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Residual birefringence in the gradient-index lenses

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The way of the calculation of the wavefront splitting caused by an isotropy induced by residual stress in the gradient lenses has been shown. Exemplified results of anisotropy measurements and the calculated splitting of the wavefronts are given.

1. Introduction

Gradient lenses of SELFOC type constitute a special type of lenses of parabolic distribution of refractive index [1]. Theoretical profile of refraction for such lenses is given by formula

$$n(r) = n_0\left[1-\frac{\mathrm{A}}{2}r^2\right], \quad 0 < r < r_0 \tag{1}$$

where: n_0 – refractive index on the axis,
A – parameter,
r_0 – radius of the lens.

Became of such a profile of the refractive index the gradient lens possesses focusing properties characteristic of a traditional lens.

The features of the lenses of SELFOC type, which decide of their applicability, are:

– small sizes and weight (diameter of order of few millimeters, length of few to tens millimeters),

– possibility of obtaining an arbitrary focal length regulated by the length of the lens.

Due to these advantages the gradient lenses are of interrest to the designers of the microoptics devices in the telecommunication systems, copying or medical devices, and the like.

The gradient lenses are produced in the ion exchange process occurring in high temperature. Its result is a glass rod of doping concentration depending on the distance from the rod axis. Some stresses appearing in the lenses produced in this way make them birefringent [2]. These stresses are due to nonuniformity of material (different doping concentrations) and to the differences between the coefficients of linear expansion in the particular regions of the lens as well as to the temperature dependent processes during production of the material. A similar effect occurs during production of the waveguide preforms.

Anisotropy evoked by these stresses causes the splitting of the input wavefront W_0 into two wavefronts $W_r(r)$ and $W_\theta(r)$ of linear polarization (of radial and tangential types, respectively), Fig. 1.

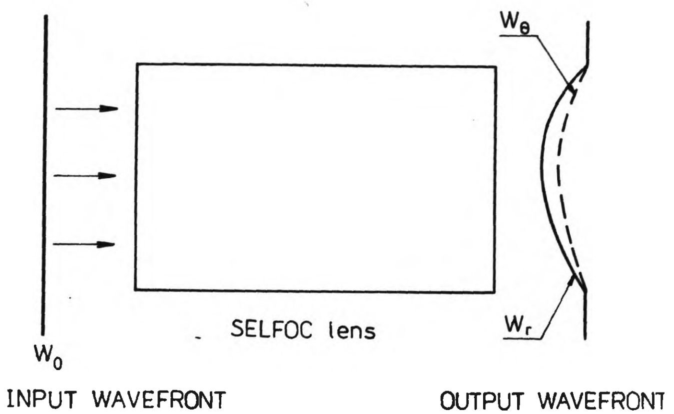

Fig. 1. Split of the input wavefront W_0 into two wavefronts W_r and W_θ of respective radial and tangential linear polarization by the gradient lens

Consequently the following effects occur:

– Worsening of the imaging quality. Similar effects caused by the residual internal stresses appear in the classical lenses [3].

– Varying polarization state of the output beam (depending on θ, r) which may exclude the application of this type ol lenses, when the transformation of the state of polarization is undesirable (for instance, in the junctions with the single mode polarization fibres).

In the present work, the way of determining the split of the wavefronts $D = W_r - W_\theta$ is presented. The advantage of the method lies in a simultaneous and nondestructive measurement of the refractive index distribution and its anisotropy in one cross-section of the gradient rod. This enables the calculation of the optical path differences d for the lenses of different lengths (and thus of different focal lengths) cut of measured gradient rod.

2. Method of refractive index distribution

Under the influence of internal stress the lens becomes an anisotropic object of principal refractive indices $n_z(r)$, $n_r(r)$, $n_\theta(r)$. For this reason a plane illuminating wave $W(x)$ suffers from splitting into two waves: $W_z(x)$ and $W_x(x)$ (Fig. 2) of linear polarizations consistent with the directions of z and x axes, respectively. The measurement of the difference $R(x) = W_z(x) - W_x(x)$ called the retardation function

