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# Letter to the Editor

## Image contrast in the coherent, aberration, apodized optical system. Rotating aperture at the Fourier transform plane

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It has been shown that the introduction of an amplitude phase apodizer into a coherent aberration optical system an imaging of periodical amplitude or phase object results in the change of the contrast which, in turn, depends on the test modulation depth and on the shape of the amplitude part of the function describing the apodizing filter. The change of contrast has been examined with respect to the function of apodizing filter as well as to the system aberration for amplitude apodizers of the types: $[1/2(1+r^2)]^p$, $(1-|r|^2)^p$ for $p = 1, 2, 3, 4$. In the next part of the text, the speckle-contrast has been shown in coherent optical system with time-varying pupil function and diffuse object.

### 1. Introduction

Let us assume that in the exit pupil of a coherent optical system there is an amplitude-phase apodizer of the transmittance

$$A(r) = t(r)e^{i\Phi(r)}, \qquad 0 < r \leqslant 1.$$

If we admit wave aberration in the optical system $W(x, y)$, then the total phase change in the pupil will equal

$$W(x; y) = w(x, y) + \Phi(r), \qquad r = \sqrt{x^2+y^2}. \tag{1}$$

As it is known, a coherent optical system is a linear filter with respect to the amplitude harmonic [1]. Coherent transfer function of such a system is

$$f(f_x, f_y) = P(\lambda f_x R, \lambda f_y)\exp\{ikW(\lambda f_x R, \lambda f_y R)\}. \tag{2}$$

where:

$k = \dfrac{2\pi}{\lambda}$ ($\lambda$ – light waveguide),

$f_x, f_y$ – spatial frequencies,

$R$ – reference sphere radius,

$$P(x, y) - \begin{cases} t(r) & \text{– pupil function within the pupil,} \\ 0 & \text{– beyond the pupil.} \end{cases}$$

To describe the optical system with quadratic detection, we shall apply the method employed in papers [2] and [3].

## 2. Amplitude and phase test

Assume that in the object space of an optical system there is a test of the amplitude transmittance

