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Phase conjugation through multiple gratings in photorefractives. Role of unequal coupling strengths and absorption

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Phase conjugation via degenerate four-wave mixing is considered when four different types of index gratings of unequal amplitudes are operative in photorefractive crystals. Effects of pump depletion and absorption are also included in the analysis. The shooting method is employed to obtain numerical solutions of the coupled-wave equations in the weak coupling regime. There are emphasized the roles of unequal coupling strengths and absorption in the multigrating operation and the influence of grating competition on the phase conjugate reflectivity (PCR). It is shown that for weak couplings, the equal strength multiple gratings have higher PCR than gratings of unequal strength, if the amplitudes of reflection and transmission of gratings are larger than those of pump and signal gratings. The presence of absorption reduces the PCR more significantly when all the gratings are operative than when a single grating is allowed. Computed results are presented in a graphical form.

1. Introduction

Over the last few years optical phase conjugation (OPC) via degenerate four-wave mixing (DFWM) in photorefractive crystals has become a subject of feverish activities in real-time holography, optical signal processing, adaptive optics (correction of aberrations) and holographic memory systems, to mention just a few areas of applications [1]–[3]. Great progress in this area, both in theoretical and experimental aspects has been made using sensitive photorefractive crystals such as BSO, BGO, $BaTiO_3$, SBN, etc. Theoretical description of stationary DFWM in OPC is usually based on a set of four first-order nonlinear coupled-wave equations for the slowly varying amplitudes of the interacting waves. In dealing with the DFWM or even two wave mixing (TWM) in reflection geometry [4] numerical techniques are commonly used to solve the boundary value problems of the coupled-wave equations.

Considerable efforts have been made to obtain an exact solution of the coupled wave equations for DFWM in the presence of either pump depletion or absorption or both. For example, CRONIN–GOLOMB et al. [1] presented such a solution in the case of negligible linear absorption. JA [5] treated the problem extensively through numerical computations for the case of a reflection geometry which includes both the effects of pump depletion and absorption. BELIC and LAX [6], and BELIC [7] separately solved analytically the coupled-wave equations with non-zero absorption in the transmission and reflection geometries, respectively. KUKHTAREV et al. [8] adopted method of CRONIN–GOLOMB et al. [1] to solve the problem in reflection geometry for an arbitrary photorefractive phase-shift between index grating and the light interference pattern. BELIC [9] has solved analytically the problem of energy and phase transfer of the interacting waves in the transmission geometry. Recently, BLEDOWSKI and KROLIKOWSKI [10] have reported a novel analytic method which treats exactly the effects of nonequal angles of incidence or different refractive index modulations of the interacting waves in the case of reflection geometry disregarding absorption.

The common feature of all these works lies in the fact that only one type of grating (either reflection or transmission) has been considered. However, in some cases the contributions of both transmission and reflection gratings to the phase-conjugate reflectivity may be comparable with each other. In general, four different index gratings may exist in the crystal and, in fact, single grating is present only when other gratings are suppressed by appropriate choices of polarization, coherence relationship of the beams and orientation of the crystal. For example, in the geometry of self-pumped phase conjugate mirror [3], [11] the formation of various gratings is unavoidable. The problem of multigrating operation has been investigated by KROLIKOWSKI [12] in the case of no pump depletion and no absorption, and by KROLIKOWSKI and BELIC [13] in the case of no pump depletion but with absorption. It should be noted that when the pump waves are much stronger than the signal and its conjugate, the contribution to the mixing of signal and its conjugate can be safely ignored so that only three types of grating mechanisms, namely, reflection, transmission and pump, will remain. Such a situation makes the theory somewhat simpler but still a complete analytic treatment is rather difficult. KROLIKOWSKI [12] solved the problem by assuming three special relationships between the coupling constants, while the authors in [13] treat it in a more general manner. Recently, BELIC [14] considered all the four grating mechanisms and he tried to reduce numerics to a minimum by assuming no absorption and two equal coupling strengths out of the four grating mechanisms. Equal coupling strengths of the mixing beams in DFWM occur when the counterpropagating waves see the same refractive index modulation or, in other words, when the interacting waves have the same polarization. On the other hand, coupling constants of mixing beams may differ owing to different angles of incidence of the beams or due to anisotropy of the modulation of refractive index.

The aim of the present paper is to consider the reflectivity in phase conjugation in the more general case when four index gratings with unequal amplitudes owing to different index modulations are operative, and when the effects of pump depletion and absorption are included. With more than one type of grating to be considered, a complete analytic treatment does not seem to be feasible if one considers the depletion effect of the pump beams and absorption in the medium. Such a situation makes the theory strongly nonlinear and it follows that, in general, numerical methods have to be used to obtain solutions of the coupled-wave equations. In our recent publication [15], we have reported a numerical method of solving the coupled nonlinear equations which is a simple and easy approach of shooting and matching technique. Emphasis was placed on the numerical method, so as regards the results of the calculations only intensities were given. In this paper we present the results of calculations of the phase conjugate reflectivity (PCR) which are of experimental interest (e.g., description of the DFWM scheme as a phase conjugate mirror with given reflectivity) and have not been reported earlier. As far as full multigrating operation is concerned, there exist many possible combinations of the relevant parameters for studying the overall impact in the governing process. It is then of interest to examine the role played by the intensity ratios at the competition of all couplings and absorption in PCR. Furthermore, the competition between the equal and unequal strengths of couplings will be another important part of the discussion in the paper. We have used a standard NAG library routine [16] which integrates the equations by Merson's method with a form of Newton iteration in a shooting and matching technique. This routine assures convergence only for weak couplings within few iterations from good initial estimates of unknown boundary values and a proper choice of matching point.