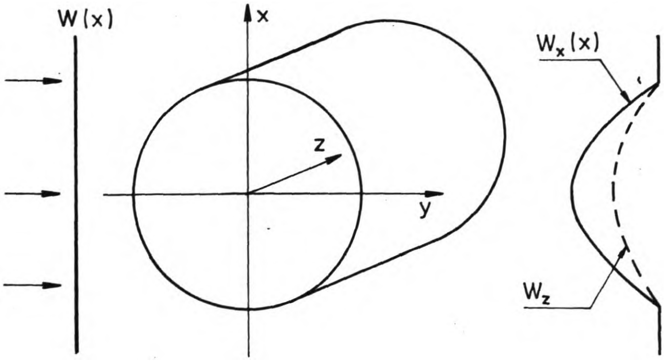

Fig. 2. Split of the plane illuminating wave $W(x)$ perpendicular to the axis of symmetry by an anisotropic gradient lens. $W_x(x)$ and $W_z(x)$ denote the linearly polarized waves of azimuths consistent with the x and z axes, respectively

enables the calculation of both the stress components, which in the cylindric coordinate system [4], [5], [6] are:

$$\delta_z(r) = \frac{1}{\pi c}\int_r^{r_0} \frac{\frac{dR(x)}{dx}}{\sqrt{x^2-r^2}}\,dx, \tag{2a}$$

$$\delta_r(r) = \frac{1}{r^2}\int_0^r \delta_z(r)\,r\,dr, \tag{2b}$$

$$\delta_\theta(r) = \delta_z(r)-\delta_r(r), \tag{2c}$$

and the refractive index anisotropy evoked by the stress, i.e.:

$$n_z-n_r = C(\delta_z-\delta_r) = C\delta_\theta, \tag{3a}$$

$$n_z-n_\theta = C(\delta_z-\delta_\theta) = C\delta_r, \tag{3b}$$

$$n_r-n_\theta = C(\delta_r-\delta_\theta) = C(2\delta_r-\delta_z) \tag{3c}$$

where: C – photoelastic constant,
r, z, θ – indices indicating the respective radial, axial and tangential components.

It is worth noting that, when substituting Eqs. (2) to (3), the knowledge of photoelastic constant C is not necessary to calculate the anisotropy of the refractive index.

The profile of the refractive index $n(r)$ of the examined lens has been determined with the help of a method involving also the transillumination of the lens with the plane wave in the direction perpendicular to its axis of symmetry (Fig. 3). The

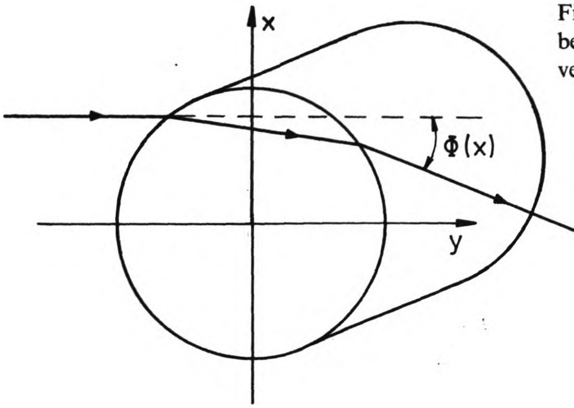

Fig. 3. Deflection of the ray $\Phi(x)$ after having been passed through a gradient lens — transversal illumination

distribution of the refractive index inside the lens is calculated from the formula [7]

$$\frac{n(r)-n_i}{n_i} = -\frac{1}{\pi}\int_r^{r_0} \frac{\Phi(x)}{\sqrt{x^2-r^2}}dx \tag{4}$$

where: n_i — refractive index of immersion in which the lens is located,
$\Phi(x)$ — angle of the ray deflection.

In reality, due to the existence of the residual anisotropy the measured refractive index is equal to an average [6]

$$n(r) = \frac{n_\theta(r)+n_r(r)}{2}. \tag{5}$$

The angle of deviation $\Phi(x)$ may be determined most conveniently by using the dynamic spatial-filtering technique [8]. The results of measurements for a lens taken by way of example are shown in Fig. 4. The measurements of both the angle of deviation Φ and the optical path difference $R(x)$ were made in a setup described in work [6].