$$H(x, y) = a + b\cos(2\pi f_x x). \tag{3}$$

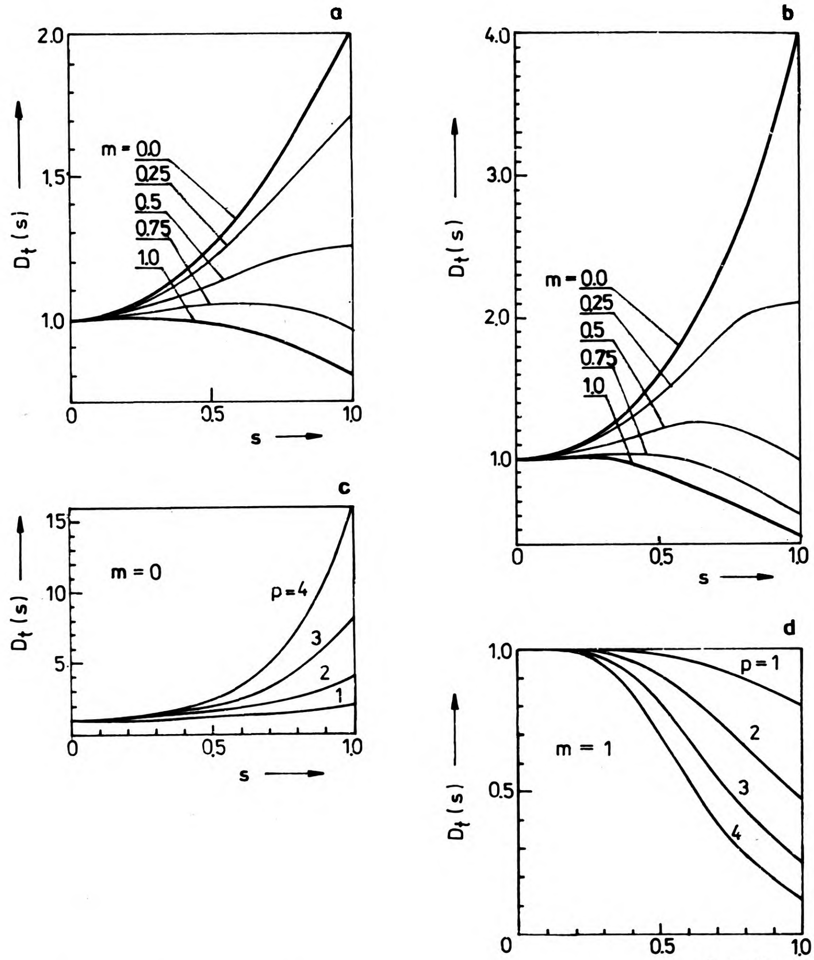


Fig. 1. Effect of amplitude apodization $t(r)$ on the image contrast of amplitude test for: $t(r) = \frac{1}{2}(1+r^2)$ (**a**); $t(r) = \left[\frac{1}{2}(1+r^2)\right]^2$ (**b**); $t(r) = \left[\frac{1}{2}(1+r^2)\right]^p$ (**c**); $t(r) = \left[\frac{1}{2}(1+r^2)\right]^p$ (**d**)

Michelson's contrast of the test equals

$$K(f_x) = \frac{2ab}{a^2+b^2}. \tag{4}$$

Energy contrast in the image is

$$K'(f_x) = \frac{2abt(0)t(s)}{a^2t^2(0)+b^2t^2(s)} \cos\left\{k\left[\frac{W(s)+W(-s)}{2} - W(0)\right]\right\} \tag{5}$$

where $s = \lambda f_x R/f_g$ ($f_g$ – cut-off frequency, $R$ – reference sphere radius, $f_x$ – spatial frequencies).

