# Interaction between cw gasdynamic source-laser with passive and active target 

M. P. Brunné<br>Institute of Fluid-Flow Machines, Polish Academy of Sciences, ul. Gen. J. Fiszera 14, 80-952 Gdańsk, Poland.


#### Abstract

The aim of this paper consists in estimating the influence of the radiation intensity coming back to the cavity of a cw gasdynamic source-laser, after being reflected by a passive target or supplied by a second interacting laser (active target) on laser energetic characteristics. The passive target denotes the target which cannot send back the light beam of intensity exceeding that generated by the source-laser. The mentioned intensity is attenuated on its way to the target and back to the laser. The active target denotes the target on which the second arbitrary high-power laser is installed. The response light beam intensity generated by this second laser can overcome losses of the light energy transport through the atmosphere, and its value entering the source-laser cavity can be much greater than the intensity initially sent to the target. An interaction between cw gasdynamic source-laser and active target can eventually lead to the gasdynamic laser destruction.


## 1. Introduction. Equations and boundary conditions

In the applications consisting in an interaction of the high-power laser with the targets, attention is given to prevention of the coming back light beam from entering the source-laser active cavity. Nevertheless, such a situation cannot be always completely eliminated. Investigators [1], [2] dealing with laser treatment of metals (welding, cutting, metal surface processing) do notice an increase of the source-laser output power which sometimes needs an appropriate change of resonator quality for ensuring the attempted results of a carried-out technological operation. It seems justified to predict that for the laser-active target interaction this effect can be even more important.

The purpose of this paper consists in estimating the influence of the beam coming back from the passive or active target on the optical efficiency of a cw gasdynamic laser active cavity. The problem is treated in terms of Rigrod-Marlow rate equation theory [3], [4] describing thermally excited lasers; the gain (population inversion) at its cavity entrance being created by a supersonic flow-forced cooling due to the difference $a$ of the rate constant values responsible for the upper and lower vibrational laser level relaxation. In what follows, it is assumed that due to a fast exchange of energy the rotational distribution function remains Maxwellian and only the maximum gain radiative transition from among $J \rightarrow J^{\prime}(\Delta J= \pm 1)$ is considered thus forming a two-level laser system. Additionally, the calculations are carried out under the premise that fast, transverse to the optical axis, flow-forced
convention can be regarded as a most efficient physical mechanism responsible for supplying and removing the vibrational energy from the two-dimensional flat parallel mirrors cavity. Under the above assumptions the equations describing cw gasdynamic laser cavity of the geometry shown in Fig. 1 can be written in the following form [5]:


Fig. 1. Geometry of a cw gasdynamic laser cavity interacting with the target(s)

$$
\begin{align*}
& v \frac{\partial N_{m}}{\partial x}=\left[\bar{B}_{n m} f_{n}\left(J_{\max }^{\prime}\right) N_{n}-\bar{B}_{m n} f_{m}\left(J_{\max }\right) N_{m}\right]\left(N_{f}^{+}+N_{f}^{-}\right),  \tag{1}\\
& v \frac{\partial N_{n}}{\partial x}=\left[\bar{B}_{m n} f_{m}\left(J_{\max }\right) N_{m}-\bar{B}_{n m} f_{n}\left(J_{\max }^{\prime}\right) N_{n}\right]\left(N_{f}^{+}+N_{f}^{-}\right)  \tag{2}\\
& c \frac{\partial N_{f}^{+}}{\partial y}=\left[\bar{B}_{m n} f_{m}\left(J_{\max }\right) N_{m}-\bar{B}_{n m} f_{n}\left(J_{\max }^{\prime}\right)\right] N_{f}^{+}  \tag{3}\\
& c \frac{\partial N_{f}^{-}}{\partial y}=\left[\bar{B}_{n m} f_{n}\left(J_{\max }^{\prime}\right) N_{n}-\bar{B}_{m n} f_{m}\left(J_{\max }\right)\right] N_{f}^{-} \tag{4}
\end{align*}
$$

where $N_{m}$ and $N_{n}$ are the populations of the upper and lower vibrational laser levels, $f_{m}\left(J_{\max }^{\prime}\right)$ and $f_{n}\left(J_{\max }^{\prime}\right)$ are the Maxwell-Boltzmann distribution function values for the rotational sublevels of the $m$ and $n$ vibrational levels, $N_{f}^{+}$and $N_{f}^{-}$indicate the densities of photons moving within cavity region $\boldsymbol{G}$ (Fig. 1) along the rays $s=y$ in the direction parallel $\left(N_{f}^{+}\right)$and antiparallel $\left(N_{f}^{-}\right)$to the $y$ coordinate, $\bar{B}_{m n}$ and $\bar{B}_{n m}$ are the modified Einstein coefficients [5] of stimulated emission and absorption, respectively, and finally $v$ and $c$ indicate the flow and light velocities. The $x$,
$y$ coordinates describe the plain cross-section of the cavity region $\boldsymbol{G}$, perpendicular to the $z$ coordinate (Fig. 1). In Figure 1 the length, width and the distance between two flat parallel mirrors $\boldsymbol{Z}^{+}$and $\boldsymbol{Z}^{-}$are indicated by $L_{x}, L_{z}$ and $L_{y}$. The dissipative properties of mirrors are measured by their reflectivity coeffcients $R^{+}$(reflecting mirror) and $R^{-}$(transmitting mirror).

