
\begin{equation*}
\varrho_{j}:=\frac{k_{0}^{2} \varepsilon_{r}^{\prime \prime}}{2 k_{j}}, \quad \gamma_{j}:=\frac{\mu_{0} \omega^{2} \chi}{4 k_{j}}, \quad j=1,2 . \tag{16}
\end{equation*}
$$

The system (15) has the following solution:

$$
\begin{align*}
& \xi_{j}(z)=\xi_{j}(0) \exp \left(-\varrho_{j} z\right)  \tag{17a}\\
& \Phi_{j}(z)-\Phi_{j}(0)=-\gamma_{j} z\left[\xi_{j}^{2}(0)+2 \xi_{3-j}^{2}(0)\right] \quad \text { if } \varepsilon_{r}^{\prime \prime}=0 \\
& \Phi_{j}(z)-\Phi_{j}(0)= \gamma_{j}\left\{\frac{1}{2 \varrho_{j}}\left[\exp \left(-2 \varrho_{j} z\right)-1\right] \xi_{j}^{2}(0)\right. \\
&\left.+\frac{1}{\varrho_{3-j}}\left[\exp \left(-2 \varrho_{3-j} z\right)-1\right] \xi_{3-j}^{2}(0)\right\} \quad \text { if } \varepsilon_{r}^{\prime \prime} \neq 0, \quad j=1,2 \tag{17b}
\end{align*}
$$

## 4. Boundary conditions and bistability of transmitted light

The mirrors 1-4 are used to achieve feedback of the wave field in the nonlinear medium. The wave leaving the medium at the plane $z=L$ is split by the mirror 2 . Its reflected part is directed by the mirrors 3 and 4 to the mirror 1 where it is split and its reflected part is incident onto the nonlinear medium at the plane $z=0$. Hence, the boundary conditions at the plane $\Sigma=0$ are as follows:

$$
\begin{align*}
\sqrt{2 T_{\|}} E_{0}+R_{\|} \exp \left(i \Phi_{\|}\right)\left\{\xi_{1}(L) \exp \right. & {\left.\left[i\left(\Phi_{1}(L)-k_{1} L\right)\right]+\xi_{2}(L) \exp \left[i\left(\Phi_{2}(L)-k_{2} L\right)\right]\right\} } \\
& =\xi_{1}(0) \exp \left[i \Phi_{1}(0)\right]+\xi_{2}(0) \exp \left[i \Phi_{2}(0)\right] \\
R_{\perp} \exp \left(i \Phi_{\perp}\right)\left\{\xi _ { 2 } ( L ) \operatorname { e x p } \left[i \left(\Phi_{2}(L)-\right.\right.\right. & \left.\left.\left.k_{2} L\right)\right]-\xi_{1}(L) \exp \left[i\left(\Phi_{1}(L)-k_{1} L\right)\right]\right\}  \tag{18}\\
= & \xi_{2}(0) \exp \left[i \Phi_{2}(0)\right]-\xi_{1}(0) \exp \left[i \Phi_{1}(0)\right]
\end{align*}
$$

The phases of the field components, having the electric vector parallel and transversal to the plane of incidence of the mirrors. increase respectively $\Phi$ and $\Phi_{\perp}$ on their way from the plane $z=L$ to the plane $z=0$.

The following symbols are introduced:

$$
\begin{aligned}
& \eta_{j}:=\exp \left(\varrho_{j} L\right), \quad j=1,2, \\
& \Delta \Phi_{j}:=\Phi_{j}(L)-\Phi_{j}(0), \quad j=1,2 \\
& \Delta \Phi:=\Phi_{1}(L)-\Phi_{2}(L)+\left(k_{2}-k_{1}\right) L
\end{aligned}
$$

$I_{0}$-intensity of the wave incident onto the mirror 1 from outside,
$I_{\mathrm{tr}}$-intensity of the wave transmitted through the mirror 2 ,
$I_{1}$-intensity of the right-circularly polarized component of the wave leaving the crystal at the plane $z=L$,
$I_{2}$ - intensity of the left-circularly polarized component of the wave leaving the crystal at the plane $z=L$.
Now, we can rewrite the boundary conditions:

$$
\begin{align*}
& I_{j}=\frac{2}{|\Omega|^{2}} T_{\|} I_{0}\left[R_{\perp}^{2}+\eta_{3-j}^{2}-2 R_{\perp} \eta_{3-j} \cos \left(\Delta \Phi_{3-j}+\Phi_{\perp}-k_{3-j} L\right)\right], \quad j=1,2  \tag{19a}\\
& \cos (\Delta \Phi)=\sqrt{\frac{I_{2}}{I_{1}} \frac{L_{1}}{M}, \quad \sin (\Delta \Phi)=\sqrt{\frac{I_{2}}{I_{1}} \frac{L_{2}}{M}}} \begin{array}{l}
I_{\mathrm{tr}}=\frac{1}{2}\left(T_{\|}+T_{\perp}\right)\left(I_{1}+I_{2}\right)+\left(T_{\|}-T_{\perp}\right) \sqrt{I_{1} I_{2}} \cos (\Delta \Phi) \\
\Delta \Phi_{j}=\gamma_{j 1} I_{1}+\gamma_{j 2} I_{2}, \quad j=1,2
\end{array}, l \tag{19b}
\end{align*}
$$