Contrast change in the image with respect to the object is

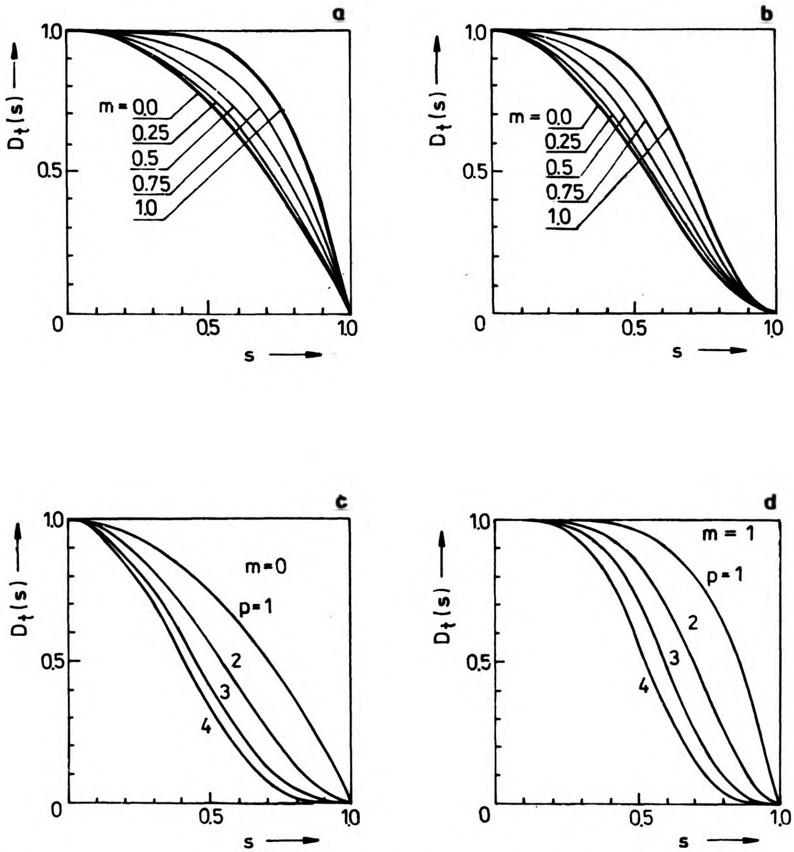


Fig. 2. Effect of apodization $t(r)$ on the image contrast of amplitude test for: $t(r) = 1-r^2$ (**a**); $t(r) = (1-r^2)^2$ (**b**); $t(r) = (1-r^2)^p$ (**c**); $t(r) = (1-r^2)^p$ (**d**)

$$D(f_x) = \frac{K'(f_x)}{K(f_x)} = \frac{\dfrac{t(s)}{t(0)}(1+m^2)}{1+m^2\dfrac{t^2(s)}{t^2(0)}} \cos\left\{k\left[\frac{W(s)+W(-s)}{2} - W(0)\right]\right\} \tag{6}$$

($m = b/a$ – test modulation depth).

The phase shift appearing in the image will have the form

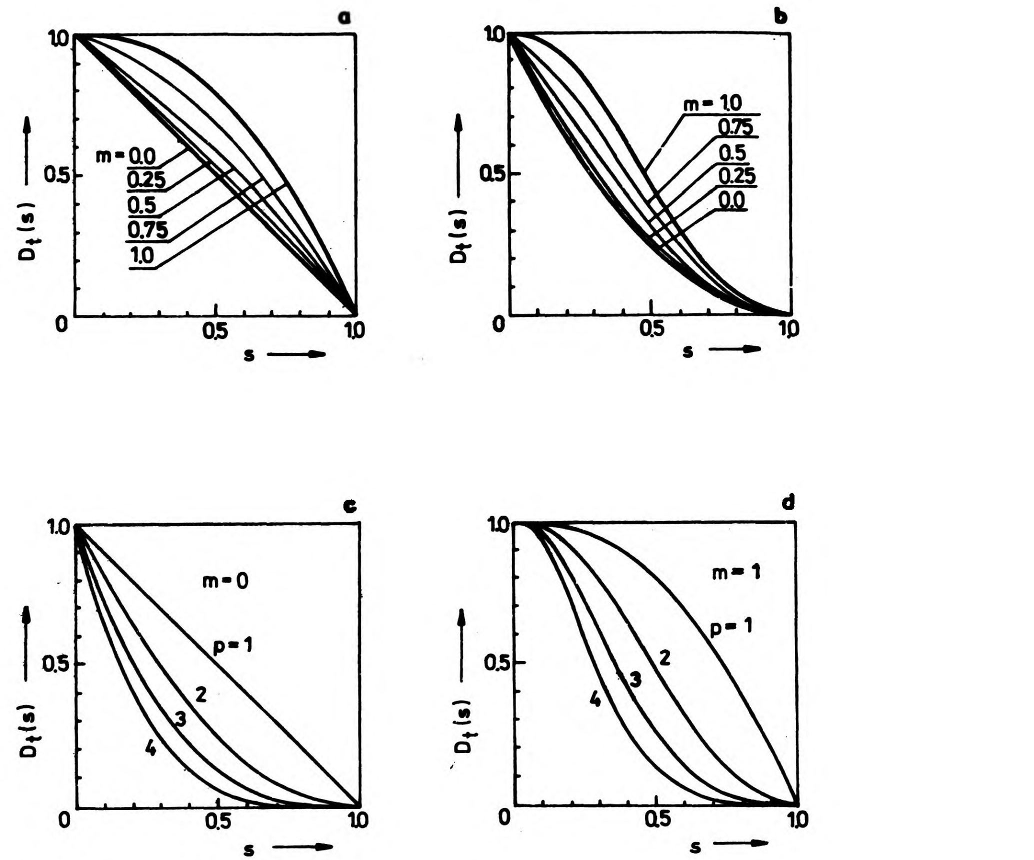


Fig. 3. Effect of apodization $t(r)$ on the image contrast of amplitude test for: $t(r) = 1-|r|$ (**a**); $t(r) = (1-|r|)^2$ (**b**); $t(r) = (1-|r|)^p$ (**c**); $t(r) = (1-|r|)^p$ (**d**)

$$\Theta(f_x) = k\,\frac{W(s)-W(-s)}{2} \tag{7}$$

($k = 2\pi/\lambda$, $\lambda$ – light wavelength).