In the analysis that follows we begin with the governing coupled-wave equations needed to solve the problem of multiple gratings in a more general manner. Applying shooting and matching technique from a standard routine [16] to coupled-wave equations, we compute the PCR as a function of four different coupling strengths and absorption in the medium. We study the dependence of the PCR on the ratio of two pump beams and signal-to-reference beam ratio with coupling strengths and absorption as varying parameters. We show the influence of grating competition on the PCR for various combinations of coupling mechanisms and the absorption with special emphasis put on the competition between equal and unequal strengths of coupling. The results of numerical calculations will be presented here only for the case of weak coupling. We compare out results with the cases when either pure reflection [5], [7] or transmission [1], [6] gratings are responsible for the phase conjugation and also when equal strength multiple gratings [14] are operative. Numerical results obtained from computer calculations are discussed and presented in the graphical form.

2. Description of coupled-wave equations in multiple gratings

The basic interaction geometry of the FWM process is shown in Fig. 1. The photorefractive crystal, situated between the planes $z = 0$ and $z = d$, is illuminated by two counterpropagating pump waves A_1 and A_2. Because of the nonlinear coupling of the waves in the medium, a signal wave A_4, incident from the side of A_1, causes generation of the wave A_3, which is a phase-conjugate replica of the signal wave.

As mentioned in the Introduction, four different grating mechanisms exist in a standard DFWM geometry. These are i) a large spaced transmission grating (T) due to mixing of A_4 with A_1 and A_3 with A_2, ii) a small spaced reflection grating (R) due to mixing of A_3 and A_1, and A_2 and A_4, iii) mixing (P) of two pumps A_1 and A_2, and iv) mixing (S) of signal A_4 with its conjugate A_3. Assuming all the waves to

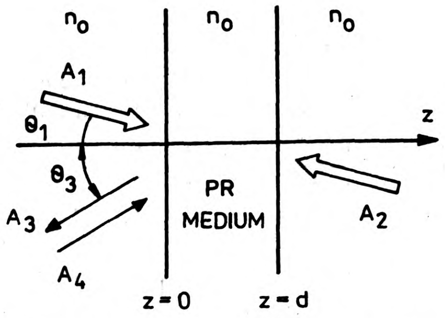


Fig. 1. Basic configuration for the DFWM process. A_1 and A_2 are the pump beams, A_4 is the signal beam and A_3 is the generated phase-conjugate beam. θ_1 and θ_3 are incident angles. It is assumed that the average refractive index n_o is the same in the different regions

be plane, the interaction between them is described by the set of four first-order nonlinear differential equations. These equations are derived from the combination of the Maxwell equations and the material equations [17], in a usual slowly varying amplitude approximation. Here we follow the treatment of CRONIN–GOLOMB et al. [1] and consider only the steady state situation. Taking absorption into account and considering the interaction of all the relevant mechanisms the equations take the following form:

$$\begin{aligned} IA_1' &= g_T A_T A_4 + g_R^* A_R A_3 + g_P A_P A_2 - (\alpha/2) I A_1, \\ IA_2^{*\prime} &= -g_T A_T A_3^* + g_R^* A_R A_4^* + g_P A_P A_1^* + (\alpha/2) I A_2^*, \\ IA_3' &= g_T A_T A_2 + g_R A_R^* A_1 + g_S A_S A_4 + (\alpha/2) I A_3, \\ IA_4^{*\prime} &= g_T A_T A_1^* + g_R A_R^* A_2^* + g_S A_S A_3 - (\alpha/2) I A_4^*. \end{aligned} \tag{1}$$

Here A_j $(j = 1, 2, 3, 4)$ represent the complex amplitudes of the waves and depend only on the propagation distance z (Fig. 1). The prime indicates the derivative with respect to z and the asterisk denotes the complex conjugation. $I = \sum_j |A_j|^2$ is the total intensity. $A_T = A_1A_4^* + A_2^*A_3$, $A_R = A_1A_3^* + A_4A_2^*$, $A_P = A_1A_2^*$ and $A_S = A_3A_4^*$ are the interference terms. $\alpha(= \alpha_I/\cos\theta_j)$ is the actual absorption coefficient, α_I is the intensity absorption coefficient at the wavelength of the incident beams and θ_j is the angle of incidence of the j-th wave. g_T, g_R, g_P and g_S represent the coupling parameters describing different ways in which light beams can combine to build the interference patterns. Their general form in Eqs. (1) is given by

$$g_k = i n_k \exp(i\Phi_k) \tag{2}$$

where $i = \sqrt{-1}$ and subscripts $k = $ T, R, P and S represent the four types of wave mixings, and n_k and Φ_k are respectively the amplitudes and phases of the index gratings, as described earlier. Expressions for n_k and Φ_k can be found elsewhere [1], [3], [17]. From expression (2) it follows that the change of signs and magnitudes of the coupling constants g_k or n_k can be achieved through the choice of an appropriate orientation of the crystal and/or polarization of the interacting waves. In the analysis that follows, a slight difference in the values of α and g_k in Eq. (2) owing to the different angles of incidence will be ignored.

