# Comparative studies on geometrical-optical performance of discrete mirror seasonally adjusted linear solar concentrator and continuous profile compound parabolic concentrator 

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#### Abstract

An attempt has been made to compare the geometrical concentration ratio and distribution of local concentration ratio over the flat horizontal absorber surfaces for geometries of the profiles of discrete mirror seasonally adjusted linear solar concentrator and continuous profile compound parabolic concentrator, using ray tracing procedure.


## 1. Introduction

The production of continuous profile compound parabolic concentrator, developed by Winston [1] and Winston and Hinterberger [2] for the concentration of solar radiation is difficult in the developing countries. The mirror profile composed of flat mirror elements can be generated [3], by using the basic principle of optics, simplifying the production and assembly procedure. In this work, the geometrical design, the geometrical concentration ratio CR and the distribution of local concentration ratio LCR over the flat-horizontal absorber surfaces of the discrete mirror seasonally adjusted linear solar concentrator and the continuous profile compound parabolic concentrator have been compared by using the ray tracing procedure.

## 2. Generation of a discrete mirror reflector profile

Figure 1 shows the geometry used in the generation of a discrete mirror seasonally adjusted linear solar concentrator. If $\theta_{\mathrm{m}}$ is the desired half maximum acceptance angle, then the angle made by the $j$-th mirror element with the horizontal is chosen so that a ray incident on the aperture plane at an angle equal to the half acceptance angle $\theta_{\mathrm{m}}$ and incident on the top edge of the mirror element is reflected back on the absorber to such a point that all rays incident at angles less than the acceptance angle will be intercepted by the absorber after one or more reflections. An implicit equation relating the slope $m(j)$ of the reflected ray and the slope of the mirror element is established and given by

$$
\begin{equation*}
m(j)=[y(j-1)+w \sin \psi(j)] /\left[x(j-1)+w \cos \psi(j)+d_{2} / 2\right] . \tag{1}
\end{equation*}
$$



Fig. 1. Geometry of discrete mirror seasonally adjusted linear solar concentrator
( $w$-width of the mirror element $(m), \psi(j)$-angle of the $j$-th mirror element with the horizontal (degrees), $d_{2}$-absorber width ( $m$ ).

This equation is solved for $\psi(j)$ using the numerical iterative procedure for the known values of acceptance angle and width of the mirror element. The width of the first mirror element on either side of the flat horizontal absorber is selected so that the angle of incidence on the receiver be less than $60^{\circ}$. As we know, the transmittance of glass cover for an incidence angle larger than $60^{\circ}$ is considerably reduced and the absorption is poor. The width $w_{1}$ of this mirror element is obtained from the following relation:

$$
\begin{equation*}
w_{1}=\left[d_{2} \sin \left\{2 \psi(1)-90-\theta_{m}\right\}\right] /\left[\sin \left\{90+\theta_{\mathrm{m}}+\psi(1)\right\}\right] . \tag{2}
\end{equation*}
$$

$\psi(1)-$ angle between the 1 st mirror element and the horizontal (degrees), $\theta_{\mathrm{m}}$-half maximum acceptance angle (degrees).

The rest of the profile is generated with the help of the Eq. (1). The largest possible angle of the mirror at the top is $90^{\circ}$ limiting the concentration and the height of the reflector profile. If $d_{1}$ is the entrance aperture, the geometrical concentration ratio is given by

$$
\begin{equation*}
C R=d_{1} / d_{2} . \tag{3}
\end{equation*}
$$

## 3. Generation of CPC profile

Figure 2 shows the geometry of the compound parabolic concentrator CPC with flat horizontal absorber, constructed according to the principle outlined by Winston and Hinterberger [2]. For the convenience of design and ray trace formulation, the equation of the parabola with origin $E\left(x_{1}, x_{1}\right)$ has been shifted to the point $0(x, y)$ at


Fig. 2. Geometry of continuous profile compound parabolic concentrator
the axis of CPC through a coordinate system $B\left(x_{2}, y_{2}\right)$ parallel to $0(x, y)$ and to another coordinate system $B\left(x_{3}, y_{3}\right)$ simply rotated by the acceptance angle $\theta_{m}$ clockwise to $0(x, y)$. The equations representing the profiles of the left and right hand sides of the CPC with origin at 0 can be expressed respectively, as follows:

$$
\begin{equation*}
\left[\left(x-d_{2} / d\right) \sin \theta_{m}+y \cos \theta_{\mathrm{m}}+1 / 4 g\right]-g\left[\left(x-d_{2} / 2\right) \cos \theta_{\mathrm{m}}-y \sin \theta_{\mathrm{m}}\right]^{2}=0, \tag{4}
\end{equation*}
$$

and

$$
\begin{equation*}
\left[-\left(x+d_{2} / 2\right) \sin \theta_{\mathrm{m}}+y \cos \theta_{\mathrm{m}}+1 / 4 g\right]-g\left[\left(x+d_{2} / 2\right) \cos 0_{\mathrm{m}}+y \sin \theta_{\mathrm{m}}\right]^{2}=0 \tag{5}
\end{equation*}
$$

where $g=1 /(4 f)$ and $f$-focal length of the parabola.