For a phase test of the transmittance

$H(x, y) \simeq 1 + im\sin\, x$

the change of contrast with respect to the object equals

$$D(f_x) = \frac{\dfrac{t(s)}{t(0)}(1+m^2)}{1+m^2\,\dfrac{t^2(s)}{t^2(0)}}\,\sin\left\{k\left[\frac{W(s)+W(-s)}{2} - W(0)\right]\right\}. \tag{8}$$

From Equation (8) it results that for low-contrast object, at $m \to 0$, when $W(0) = \pi/2$ and $W(s) = 0$, the change of the contrast is the strongest one. In functions

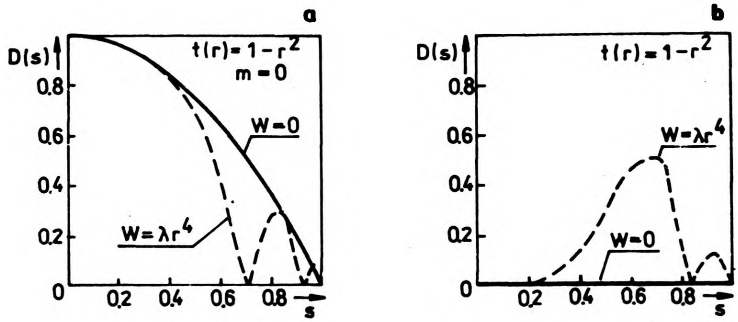


Fig. 4. Contrast change $D(s)$ for amplitude test (**a**) and phase test (**b**) for the optical system with spherical aberration $W(r) = \lambda r^2$ apodized with the function $t(r) = 1 - r^2$

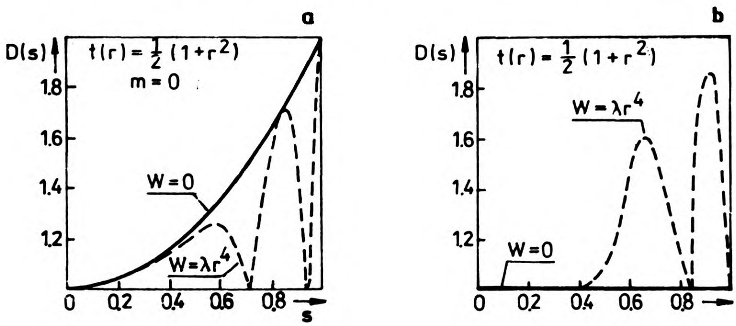


Fig. 5. Contrast change $D(s)$ for amplitude test (**a**) and phase (**b**) test for the optical system with spherical aberration $W(r) = \lambda r^4$ apodized with the function $t(r) = (1 + r^2)0.5$