Introducing $A_j = \sqrt{I_j}\exp(i\Psi_j)$ in the form of intensities I_j and the relative phases Ψ_j and using expression (2) for g_k, Eqs. (1) can be readily transformed into the following set of intensity equations:

$$
\begin{aligned}
dI_1/dz &= (2I_1/I)[n_T I_4 \sin\Phi_T - n_R I_3 \sin\Phi_R - n_P I_2 \sin\Phi_P) \\
&\quad + (2/I)(I_1I_2I_3I_4)^{1/2}[n_T \sin(\Phi_T+\Psi) - n_R \sin(\Phi_R - \Psi)] - \alpha I_1, \\
dI_2/dz &= (2I_2/I)[n_T I_3 \sin\Phi_T - n_R I_4 \sin\Phi_R - n_P I_1 \sin\Phi_P) \\
&\quad - (2/I)(I_1I_2I_3I_4)^{1/2}[n_T \sin(\Psi-\Phi_T) + n_R \sin(\Psi + \Phi_R)] + \alpha I_2, \\
dI_3/dz &= (2I_3/I)[-n_T I_2 \sin\Phi_T - n_S I_4 \sin\Phi_S - n_R I_1 \sin\Phi_R] \\
&\quad - (2/I)(I_1I_2I_3I_4)^{1/2}[n_R \sin(\Phi_R-\Psi) + n_T \sin(\Phi_T - \Psi)] + \alpha I_3, \\
dI_4/dz &= (2I_4/I)[-n_T I_1 \sin\Phi_T - n_R I_2 \sin\Phi_R - n_S I_3 \sin\Phi_S) \\
&\quad - (2/I)(I_1I_2I_3I_4)^{1/2}[n_T \sin(\Phi_T+\Psi) + n_R \sin(\Phi_R + \Psi)] - \alpha I_4
\end{aligned}
\tag{3}
$$

where I_1, I_2, I_3 and I_4 are intensities of two pumps, the generated phase-conjugate beam and the signal beam, respectively, and $\Psi = \Psi_4 + \Psi_3 - \Psi_2 - \Psi_1$ represents the phase mismatch.

We assume that all the gratings have the same phase shift with respect to their light interference pattern, i.e., $\Phi_T = \Phi_R = \Phi_P = \Phi_S = \Phi$ considering the most interesting case, in which $\Phi = \pi/2$. Such a phase shift appears when diffusion of photocarriers is dominant in the photorefractive process and it has the greatest practical importance [3]. We shall further restrict our attention to the case of exact phase conjugation (i.e., $\Psi = 0$ or π) in which case the variation the phases of the interacting waves with propagation distance (so-called phase transfer) has been dropped out from the system of Eqs. (3). On these assumptions, Eqs. (3) take the following simplified from for describing only the stationary energy transfer:

$$
\begin{aligned}
dI_1/dz &= (2I_1/I)[n_T I_4 - n_R I_3 - n_P I_2] + (2/I)(I_1I_2I_3I_4)^{1/2}[n_T - n_R] - \alpha I_1, \\
dI_2/dz &= (2I_2/I)[n_T I_3 - n_R I_4 - n_P I_1] + (2/I)(I_1I_2I_3I_4)^{1/2}[n_T - n_R] - \alpha I_2, \\
dI_3/dz &= (2I_3/I)[-n_T I_2 - n_S I_4 - n_R I_1] - (2/I)(I_1I_2I_3I_4)^{1/2}[n_T + n_R] + \alpha I_3, \\
dI_4/dz &= (2I_4/I)[-n_T I_1 - n_R I_2 - n_S I_3] - (2/I)(I_1I_2I_3I_4)^{1/2}[n_T + n_R] - \alpha I_4.
\end{aligned}
\tag{4}
$$

We note that Equations (4) under appropriate conditions are reduced to the case when only one type of grating is present. For example, a pure transmission grating occurs when $n_R = 0$, [4], while a pure reflection grating occurs when $n_T = 0$, [5], [8] and, as usually, when two-wave terms are ignored (i.e., $n_S = n_P = 0$). These equations also reduce in the case of multiple gratings [12], [13] with negligible signal coupling ($n_S = 0$) and undepleted pumps (I_1, $I_2 > I_3$, I_4). Our main purpose here, is to investigate the influence on the PCR of various beam couplings when all the gratings are of unequal strengths i. e. when $n_R \neq n_T \neq n_S \neq n_P \neq 0$. Such a situation may exist commonly in some phase conjugate experiments. In this case it seems impossible to obtain analytic solutions of Eqs. (4). Even for equal strengths of couplings ($n_R = n_T$, $n_S = n_P$) solutions are not fully analytical [14].

Our task is thus to solve Equations (4) numerically with split boundary conditions specified on two opposite faces of the crystal. In general, a system of nonlinear differential equations with boundary conditions given at two (or more) points cannot be guaranteed to have an exact solution. It has to be solved iteratively. In the present calculations we use a standard NAG library routine [16] which integrates Eqs. (4) by means of Runge–Kutta–Merson method with a form of Newton iteration in a schooting and matching technique. The numerical steps in this routine follow closely the treatment used by JA [5].