where the symbols $\Omega, L_{1}, L_{2}, M$ denote:

$$
\begin{align*}
\Omega= & \left\{\eta_{1} \exp \left(-i \Delta \Phi_{1}\right)-R_{\|} \exp \left[i\left(\Phi_{\|}-k_{1} L\right)\right]\right\}\left\{R_{\perp} \exp \left[i\left(\Phi_{\perp}-k_{2} L\right)\right]\right. \\
& \left.-\eta_{2} \exp \left(-i \Delta \Phi_{2}\right)\right\}+\left\{\eta_{1} \exp \left(-i \Delta \Phi_{1}\right)-R_{\perp} \exp \left[i\left(\Phi_{\perp}-k_{1} L\right)\right]\right\} \\
& \left\{R_{\|} \exp \left[i\left(\Phi_{\|}-k_{2} L\right)\right]-\eta_{2} \exp \left(-i \Delta \Phi_{2}\right)\right\}, \\
L_{1}= & R_{\perp}^{2}-R_{\perp}\left[\eta_{1} \cos \left(\Delta \Phi_{1}+\Phi_{\perp}-k_{1} L\right)+\eta_{2} \cos \left(\Delta \Phi_{2}+\Phi_{\perp}-k_{2} L\right)\right] \\
& +\eta_{1} \eta_{2} \cos \left(\Delta \Phi_{1}-\Delta \Phi_{2}+k_{2} L-k_{1} L\right), \\
L_{2}= & \eta_{1} \eta_{2} \sin \left(\Delta \Phi_{1}-\Delta \Phi_{2}+k_{2} L-k_{1} L\right)+R_{\perp} \eta_{2} \sin \left(\Delta \Phi_{2}+\Phi_{\perp}-k_{2} L\right) \\
& -R_{\perp} \eta_{1} \sin \left(\Delta \Phi_{1}+\Phi_{\perp}-k_{1} L\right), \\
M= & R_{\perp}^{2}+\eta_{1}^{2}-2 R_{\perp} \eta_{1} \cos \left(\Delta \Phi_{1}+\Phi_{\perp}-k_{1} L\right), \tag{20}
\end{align*}
$$

and the matrix $\Gamma=\left[\gamma_{j k}\right]_{j, k=1,2}$ is defined below:

$$
\gamma_{j k}=\left\{\begin{array}{l}
-2 \gamma_{j} L \sqrt{\mu_{0} / \varepsilon_{0}}\left(2-\delta_{j k}\right) \quad \text { if } \varepsilon_{\mathrm{r}}^{\prime \prime}=0,  \tag{21}\\
2 \frac{\gamma_{j}}{\varrho_{k}} \sqrt{\mu_{0} / \varepsilon_{0}}\left(1-\eta_{k}^{2}\right)\left(2-\delta_{j k}\right) \quad \text { if } \varepsilon_{\mathrm{r}}^{\prime \prime} \neq 0, \quad j, k=1,2 .
\end{array}\right.
$$

Expressions (19a) differ from the classical ones [13], [14] which describe the course of bistability in a ring cavity. In the formulas (19a) $I_{1}$ depends not only on $I_{1}$ but also on $I_{2}$ and inversely:

$$
\begin{align*}
& I_{1}=f_{1}\left(I_{1}, I_{2}, I_{0}\right), \\
& I_{2}=f_{2}\left(I_{1}, I_{2}, I_{0}\right), \tag{22}
\end{align*}
$$

$f_{1}$ and $f_{2}$ denote nonlinear functions.

The system of algebraic equations (19) can be interpretted by means of numerical analysis. The method used in this paper consists of two steps. In the first step the analysis of the equation

$$
\begin{align*}
& I_{1}\left[R_{\perp}^{2}+\eta_{1}^{2}-2 \eta_{1} R_{\perp} \cos \left(\Delta \Phi_{1}+\Phi_{1}-k_{1} L\right)\right] \\
& \quad=I_{2}\left[R_{1}^{2}+\eta_{2}^{2}-2 R_{\perp} \eta_{2} \cos \left(\Delta \Phi_{2}+\Phi_{\perp}-k_{2} L\right)\right] \tag{23}
\end{align*}
$$

allows us to approximate the curve describing interdependence of $I_{1}$ and $I_{2}$ by respective segments. This curve is used in the second step to find a starting point $\left(I_{1}^{(0)}\right.$, $\left.I_{2}^{(0)}\right)$ for the Newton's two-dimensional method, based on solving two nonlinear Eqs. (19c) and (23) for each given value $I_{\text {r. }}$. Having found the exact values ( $I_{1} \cdot I_{2}$ ) one can easily determine $I_{0}$ (see Eq. (19a)). It is also possible to describe the state of polarization of the wave field transmitted through the mirror 2 as a function of $\left(I_{1}, I_{2}\right)$.