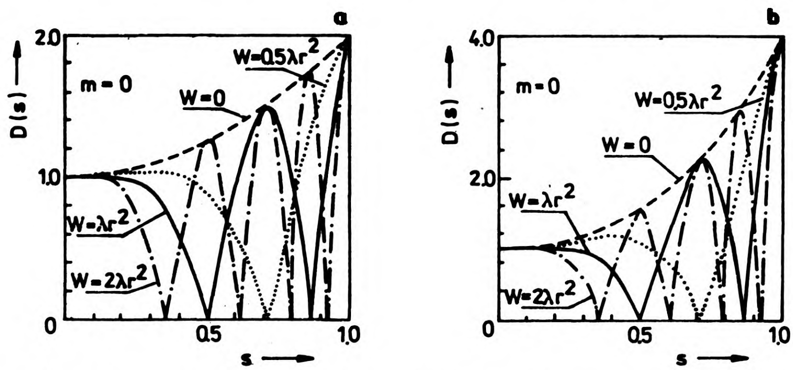


Fig. 6. Contrast $D(s)$ for amplitude test in the optical system with aberrations $W = 0.5r^2\lambda$, $\lambda r^2$, $2\lambda r^2$ apodized with the function $t(r) = \frac{1}{2}(1+r^2)$ (**a**); $t(r) = \left[\frac{1}{2}(1+r^2)\right]^2$ (**b**)

describing the fall contrast for amplitude (6) and phase (8) tests, two parts may be distinguished; namely, a part depending solely on the shape of apodizing function $t(r)$ and a part which depends on the wave aberration of the system $W(x, y)$. Let $D_t$ denote first part of the function, it will amount to

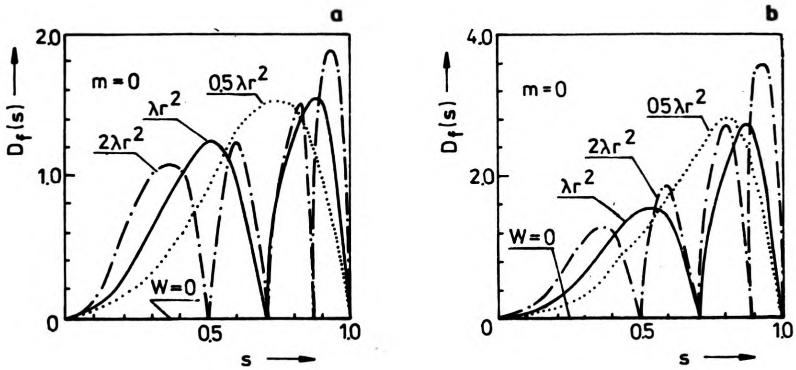


Fig. 7. Contrast change $D(s)$ for phase test in the optical system with aberrations $W = 0.5\lambda r^2$, $\lambda r^2$, $2\lambda r^2$ apodized with functions: $t(r) = \frac{1}{2}(1+r^2)$ (**a**); $t(r) = \left[\frac{1}{2}(1+r^2)\right]^2$ (**b**)

Contrast change $D_t(s)$ for low-contrast objects and apodizers: $\left[\frac{1}{2}(1+r^2)\right]^p$, $p = 1, 2, 3, 4$; $(1-r^2)^p$, $p = 1, 2, 3, 4$; $(1-|r|)^p$, $p = 1, 2, 3, 4$,

| | $D_t$ | | |
|---|---|---|---|
| $r(t)$ | $s = 0$ | $s = 1,\ m = 0$ | $s = 1,\ m = 1$ |
| $\frac{1}{2}(1+r^2)$ | 1 | 2.0 | 0.8 |
| $\left[\frac{1}{2}(1+r^2)\right]^2$ | 1 | 4.0 | 0.5 |
| $\left[\frac{1}{2}(1+r^2)\right]^3$ | 1 | 8.0 | 0.2 |
| $\left[\frac{1}{2}(1+r^2)\right]^4$ | 1 | 16.0 | 0.1 |
| $(1-r^2)$ | 1 | 0 | 0 |
| $(1-r^2)^2$ | 1 | 1 | 0 |
| $(1-r^2)^3$ | 1 | 0 | 0 |
| $(1-r^2)^4$ | 1 | 0 | 0 |
| $(1-\lvert r\rvert)$ / | 1 | 0 | 0 |
| $(1-\lvert r\rvert)^2$/ | 1 | 0 | 0 |
| $(1-\lvert r\rvert)^3$/ | 1 | 0 | 0 |
| $(1-\lvert r\rvert)^4$/ | 1 | 0 | 0 |