3. Numerical results and discussions

In the numerical method, we estimate the unknown boundary values at each end point, namely, $I_1(d)$, $I_3(o)$, $I_4(d)$ and $I_2(o)$ and Eqs. (4) are integrated backwards from $z = d$ to $z = 0$ (d represents the thickness of the medium). An attempt is then made to match the computed solution with the known boundary conditions. During the backward integration, we use very small value of $I_3(d) = 10^{-13}$, instead of the exact boundary value $I_3(d) = 0$, in order to overcome the inaccuracies in the computed value of $I_3(o)$. At our first attempt, in order to verify the convergence with good starting guesses, we solved the equations for a pure reflection grating, disregarding absorption and we obtained the same results as those presented by JA [5]. Usually in solving Eqs. (4), the present routine [16] takes 6–10 iterations to obtain the solution with the accuracy of about 0.1%. Only in few situations, for saving the computation time, a proper matching point with a suitable step length of integration was chosen. The routine is found to be very efficient in the weak coupling regime. In the vicinity of the strong coupling regime or above the threshold value of the coupling strengths, multiple solutions are possible. Consequently, the present routine is not adequate and alternative technique should then be used. We believe that the general parameter mapping technique [18] can be used to establish the existence of multiple solutions.

The quantity of interest in our calculations is the phase conjugate reflectivity, PCR $= I_3(o)/I_4(o)$. Besides the coupling strengths (n_T, n_R, n_P, and n_S) and absorption (α), we consider two more parameters defined basing on the known boundary conditions. These are the intensity ratio of pumps $\gamma = I_2(d)/I_1(o)$, and the signal-to-reference beam ratio which can be defined in two ways [5]:

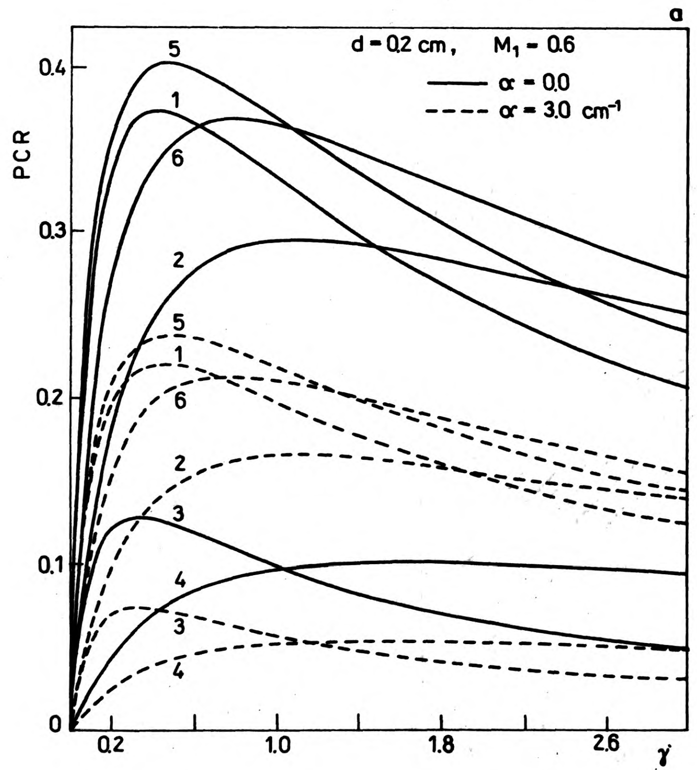
$$M_1 = I_4(o)/I_2(d), \qquad M_2 = I_4(o)/[I_1(o) + I_2(d)] = M_1\gamma/(1+\gamma). \tag{5}$$

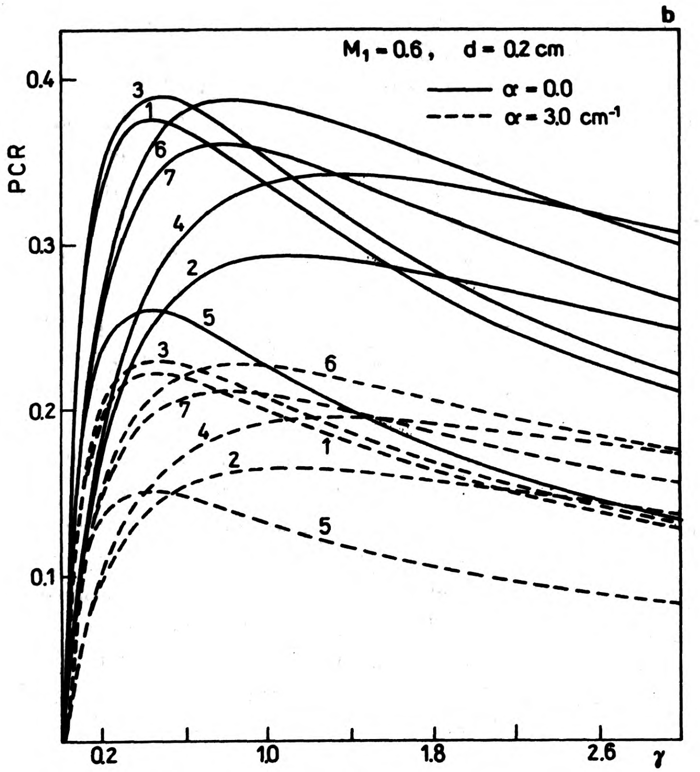
We must note here that the results are obtained only for weak couplings, i.e., when