Making use of the $x$ and $y$ coordinates of the point $A\left(d_{2} / 2,0\right)$, Eqs. (4) and (5) can be expressed in the following simplified form:

$$
\begin{equation*}
\left.\left(4 d_{2}^{2} \cos ^{2} \theta_{\mathrm{m}}\right) g^{2}+\left(d_{2} \sin \theta_{\mathrm{m}}\right) g-1\right)=0 . \tag{6}
\end{equation*}
$$

Thus, for the known values of $d_{2}$ and $\theta_{\mathrm{m}}$ the above equation defines $g$, and hence the parabola. Another valuable approach is to select arbitrarily the values of $d_{2}$ and $\theta_{\mathrm{m}}$, according to the design requirement. Thus, the height $(y=H)$ of the CPC for the required concentration is treated as a variable and adjusted to maximize $d_{1}$, making use of Eqs. (4) and (5), thus finding the parabola and hence the CPC with the maximum concentration ratio.

## 4. Ray trace formulation

For ray tracing the collector aperture is divided into 1000 sections, on each of them one ray is made incident. The path of each ray is followed until it strikes the absorber. The ratio of the number of rays striking a particular zone on the flat horizontal absorber surface to the number of rays incident over the entrance aperture on an equal size zone provides the LCR.

## 5. Consideration of second and third reflections

If an incident ray does not reach the flat horizontal absorber after one reflection, being instead incident on the $k$-th mirror element, as shown in Fig. 3, the ray


Fig. 3. Ray undergoing 1st, 2nd and 3rd reflections from
reflected from $j$-th mirror element will now be treated as the incident ray for the $k$-th mirror element. The solutions of the equation representing the incident ray and the equation of the $j$-th mirror element give the coordinate points for obtaining the equation representing the reflected ray (incident for $k$-th mirror element) from $j$-th mirror element. The solutions of this equation and the equation of the $k$-th mirror element provide the coordinate points for the second reflected ray striking the absorber. For some values of the acceptance angles third reflections may also occur. It may be taken into account by following the same procedure as that for the second reflection, the ray reflected from the $k$-th mirror element being considered as the incident ray for the $l$-th mirror element.

## 6. Results and discussion

Figure 4 shows the variation of the geometrical concentration ratio with the ratio of the concentrator height to the absorber width for a discrete mirror seasonally adjusted linear solar concentrator and a continuous profile compound parabolic concentrator. It is observed that in the case of discrete mirror concentrator, the concentration ratio is smaller than that of CPC for the same ratio of height to absorber width, but still the concentration is within the useful range having the maximum value of about 6 .

The distributions of LCR on the flat horizontal absorber surfaces of the discrete mirror seasonally adjusted linear solar concentrator and continuous profile com-


Fig. 4. Variation of the concentration ratio with the height of the concentrators for a discrete mirror concentrator and a continuous profile CPC. Half acceptance angle $-8^{\circ}$ ( $\square$-compound parabolic concentration, $\Delta$-discrete mirror concentrator)
pound parabolic concentrator are shown in Fig. 5 and Fig. 6, respectively, the incidence angle ranging from $0^{\circ}$ to $8^{\circ}$. For angle of incidence $0^{\circ}$, the distribution is symmetrical for both the concentrators, but in the case of discrete mirror concentrator one observes a peak at the middle of the flat horizontal absorber surface, whereas the continuous profile CPC produces two peaks at the middle and other two at the ends of the absorber surface. The heights of the peaks at the middle of the absorber surface gradually increase and shift towards the left hand side, in both the cases the angles of incidence increase, while in the case of continuous profile CPC






Fig. 5. Local concentration ratio distributions over the flat-horizontal absorber of discrete mirror concentrator


Fig. 6. Local concentrator ratio distributions over the flat-horizontal absorber of continuous profile CPC

those near the ends of the absorber surface diminish rapidly as the angle of incidence increases. From the above it follows that the local concentration ratio distribution in the case of a discrete mirror seasonally adjusted linear solar concentrator is more uniform than that of continuous profile compound parabolic concentrator.

The average number of reflections $\langle n\rangle$ and first, second and third reflections from the reflector walls for CPC and discrete mirror concentrator are shown in the Table. It is observed that the average number of reflections $\langle n\rangle$ in the case of continuous profile compound parabolic concentrator is higher than that for the discrete mirror

Average number of reflections $\langle n\rangle$ and the percentage of rays undergoing 1st, 2nd and 3rd reflections

| Concentrator <br> type | Average <br> reflection <br> $\langle n\rangle$ | Percentage <br> of direct <br> reflection | Percentage <br> of 1st <br> reflection | Percentage <br> of 2nd <br> reflection | Percentage <br> of 3 rd <br> reflection |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Discrete mirror <br> concentrator | 0.87 | 13.31 | 86.69 | 0 | 0 |
| Continuous <br> profile CPC | 0.96 | 16.69 | 73.48 | 8.01 | 1.18 |

seasonally adjusted linear solar concentrator, although the discrete mirror concentrator has losses at the mirror junctions. It is further noticed that there are no second and third reflections in the case of a discrete mirror concentrator, and chat they occi in the case of CPC.

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## References

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## Сравнительное геометрико-аналитическое изучение действия линейного солнечного концентратора, состоящего из аддитивных зеркал, сезонно регулируемого, а также параболического концентратора постоянного профила

При помощи расчета хода лучей предпринят опыт геометрического сравнения локального распределения коэффициента концентрации на плоской горизонтальной поверхности абсорбера, геометрии профилей солнечного концентратора дискретного профиля зеркала регулируемого сезонно, а также параболического концентратора постоянного профиля.