$$D_t = \frac{\frac{t(s)}{t(0)}(1+m^2)}{1+m^2\frac{t^2(s)}{t^2(0)}}. \tag{9}$$

For the test of small modulation depth ($m \to 0$), the run of the function is given by the formula

$$D_{t,m\to 0} = \frac{t(s)}{t(0)} \,. \tag{10}$$

Figures 1–3 present the functions $D_t(s)$ for the apodizers $\left[\frac{1}{2}(1+r^2)\right]^p$, $(1-r^2)^p$, $(1-|r|)^p$, where $p = 1, 2, 3, 4$. For low-contrast object, the contrast increases with $p$ for apodizer $\left[\frac{1}{2}(1+r^2)\right]^p$, see the Table. From the graphs, it follows that the apodizer of this type strongly improves the contrast within the range of high frequencies for the tests of rather small modulation depth. The smaller the modulation depth the stronger is the contrast improvement. On the other hand, for great modulation depths ($m \to 1$) the contrasts become weaker.

The introduction of the aberration (Figs. 4–7) deteriorates the contrast in the case of the amplitude test, and that in the case of the phase test this contrast is improved. In the next part of the text, the speckle-contrast has been shown in coherent optical system with time-varying pupil function and diffuse object [4].

## 3. Statistical properties of the time-averaged image speckle pattern

Figure 8 shows schematically an optical system for coherent image formation of a uniform diffuse object, i.e., a stationary random phase object with no signal. It is equivalent to a double-diffraction imaging system used for spatial filtering and is employed here to vary a pupil in time. In particular, an aperture is rotated at the Fourier transform plane of the object corresponding to the pupil

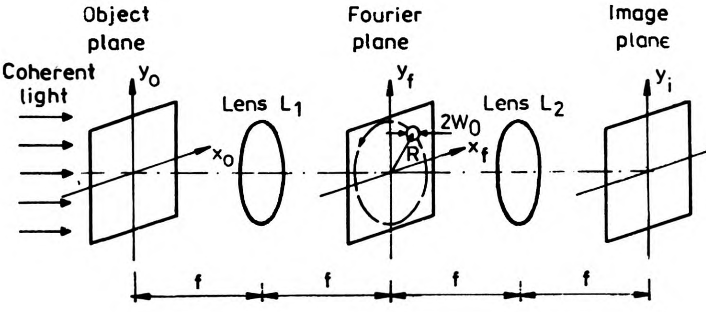


Fig. 8. Optical system for coherent imaging of a uniform diffuse object through a time-varying pupil at the Fourier transform plane of the object. The rotating circular aperture with rotating radius $R$ and aperture width $W_0$ is set at the Fourier transform plane and the lenses are assumed to have focal length $f$

plane. For mathematical simplicity, two-dimensional coordinates at the object, Fourier transform and image planes, are denoted by the position vectors of $\vec{x}_o = (x_o, y_o)$, $\vec{x}_f = (x_f, y_f)$ and $\vec{x}_i = (x_i, y_i)$, respectively.

When a uniform transparent diffuse object is normally illuminated by coherent light of unit intensity, the object amplitude may be expressed by

$$U_o(\vec{x}_o) = \exp[i\Phi(\vec{x}_o)] \tag{11}$$

where $\Phi(x_o)$ is a random phase shift due to the surface roughnes of the diffuse object. With a magnification of unity, the point spread function is a Fourier transform of the time-averaging pupil function $P(x_f, t)$ which is given by

$$h(\vec{x}_o, \vec{x}_i; t) = \frac{1}{\lambda^2 f^2} \int_{-\infty}^{\infty} P(\vec{x}_f, t)\exp\left[-i\frac{k}{f}(\vec{x}_o+\vec{x}_i)\vec{x}_f\right]d\vec{x}_f \tag{12}$$