The wave field transmitted through the mirror 2 has the following components:

$$
\begin{align*}
E_{x}^{\mathrm{tr}}(z, t)= & \sqrt{\frac{T_{\|}}{2}}\left\{\xi_{1}(L) \exp \left[i\left(\Phi_{1}(L)-k_{1} L\right)\right]+\xi_{2}(L) \exp \left[i\left(\Phi_{2}(L)-k_{2} L\right)\right]\right\} \\
& \times \exp \left[i\left(\omega t-k_{0} z\right)\right] . \\
E_{y}^{\mathrm{Ir}}(=, t)= & i \sqrt{\frac{T_{1}}{2}}\left\{\xi_{2}(L) \exp \left[i\left(\Phi_{2}(L)-k_{2} L\right)\right]-\xi_{1}(L) \exp \left[i\left(\Phi_{1}(L)-k_{1} L\right)\right]\right\} \\
& \times \exp \left[i\left(\omega t-k_{0} z\right)\right] . \tag{24}
\end{align*}
$$

It means that the matrix of coherence [15]

$$
\begin{equation*}
\mathbf{M}=\left[M_{j k}\right]=\left[\left\langle E_{j} E_{k}^{*}\right\rangle\right], \quad j, k=x, y, \tag{25}
\end{equation*}
$$

has the determinant $|\mathbf{M}|=0$. Hence, the parameter of polarization $P[15]$ is equal to 1 what means that the transmitted light is completely polarized. The polarization ellipse is characterized by two parameters $\Theta$ and $\Psi$. The absolute value $|\tan \Theta|$ is equal to the ratio of semiaxes of the ellipse, the sign of $\tan \Theta$ determines the direction of the electric vector rotation-right if $\tan \Theta>0$, left if $\tan \Theta<0 . \Psi$ is the angle between the major semiaxis and the $0 x$-axis. The parameters can be calculated from the following formulas:

$$
\begin{align*}
& \sin 2 \Theta=\frac{\left(M_{y x}-M_{x y}\right)}{M_{x x}+M_{y y}}=\frac{\left(I_{1}-I_{2}\right) \sqrt{T_{\|} T_{\perp}}}{I_{\mathrm{tr}}},  \tag{26}\\
& \tan 2 \Psi=\frac{M_{x y}+M_{y x}}{M_{x x}-M_{y y}}=\frac{4 \sqrt{T_{\|} T_{\perp}} \sqrt{I_{1} I_{2}} \sin (\Delta \Phi)}{\left(I_{1}+I_{2}\right)\left(T_{\|}-T_{\perp}\right)+2\left(T_{\|}+T_{\perp}\right) \sqrt{I_{1} I_{2}} \cos (\Delta \Phi)} . \tag{27}
\end{align*}
$$

The nonlinear interdependence of $I_{1}$ and $I_{2}$ (22) allows to expect bistable changes of $\tan \Theta$ and $\Psi$.

## 5. Graphical illustration of the received results

The interpretation of the previously discussed results is based on numerical computations executed on an IBM PC/XT compatible microcomputer. The values of the used material constants refer to $\mathrm{CS}_{2}$ [11], [16]. The medium is assumed to reveal no energy dissipation $\left(\varepsilon_{\mathrm{r}}^{\prime \prime}=0\right)$. The geometrical parameters of the cavity are as follows: $L=0.01 \mathrm{~m}, R_{\|}=0.4, R_{\perp}=0.43$.


Fig. 2. Intensity bistability in the ring resonator referring to the phase increases: $\Phi_{\perp}=\Phi_{\|}=\pi / 6$ (a), and $\Phi_{\perp}=\pi / 3, \Phi_{\|}=\pi / 2$ (b)


Fig. 3. Bistability of the ratio of the polarization ellipse semiaxes referring to the phase increases: $\Phi_{\perp}=\Phi_{\|}=\pi / 6$ (a), $\Phi_{\perp}=\pi / 3, \Phi_{\| \mid}=\pi / 2$ (b)

Figures 2-4 present the course of bistability in the ring cavity when the external magnetic field is constant and reaches $B_{0}=0.01 \mathrm{~T}$. The figures denoted by the letter a refer to the phase increases $\Phi_{\|}=\Phi_{\perp}=\pi / 6$, the ones denoted by $\mathbf{b}-$ to $\Phi_{\perp}=\pi / 3$, $\Phi_{\|}=\pi / 2$.