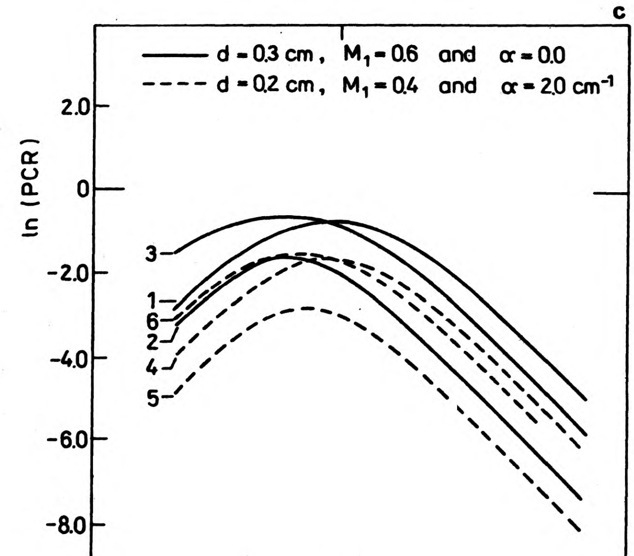
$n_k d \leqslant 1$. Their interpretations are thus no more valid in the strong coupling regime. The results of our numerical calculations are presented in Figs. 2**a**–**d**, 3 and 4. The competitive phenomena of equal and unequal strengths of couplings are the major concern of the present investigation. Besides, the presence of single grating (either reflection or transmission) is shown for comparison. The presence of absorption and its competition with various couplings is especially influential on the PCR.

In Figure 2**a**, **b** we show the results of the PCR as a function of the pump ratio γ, for various combinations of the coupling parameters. All the plots are made disregarding absorption (solid curves) as well as when the absorption is present (dashed curves). The crystal thickness is set to $d = 0.2$ cm, and the strength of all couplings $n_k d$ is kept smaller than unity. As a whole, the PCR increases rapidly for small values of γ, reaches a maximum and then decreases. The maximal value of the PCR does not necessarily occur at $\gamma = 1$. This may be due to the asymmetry between the roles of the two pumps. Presence of absorption reduces the PCR considerably in multiple gratings, regardless of whether the gratings are of equal or unequal strengths. It is significant when a single grating is present.

The most characteristic feature of the competition between equal and unequal strengths of the gratings in Fig. 2**a** is that the equal strength multiple gratings (curves 5) can show higher PCR than the gratings of unequal strengths (curves 1), if the reflection and transmission couplings become stronger than the pump and signal




b
$M_1 = 0.6$, $d = 0.2$ cm
$\alpha = 0.0$
$\alpha = 3.0$ cm^{-1}
PCR
0.4
0.3
0.2
0.1
0.2
1.0
1.8
2.6
γ
3
1
6
7
4
2
5
3
6
7
4
1
2
5


c
$d = 0.3$ cm, $M_1 = 0.6$ and $\alpha = 0.0$
$d = 0.2$ cm, $M_1 = 0.4$ and $\alpha = 2.0$ cm^{-1}
ln (PCR)
2.0
0
-2.0
-4.0
-6.0
-8.0
-4.0
-2.0
0
2.0
4.0
6.0
ln γ
3
1
6
2
4
5