where $k = 2\pi/\lambda$ is the wave number, $\lambda$ being the wavelength of light, and $f$ is the focal length of lenses. Then, the speckle amplitude at the image point $x_i$ and the time $t$ is reperesented by a convolution integral

$$U_i(\vec{x}_i, t) = \int_{-\infty}^{\infty} U_o(\vec{x}_o)h(\vec{x}_o, \vec{x}_i; t)d\vec{x}_o, \tag{13}$$

and the time-averaged speckle intensity actually recorded by photosensitive detectors, such as a TV system, and a film system over an exposure time $T$ can be written as

$$I(\vec{x}_i) = \frac{1}{T}\int_0^T U_i(\vec{x}_i, t)U_i^*(\vec{x}_i, t)dt \tag{14}$$

where the symbol $*$ indicates a complex conjugate.

The autocorrelation function of the speckle amplitude, defined by the following ensemble average:

$$\Gamma(\vec{x}_i, \vec{x}_i'; t, t') = \langle U_i(\vec{x}_i, t)U_i^*(\vec{x}_i', t')\rangle \tag{15}$$

plays an important role in characterising the statistical properties of the time-averaged speckle pattern at the image plane.

The autocorrelation function yields

$$\Gamma(\vec{x}_i, \vec{x}_i'; t, t') = \Delta S \int_{-\infty}^{\infty} [h(\vec{x}_o, \vec{x}_i; t)h^*(\vec{x}_o, \vec{x}_i'; t')]d\vec{x}_o, \tag{16}$$

$\Delta S$ is a correlation area of $\Phi(x_o)$.

## 4. Application to a rotating Gaussian soft aperture at the Fourier transform plane

The pupil function for the rotating Gaussian soft aperture is expressed by

$$P(\vec{x}_f, t) = \exp\left[\frac{|\vec{x}_f - \vec{a}(t)|^2}{2W_0^2}\right] \tag{17}$$

where $\vec{a}(t)$ is a position vector of the rotating aperture given by

$$\vec{a}(t) = (R\cos\omega t, R\sin\omega t), \tag{18}$$

($W_0$ – aperture width, Fig. 8).

Use of the pupil function in Eq. (17) yields the point spread function expressed by

$$h(\vec{x}_o, \vec{x}_i'; t) = \frac{2\pi W_0^2}{\lambda^2 f^2} \exp\left[\frac{1}{2}\left(\frac{kW_0}{f}|\vec{x}_o + \vec{x}_i|\right)^2\right] \exp\left[-i\frac{k}{f}(\vec{x}_o + \vec{x}_i)\vec{a}(t)\right]. \tag{19}$$

As is clear from Equation (19), the point spread function is time-dependent, while its modulus is time-independent. Therefore, the mean of the time-averaged speckle intensity is equivalent to that of the static speckle intensity, i.e.,

$$\langle I \rangle = \langle I(\vec{x}_i) \rangle = \Delta S \int_{-\infty}^{\infty} |h(\vec{x}_o, \vec{x}_i; t)|^2 \vec{dx}_o = \Delta S \frac{\pi W_0^2}{\lambda^2 f^2}. \tag{20}$$

On the other hand, the autocorrelation function of the time-averaged speckle intensity can be written by

$$R_I(\Delta\vec{x}_i) = \langle I \rangle^2 \left\{1 + V(\sigma, T) \exp\left[-\frac{1}{2}\left(\frac{kW_0}{f}|\Delta\vec{x}_i|\right)^2\right]\right\} \tag{21}$$

where $\Delta\vec{x}_i = \vec{x}_i - \vec{x}_i'$ denotes the distance vector between the two points $\vec{x}_i$ and $\vec{x}_i'$ in the image plane, and

$$V(\sigma, T) = \exp(--\sigma^2) \frac{1}{T^2} \int_0^T\int_0^T \exp[^2\cos\omega(t-t')] dt dt', \tag{22}$$