Figure 2 illustrates the bistable dependence of the intensity of the transmitted wave $I_{\text {tr }}$ upon the intensity of the incident wave $I_{0}$. The noncontinuous changes of $I_{\mathrm{tr}}$ take place when $I_{0}$ reaches the values $I_{0}^{\dagger}$ (the "jump" upward) and $I_{0}^{\downarrow}$ (the "jump"


Fig. 4. Bistability of the orientation angle of the polarization ellipse referring to the phase increases: $\Phi_{1}=\Phi_{\|}=\pi / 6$ (a), $\Phi_{1}=\pi / 3, \Phi_{\|}=\pi / 2$ (b)


Fig. 5. Width of the hysteresis cycle as a function of the induction of the external magnetic field
downward). The parameters of the polarization ellipse are presented as functions of $I_{0}$ in Fig. $3(\tan \Theta)$ and Fig. $4(\Psi)$. They both vary in bistable ways and the noncontinuous changes happen when $I_{0}$ reaches $I_{0}^{\dagger}$ and $I_{0}^{\dagger}$. It is worth mentioning that properly chosen geometrical parameters of the cavity can cause noncontinuous changes of the direction of rotation of the electric vector of the transmitted wave (see Fig. 3b).

Figures 5-8 show that the external magnetic field $B_{0}$ controls the parameters of the hysteresis cycles of the output intensity $I_{\mathrm{tr}}$ and the polarization ellipse parameters. The width of the hysteresis cycles $\Delta I_{0}=I_{0}^{\dagger}-I_{0}^{\dagger}$ as a function of $B_{0}$ is presented in Fig. 5. The dimensions of the bistable "jumps" of $I_{\text {tr }}, \tan \Theta, \Psi$ at the


Fig. 6. Heights of the nonlinear "jumps" of the transmitted light intensity as functions of the induction of the external magnetic field referring to the incident light intensities: $a-I_{0}=I_{0}^{\dagger}$ ("upward jump"), b-I $I_{0}=I_{0}^{1}$ ("downward jump")
points $I_{0}=I_{0}^{\dagger}, I_{0}=I_{0}^{\dagger}$ are also the functions of $B_{0}$. It is illustrated in Fig. $6\left(I_{\mathrm{tr}}\right)$, Fig. $7(\tan \Theta)$, and Fig. $8(\Psi)$. The figures denoted by the letter $\mathbf{a}$ and $\mathbf{b}$ refer to the changes at the point $I_{0}=\AA$ and the point $I_{0}=I_{0}^{\downarrow}$, respectively. In the range $B_{0}=0-0.05 \mathrm{~T}$ all the functions presented in Fig. $5-8$ appear to be approximately linear.


Fig. 7. Heights of the nonlinear "jumps" of the ratio of the polarization ellipse semiaxes of the transmitted light as functions of the external magnetic field referring to the incident light intensiticu: a $-I_{0}=I_{0}^{\dagger}$, $\mathbf{b}-I_{0}=I_{0}^{1}$


Fig. 8. Heights of the nonlinear "jumps" of the orientation angle of the transmitted light polarization ellipse as functions of the induction of the external magnetic field referring to the incident light intensities: $\mathbf{a}-I_{0}=I_{0}^{\dagger}, \mathbf{b}-I_{0}=I_{0}^{\downarrow}$

## 6. Summary and conclusions

The case of a dispersive optical bistability modified by the Faraday's effect is the subject of studies in this paper. The girotrophy, which is forced by an external magnetic field in a nonlinear ring cavity, causes bistable changes of the state of polarization of light transmitted through the cavity.

The external magnetic field splits the plane wave into right- and left-circularly polarized waves. The phases of those waves vary in different ways. Each phase function depends not only on its own amplitude but also on the amplitude of the other one. It changes the resonance conditions in the ring cavity and causes bistability of the intensity of the transmitted light. The intensity of each circularly polarized wave varies in a bistable way and therefore the effect of bistability of the polarization state of the transmitted light appears also in the ring cavity.

The numerical analysis proves that the parameters of the intensity and polarization hysteresis cycles can be controlled by the value of induction of the external magnetic field.

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## Течение оптической бистабильности в кольцевой нише в присутствии внешнего магнитного поля

Настоящая статья содержит результаты, относящиеся к течению дисперсионной оптической бистабильности в изотропной, нелинейной кольцевой нише. B ней было показано, что в присутствии внешнего магнитного поля линейно поляризованный свет, пропущенный через кольцевой резонатор, изменяет свою поляризацию в эллиптическую. Кроме этого, его интенсивность и состояние поляризации меняется бистабильно. Также направление оборота его электричсского вектора может изменяться разрывно.