3. Calculation method of wavefront splitting by a gradient lens

The light ray incident on a birefringent object, which may be a gradient lens due to its internal stress, suffers from wavefront splitting. Obviously the paths of these two splitted and orthogonally polarized rays inside the lens are different. The formula allowing us to determine numerically the trajectory of the ray in a gradient medium, in the absence of birefringence, is relatively simple and was repeatedly reported by many authors. For an anisotropic medium the following equations valid for the case of cylindric symmetry [9] may be used:

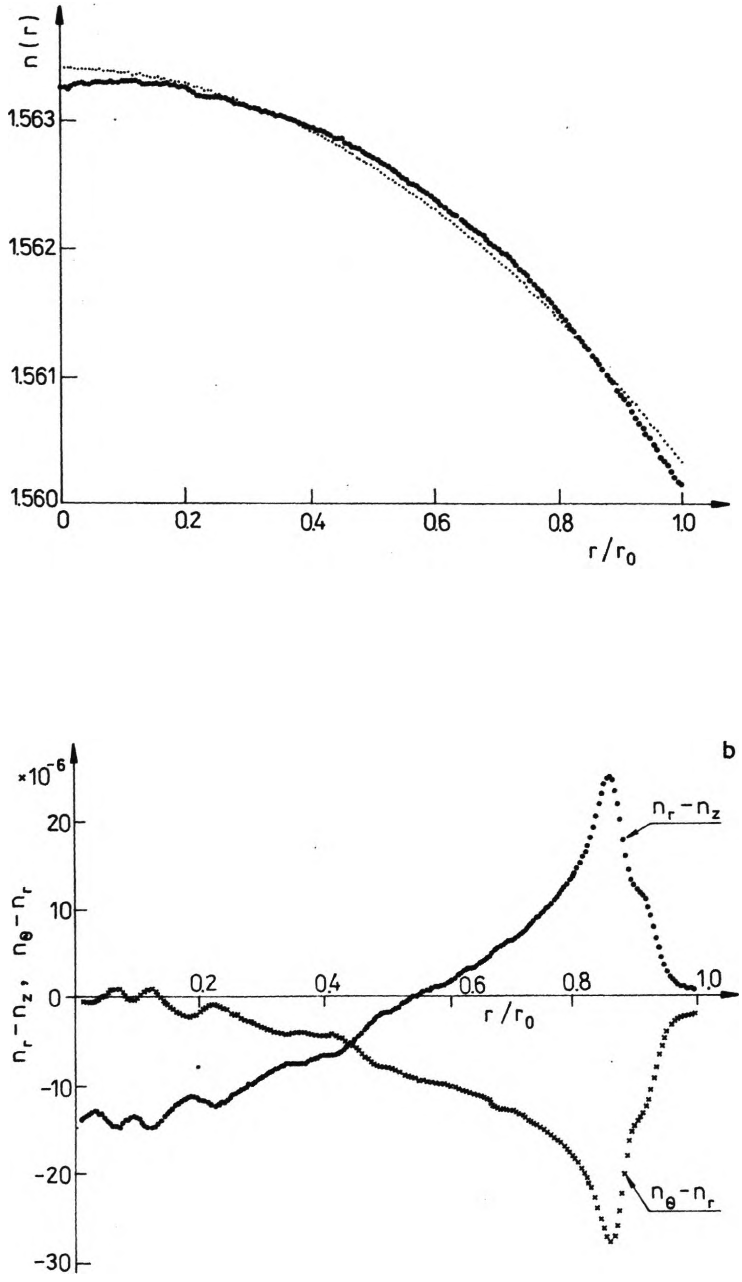

Fig. 4. Measured profile of the refractive index distribution for an exemplified gradient lens. Dotted line — approximation of the distribution by the function $n(r) = n_0\left(1-\frac{A}{2}r^2\right)$, where $n_0 = 1.5634$, $A = 3 \times 10^{-4}$ mm^{-2} (**a**); and measured differences of the principal refractive indices $n_r - n_z$ and $n_\theta - n_r$ for an exemplified gradient lens (**b**)

$$z(r) = l_{\text{in}} \int_{r_0}^{r} \frac{dr}{\pm\sqrt{m^2 - l_{\text{in}}^2}},$$
$$\theta(z) = \theta_{\text{in}} + \frac{c}{l_{\text{in}}} \int_{0}^{z} \frac{dz}{r^2} \tag{6}$$

where: $l_{\text{in}} = n_{\text{in}} \cos\gamma_{\text{in}}$, $m^2 = [n^2(r) - C^2/r^2]$,
C – constant, depending on the coordinates of the input ray, r, θ, z – coordinates in a cylindric coordinate system, indices "in" denote the input magnitudes.