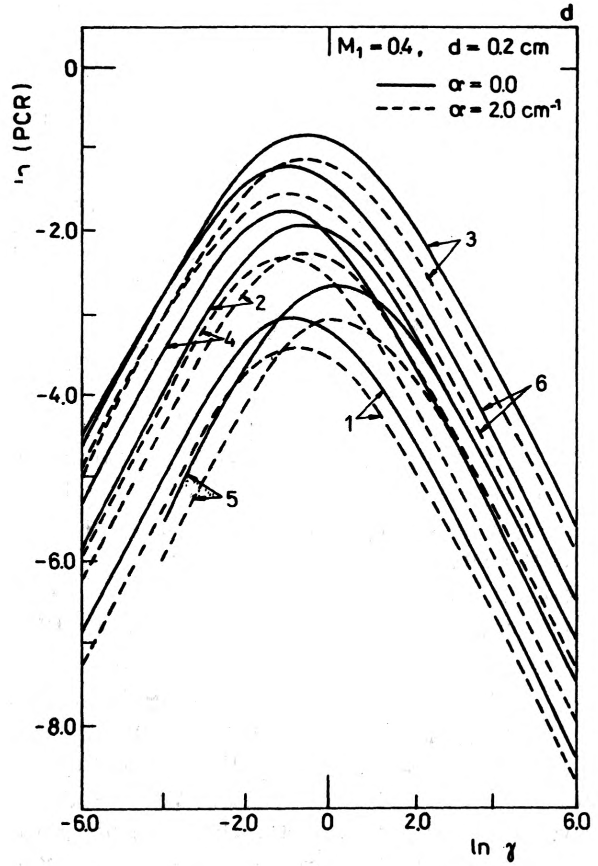


Fig. 2. PCR as a function of the pump beam ratio, γ with four coupling constants (n_R, n_T, n_P and n_S) and aberration (α) as parameters (n_k and α are given in cm^{-1}). Number in the parenthesis corresponds to the various combinations of couplings: **a** – (1) $n_R = 4.0$, $n_T = 3.6$, $n_S = 2.6$, $n_P = 3.0$; (2) $n_R = 4.0$, $n_T = 3.6$, $n_S = -2.6$, $n_P = -3.0$; (3) $n_R = 4.0$, $n_T = n_S = n_P = 0$; (4) $n_T = 4.0$, $n_R = n_S = n_P = 0$; (5) $n_R = n_T = 4.0$, $n_S = n_P = 3.0$; (6) $n_R = n_T = 4.0$, $n_S = n_P = 0.0$. **b** – (1) $n_R = 4.0$, $n_T = 3.6$, $n_S = n_P = 3.0$; (2) $n_R = 4.0$, $n_T = 3.6$, $n_S = n_P = -3.0$; (3) $n_R = n_T = 4.0$, $n_P = 3.0$, $n_S = 0.0$; (4) $n_R = n_T = 4.0$, $n_P = -3.0$, $n_S = 0.0$; (5) $n_R = n_T = 3.0$, $n_S = n_P = 4.0$; (6) $n_R = n_T = 4.0$, $n_S = 3.0$, $n_P = 0.0$; (7) $n_R = 4.0$, $n_T = 3.6$, $n_S = 3.0$, $n_P = 0.0$. **c** – (1) $n_R = n_T = 3.16$, $n_S = n_P = 0.0$; (2) $n_R = 3.16$, $n_T = n_S = n_P = 0.0$; (3) $n_R = n_T = 3.16$, $n_S = n_P = 4.64$; for $d = 0.3$ cm, $M_1 = 0.6$ and $\alpha = 0$. (4), (5) and (6) have respectively the same combinations of (1), (2) and (3) for $d = 0.2$ cm, $M_1 = 0.4$ and $\alpha = 2.0$ cm^{-1}. **d** – (1) $n_R = 1.4$, $n_T = 1.0$, $n_S = 2.0$, $n_P = 2.4$; (2) $n_R = 2.4$, $n_T = 2.0$, $n_S = 1.0$, $n_P = 1.4$; (3) $n_R = 4.4$, $n_T = 4.0$, $n_S = 1.0$, $n_P = 1.4$; (4) $n_R = 4.4$, $n_T = n_S = n_P = 0.0$; (5) $n_T = 3.16$, $n_R = n_S = n_P = 0.0$; (6) $n_R = n_T = 3.16$, $n_S = n_P = 4.64$

couplings (i.e., n_R, $n_T > n_P$, n_S). This is correct only when all the individual couplings are small enough to satisfy $n_k d \leqslant 1$. Furthermore, under this condition, a relatively small value of the PCR is observed when a single grating (curves 3 and 4) is operative. For small values of γ, a pure transmission grating (curves 4) shows a smaller PCR than a pure reflection grating (curves 3). PCR for a pure transmission grating exhibits maxima much broader than in the case of pure reflection. Opposite coupling of n_S and n_P with respect to n_R and n_T (curves 2) lead to decrease in the PCR with a broad maximum as compared to the case when coupling strengths are of the same singns (curves 1). Simultaneous presence of the reflection and transmission gratings of equal strength and without the pump and signal couplings (curves 6) may sometimes be favourable for phase conjugation for $\gamma > 1$.

Similarly as in Figure 2**a**, we see in Figure 2**b** that there is always a strong influence of the reflection and transmission couplings on the PCR. However, the contributions from the pump and signal couplings cannot be ignored in any way (curves 3 and 6). As seen from the curves 1, 3, 6 and 7, careful choices of combinations of the coupling parameters improve the generation of the phase conjugate wave.

The PCR for the wide range of γ is shown in Figs. 2**c** and **d**; a log-log scale is used. Solid curves 1 and 2 in Fig. 2**c** correspond to $\alpha = 0$, [14], and dashed curves 4–6 to $\alpha \neq 0$ with smaller values of M_1 and d. The combinations of all the couplings are kept the same in both cases. For the same combinations of coupling strengths, the PCR decreases owing to $\alpha \neq 0$ and small values of d. When $\gamma < 1$, relative variation between the curves 2 and 5 of a pure reflection grating is found to be comparatively large, probably due to the strong competition between α and n_R.

The maximal value of the PCR does not occur at the same value of γ for all combinations of n_R, n_T, n_S and n_P, as shown in Fig. 2**d**. Position of the maximal PCR shifts either to the left or to right depends on the relative magnitudes and signs of the coupling parameters. Here, it is clearly seen that for smaller amplitudes of the reflection and transmission gratings in the unequal strength multiple gratings (n_R, $n_T < n_S$, n_P; curves 1), maximal value of the PCR becomes small. PCR is found to be smaller than for the equal strength gratings ($(n_R = n_T) < (n_S = n_P)$, curves 6) and also in the case when a single grating (curves 4 and 5) is present. On the other hand, the PCR becomes large when n_R, $n_T > n_S$, n_P, e.g., the highest maximal value of the PCR can be observed in the curves 3. Thus, the strong influence of n_R and n_T on n_S and n_P always takes place when the full multigrating phase conjugation occurs for any combinations of couplings with the same signs. We note that this competitive feature of equal and unequal couplings is correct only if the couplings are weak. For strong couplings, however, the PCR is restricted to less than unity in the equal strength multiple gratings and in the presence of a pure reflection or transmission grating (with unequal pumps) the PCR can be significantly larger than one [14]. In addition, the effect of absorption is more pronounced near $\gamma = 1$ than at very large and small values of γ. For $\alpha \neq 0$, PCR depends on the relative magnitudes of the absorption and the relevant couplings.