with a parameter defined by

$$\sigma = R/W_0, \tag{23}$$

which may be called a scanning ratio. The contrast of the time-averaged speckle intensity is of primary interest; it can be derived from Eqs. (19) and (21) as

$$C = [V(\sigma, T)]^{1/2}. \tag{24}$$

This equation indicates that the contrast of the time-averaged speckle intensity depends both on the scanning ratio $\sigma$ of the rotating aperture $R$ to the aperture width $W_0$, and on the exposure time $T$, [4],

$$V(\sigma, T) = V(\sigma, \theta) = \frac{2}{\theta^2} \exp(-\sigma^2) \int_0^\theta (\theta - \varphi) \exp(\sigma^2 \cos\varphi) d\varphi$$

where $\theta = \omega T$ is the rotating angle of the aperture in the exposure time $T$.

Figure 9 shows the resultant contrast of the time-averaged speckle intensity as a function of the scanning ratio $\sigma$ for the various values of the rotating angle $\theta$. Starting from $C = 1$, the contrast decreases monotonously with an increase of the scanning ratio $\sigma$. As the rotating angle $\theta$ approaches $2\pi$ rad, the contrast rapidly decreases in the region of small values of $\sigma$. Of course, with any values of $\sigma(\neq 0)$, the contrast takes a minimum for $\theta = 2\pi$ rad.

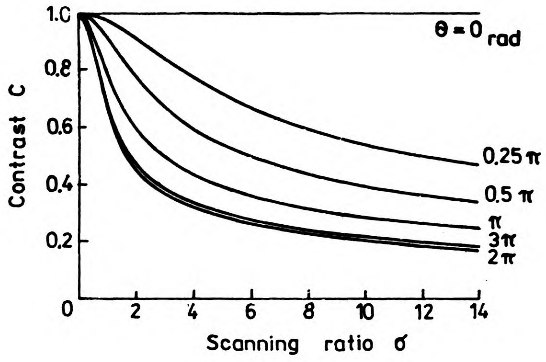


Fig. 9. Contrast $C$ of time-averaged speckle intensity as a function of the scanning ratio $\sigma$ for six values of the rotating angle $\theta$, $\sigma = R/W_0$, ($R$ – rotating aperture, $W_0$ – aperture width)

The contrast $C$ in Figure 9 and contrast $D_t(s)$ in Figures 3a–d for apodizers $t(r) = [1-|r|]^p$, $p = 1, 2, 3, 4$ (see the Table, items *) are similar. The aberration optical system $W(r) = 0.5\lambda r^2$, $\lambda r^2$, $2\lambda r^2$ for phase test object improved the contrast $D_t(s)$ (Fig. 7). The good idea is combination the pupil-aberration-apodizer function (constant in time), (in particular, apodizers $1/2[(1+r^2)]^p$) with rotating aperture time averaging in coherent optical system with diffuse object. The total pupil function in this case is

$$A(r, x_f, t) = t(r)\exp\left[\frac{|\hat{x}_f - \hat{a}(t)|^2}{2W_0^2}\right]\exp[i\Phi(r)].$$

## References

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## Изобразительный контраст когерентной, аберрационной, анодизированной оптической системы. Применение вращательной апертуры в плоскости Фурье

Было показано, что введение амплитудно-фазового аподизатора в когерентную, аберрационную отражающую оптическую систему с периодическим амплитудным или фазовым предметом вызывает изменение контраста, зависимое от модуляции теста и амплитудной части функции, описывающей аподизатор. Изменение контраста было исследовано для следующих амплитудных аподизаторов: $[0.5(1+r)^2]^p$, $(1-r^2)$, $(1-|r|)^p$; $(p = 1, 2, 3, 4)$. Показали также speckle-contrast в когерентной оптической системе со зрачковой временно изменяющейся функцией и предметом диффузии.

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