However, these equations are not too convenient due to the inverse dependence of z on r. Moreover, independently of the kind of the used equations, the ray trajectory equations should be solved with the help of the time-consuming correction [10] methods due to both anisotropy and the dependence of the effective refractive index upon the angle made by the light ray with the axis z. For small anisotropy (Δn of order of 10^{-5}) its influence on the ray trajectory may be neglected. Even such a simplifying assumption allows us to calculate the optical path difference between the rays of different polarizations. Assume that both the rays travel along a common trajectory determined by an average refractive index (5), i.e., without taking account of the corrections for the difference in trajectories connected with different states of polarization attributed to each of them. This trajectory is determined by applying the ray equation [11]

$$\frac{d}{ds}\left(n(r)\frac{dr}{ds}\right) = \nabla n(r) \tag{7}$$

which is obtained from the Maxwell equations for an isotropic medium. This differential equation may be solved relatively easily by using numerical methods. Such procedures were presented in a number of works, e.g., in [12], [13]. The optical path difference D between the rays will be determined as a product of the geometric path (common to both the rays) and the refractive index difference ΔN

$$dD = \Delta N\, ds. \tag{8}$$

For the case of meridional rays considered in this work the difference ΔN between the effective refractive index N_θ for the ray of tangential polarization (polarization plane perpendicular to the lens radius) and the effective refractive index N_r for the ray of the radial polarization (polarization plane coinciding with the plane passing through the lens axis) may be determined with the help of the principal refractive indices n_z, n_r, n_θ and the angle of the ray slope γ in a given point of trajectory (Fig. 5):

$$\Delta N = N_\theta - N_r,$$
$$N_\theta = n_\theta, \tag{9}$$
$$N_r = n_r \cos^2\gamma + n_z \sin^2\gamma$$

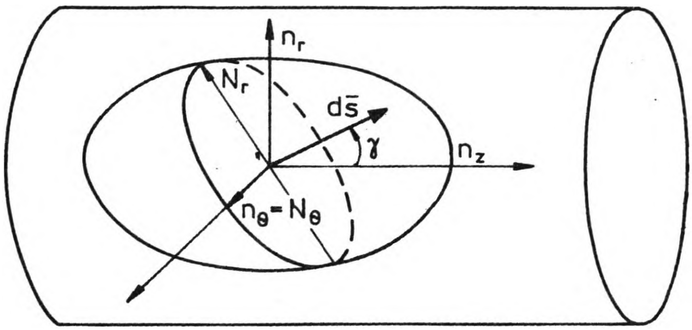

Fig. 5. Effective refractive indices N_θ and N_r for the rays of tangential and radial polarization, respectively, determined by the principal refractive indices n_r, n_z and n_θ (ds denotes the local direction of the ray in the given point of the lens)

hence it follows

$$\Delta N = (n_\theta - n_r) + (n_r - n_z)\sin^2\gamma. \tag{10}$$

Finally, the splitting of the wavefronts corresponding to the radial and tangential polarizations, respectively, is given by the formula

$$D = \int_S [(n_\theta - n_r) + (n_r - n_z)\sin^2\gamma]\, ds \tag{11}$$

where S – geometric path of both the rays along the common trajectory.

The above formula is very convenient for calculations since the differences of the principal refractive indices $n_\theta - n_r$ and $n_r - n_z$ are determined immediately when measuring the anisotropy.

4. Exemplified results and final remarks

The exemplified calculations have been performed by using Runge–Kutta algorithm [13] to determine the trajectory of the ray passing through the gradient medium. The wavefront split at the input of two gradient lenses (cut out of the same rod) of the lengths $z = 25$ mm and 90 mm (1/2 pitch) have been calculated (Fig. 6**a**,**b**). The second of these lenses images the plane input wave into a point in the output plane of the lens and that is why the value of the retardation has been given as a function of the output ray.

The wavefront split D in the exemplified lenses is great being of order of λ and 2λ, respectively. A possible influence of the accepted simplifying assumptions (common trajectory) on the accuracy of calculations of the splitting $D(r)$ has been estimated by inserting the examined lenses in a polariscope. A photo of the image from the polariscope for the lens 1 (Fig. 7) shown that the black fringe corresponding to the isochrome of first order $D = 560$ nm is positioned at the distance of about $0.8\, r_0$ from the middle. This confirms the results of the computer calculations (Fig. 6**a**).