Figure 3 shows the following characteristics. Similar to the results reported for the case of a pure reflection gratings [5] the PCR approaches saturation when $M_2^{-1} \geqslant 5.0$ without showing any maximum. As expected from the energy transfer for different types of coupling mechanisms [15] all the curves do not a similar trend. For example, the PCR increases at a slower rate for three cases: opposite couplings (curves 2), single grating (curves 5) and small values of couplings (curves 6). Over a wide range of M_2, the PCR strongly depends on the relative magnitudes and signs of the coupling parameters. A drastic increase in the PCR can be observed in the

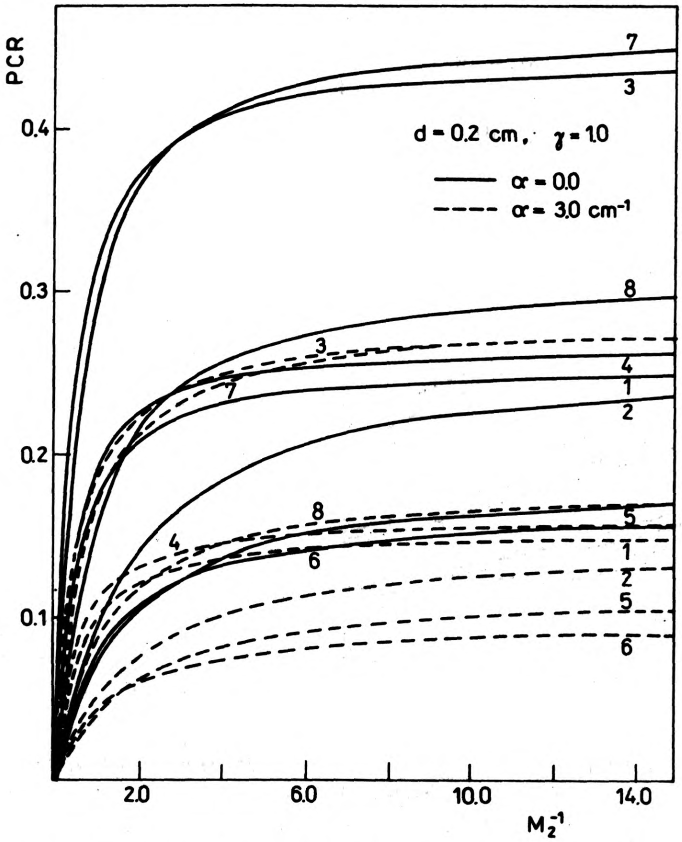


Fig. 3. PCR as a function of M_2^{-1} (Eq. (5)) with n_R, n_T, n_P and n_S and α as parameters for $d = 0.2$ cm, and $\gamma = 1.0$: (1) $n_R = 3.16$, $n_T = 3.0$, $n_S = 4.0$, $n_P = 4.64$; (2) $n_R = 3.16$, $n_T = 3.0$, $n_S = -4.0$, $n_P = -4.64$; (3) $n_R = 4.64$, $n_T = 4.0$, $n_S = 3.0$, $n_P = 3.16$; (4) $n_R = n_T = 3.16$, $n_S = n_P = 4.64$; (5) $n_R = 4.64$, $n_T = n_S = n_P = 0.0$; (6) $n_R = 2.4$, $n_T = 2.0$, $n_S = 1.0$, $n_P = 1.4$; (7) $n_R = n_T = 4.2$, $n_S = n_P = 1.4$; (8) $n_R = n_T = 3.16$, $n_S = n_P = 0.0$

curves 3 and 7, where the influence of n_R and n_T on the PCR becomes strong enough to suppress other couplings. Absorption is more effective for $M_2 < 1$ whereas it is less significant for $M_2 > 1$.

Figure 4 indicates that PCR is inversely proportional to α. The magnitude of the PCR for a particular α shifts upwards depending on the relative strengths of couplings. In general, absorption considerably influences the PCR when more than one grating is operative. In this case, the competition of absorption with all the relevant couplings plays an important role in phase conjugation. In practice, the influence of α on the PCR strongly depends on the reflection and transmission couplings. Comparison of the curves 1 and 2 indicates that the competition of α with n_S and n_P is not as effective as with n_R and n_T. Simultaneous presence of equal strengths of couplings (curve 9) leads to a decrease in the PCR owing to the comparatively small magnitudes of n_R and n_T. Comparison of the single grating cases (curves 5 and 6) indicates a rapid decrease of the PCR at large values of α for the transmission grating.

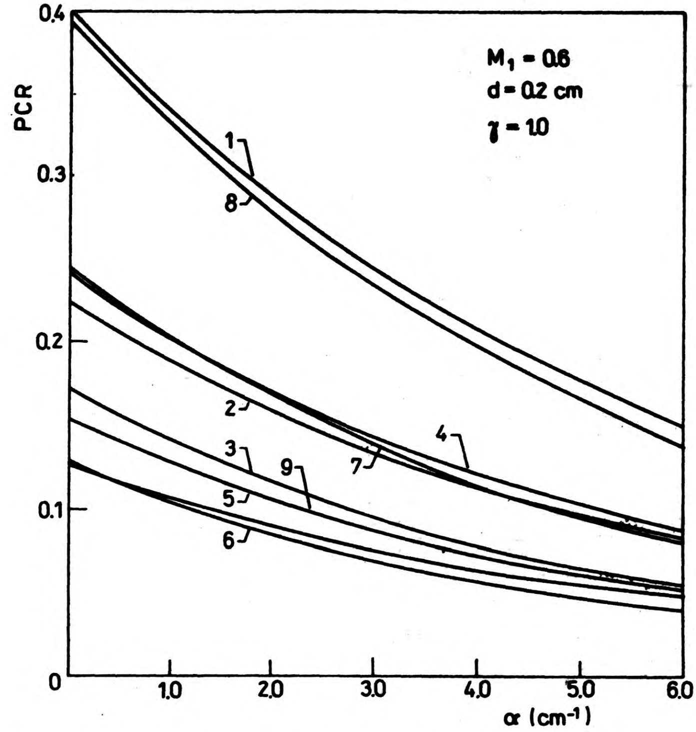


Fig. 4. PCR as a function of α in cm^{-1} for $M_1 = 0.6$, $d = 0.2$ cm and $\gamma = 1.0$: (1) $n_R = 4.64$, $n_T = 4.0$, $n_S = 3.0$, $n_P = 3.16$; (2) $n_R = 3.16$, $n_T = 3.0$, $n_S = 4.0$, $n_P = 4.64$; (3) $n_R = 3.16$, $n_T = 3.0$, $n_S = -4.0$, $n_P = -4.64$; (4) $n_R = n_T = 3.16$, $n_S = n_P = 4.64$; (5) $n_R = 4.64$, $n_T = n_S = n_P = 0.0$; (6) $n_T = 4.64$, $n_R = n_S = n_P = 0.0$, (7) $n_R = n_T = 3.16$, $n_S = n_P = 0.0$; (8) $n_R = 4.4$, $n_T = 4.0$, $n_P = 1.4$; $n_S = 1.0$; (9) $n_R = n_T = n_S = n_P = 2.4$

4. Conclusions

We have investigated on the example of PCR the effect of competition of various combinations of couplings with unequal strengths in the multigrating phase conjugation. Results of our numerical calculations are presented in the weak coupling regime (i.e., $n_k d$, up to 1), including the effects of absorption and pump depletion. It has been shown that for weak couplings, the multiple gratings of equal strength have a higher PCR than those of unequal ones provided the amplitudes of reflection and transmission gratings are kept larger than those of pump and signal gratings. Presence of absorption reduces the PCR significantly when all the relevant gratings are operative, whereas, it is less significant when a single grating is present. In addition to reflection and transmission couplings, both signal and pump couplings may play an important role in phase conjugation. Indeed, in some cases a careful choice of the combinations of all couplings may be used to improve the PCR. Thus, in the weak coupling regime a compromise should always be reached over equal and unequal amplitudes of the multiple gratings in order to optimize the PCR. In our calculations, the routine [16] is found to be very efficient with a desired accuracy for $n_k d$ up to 1. However, the routine will not be adequate for strong couplings (say, $n_k d > 10$), as in those cases the PCR becomes unstable and multiple solutions may be possible. A general parameter mapping technique [18] can then be used to establish the existence of multiple solutions. Recently, similar calculations have been reported by BELIC and KROLIKOWSKI [19] for the strong coupling regime. However, there are some basic differences in the subject and the method itself:

1. We solve the intensity coupled-wave equations, whereas in [19] the complex amplitude equations have been solved.

2. We use NAG routine [16] in the shooting and matching technique, which restricts the calculations only to the weak coupling regime ($nd \leqslant 1$). In the strong coupling regime ($nd > 1$) the routine is found not to be adequate, due to inherent instability of the solutions.

3. Belic and Krolikowski consider two out of four coupling mechanisms. In this way, the individual impact of a particular coupling mechanism on the phase conjugation has been clearly explained. We have shown the results with respect to the presence of various combinations of all the relevant couplings.

4. The role of absorption on the PCR in the strong coupling regime shows some interesting features which have not been found in the weak coupling regime. It is of particular interest for the former that the presence of absorption exhibits beneficial influence on the multigrating phase conjugation.

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Сопряжение фаз посредством мультирешеток в фоторефракционных материалах. Роль неодинаковой силы сопряжения и абсорбции

Фазовое сопряжение посредством вырожденного черырехволнового смешивания рассматривается тогда, когда четыре разных типа фазовой рефракционной неодинаковой амплитуды распространяются в фоторефракционном материале. Для анализа введены эффекты обеднения накачки, а также абсорбции. Использован метод „shooting" с целью получения численных решений уравнений для волны, сопяженной в условиях слабого сопряжения. Подчеркута роль неравной силы связи и абсорбции во многорещетчатом действии, а также влияние конкуренции решеток на понижающую способность сопряженной фазы (PCR). Было показано, что для слабого сопряжения мультирешетки равной „мощности" обладают высшим PCR чем решетки неравной мощности, если амплитуды отражения и передачи решеток больше, чем такие же величины для накачивающих и сигнальных решеток. Наличие абсорбции редуцирует PCR сильнее тогда, когда действуют все решетки, чем тогда, когда допускается лишь одна решетка. Результаты расчетов представлены графически.

Перевел Станислав Ганцаж