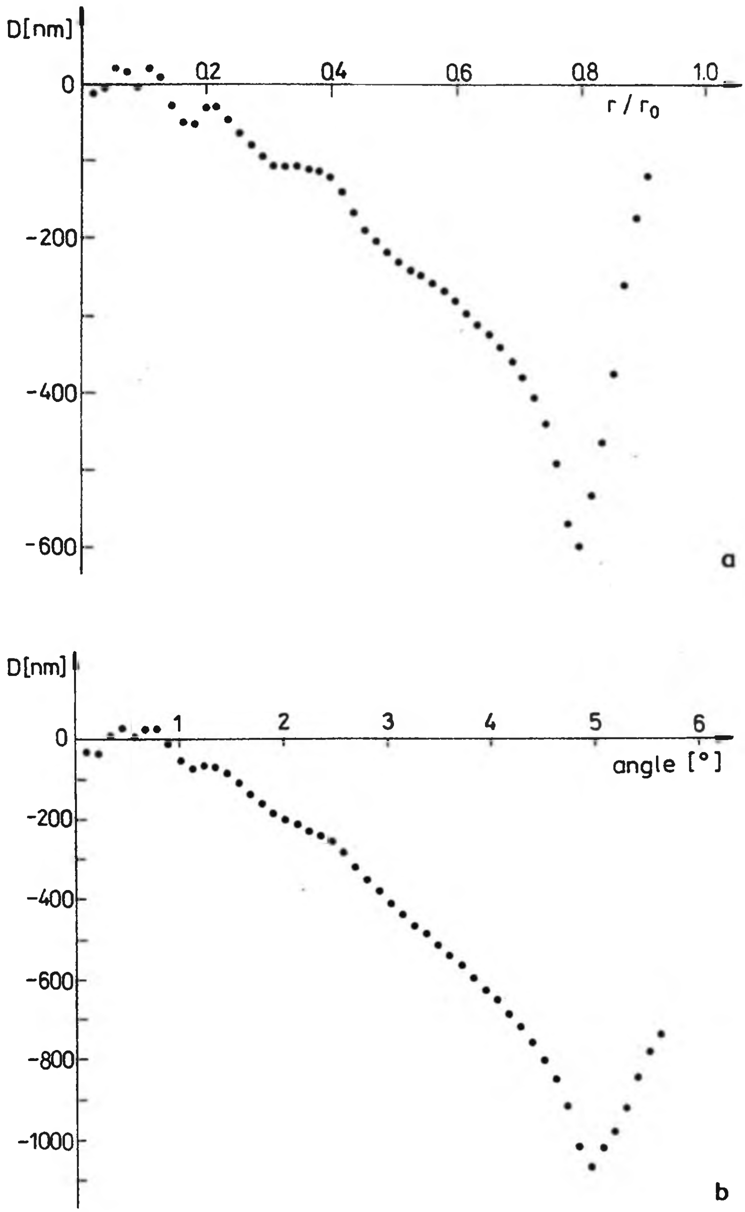

Fig. 6. Exemplified calculations of the split of the wavefronts after passing through the gradient lens: **a** – lens of the length $z = 25$ mm, **b** – lens of the length $z = 90$ mm (1/2 pitch)

In spite of the simplifying assumption concerning the common trajectory of both the rays the proposed method may be USEFUL for determining the split of the orthogonally polarized wavefronts in the selfoc type lenses at the presence of small residual birefringence. This may makes it possible to estimate the applicability of any particular lens to definite purposes.

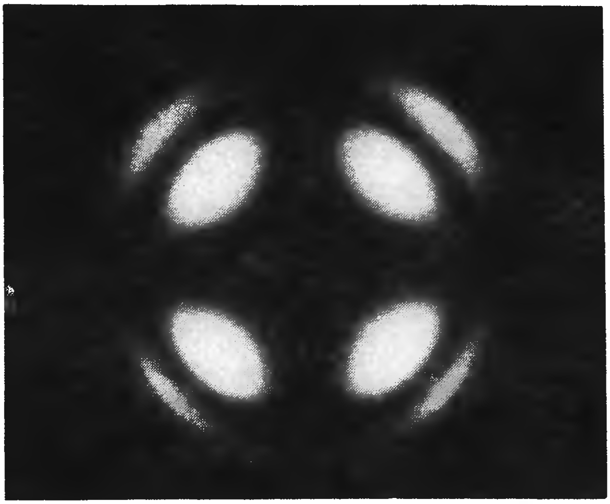

Fig. 7. Photo of a gradient lens of the length $z = 25$ mm in a polariscope. A black fringe of the first order isochrome is visible

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Остаточное двойное лучепреломлеиие градиентных линз

Представлен способ вычисления раздвоения волновых фронтов, вызванного анизотропией, индуцированной остаточными напряжениями в градиентных линзах. В работе помещены примерные результаты измерений анизотропии, а также вычислены раздвоения волновых фронтов.