For the geometry shown in Figure 1, Equations (1)-(4) should satisfy the following boundary conditions:

$$
\begin{align*}
& N_{m}(0, y)=N_{m 0}, N_{n}(0, y)=N_{n 0},  \tag{5}\\
& N_{f}^{-}\left(x, L_{y}\right)=R^{+}, N_{f}^{+}\left(x, L_{y}\right),  \tag{6}\\
& N_{f}^{+}(x, 0)=R^{-} N_{f}^{-}(x, 0)+N_{b}^{(\mathrm{p}, \mathrm{a})} . \tag{7}
\end{align*}
$$

Equation (5) sets the value of the gain at the cavity entrance. Equations (6) and (7) interrelate the incident and reflected light intensities at the resonator mirror surfaces. The additional photon number density $N_{b}^{(p, a)}$ denotes the intensity of radiation field coming back from the passive or active target which enters into the source-laser cavity through its transmitting mirror.

If the target is a passive one, the unique source of the $N_{b}^{(\mathrm{p})}$ is $N_{f}^{-}(x, 0)$. It is, therefore, easy to establish the relation between those two quantities. It has the following form:

$$
\begin{equation*}
N_{b}^{(\mathrm{p})}=\left(1-R^{-}\right)\left(1-R_{b}^{-}\right) R_{t} \exp \left[-\left(k_{\mathrm{LT}}+k_{\mathrm{TL}}\right) L_{\mathrm{LT}}\right] N_{f}^{-}(x, 0) . \tag{8}
\end{equation*}
$$

It seems also reasonable to assume that the response of the active target will be related to the sensed signal coming to it from the source-laser. Therefore

$$
\begin{equation*}
N_{b}^{(\mathrm{a})}=\delta\left(1-R^{-}\right)\left(1-R_{b}^{-}\right) \exp \left[-\left(k_{\mathrm{TL}}+k_{\mathrm{LT}}\right) L_{\mathrm{LT}}\right] N_{f}^{-}(x, 0) \tag{9}
\end{equation*}
$$

where $R_{b}^{-}$is the reflectivity of the source-laser transmitting mirror outer surface, $R_{t}$ stands for the target reflectivity, $L_{\mathrm{LT}}$ is the distance between the transmitting mirror $\boldsymbol{Z}^{-}$and the target surface, $k_{\mathrm{LT}}$ and $k_{\mathrm{TL}}$ are the attenuation coefficients measuring the light absorption and other dissipative losses of the laser beam on its way to the target ( $k_{\mathrm{LT}}$ ) and back ( $k_{\mathrm{TL}}$ ), respectively. In Eq. (9) the parameter $\delta$ indicates that the power sent to the source-laser by the active laser can arbitrarily exceed the gasdynamic laser power.

In view of Equations (8) and (9) the boundary condition (7) can formally be rewritten in a simple form

$$
\begin{equation*}
N_{f}^{+}(x, 0)=\lambda_{r} R^{-} N_{f}^{-}(x, 0) \tag{10}
\end{equation*}
$$

where the inner structure of algebraic coefficient $\lambda_{r}$ follows the relation

$$
\begin{equation*}
\lambda_{r}=1+\left(R^{-}\right)^{-1}\left(1-R^{-}\right)\left(1-R_{b}^{-}\right) R_{t} \exp \left[-\left(k_{\mathrm{LT}}+k_{\mathrm{TL}}\right) L_{\mathrm{LT}}\right] \tag{11}
\end{equation*}
$$

for a passive target, and

$$
\begin{equation*}
\lambda_{\mathrm{r}}=1+\delta\left(R^{-}\right)^{-1}\left(1-R^{-}\right)\left(1-R_{b}^{-}\right) \exp \left[-\left(k_{\mathrm{TL}}+k_{\mathrm{LT}}\right) L_{\mathrm{Lr}}\right] \tag{12}
\end{equation*}
$$

for an active one.

In the calculations the assumed arbitrary response of the active target permits us to use the boundary condition in the simpler form (10) and apply Eq. (9) to recover the coefficient $\delta$ which, following Eq. (10), allows us to calculate the power generated by this target and needed to attain some $\lambda_{r}$ values particularly important for applicative reasons. The coefficient(s) of the light-beam intensity, decrease occurring during its propagation through the atmosphere, follow the known relation [6]

$$
\begin{equation*}
k_{\mathrm{LT}}=k_{\mathrm{TL}}=k(\lambda)=N \int_{0}^{\infty} \sigma(r, \lambda) f(r) d r \tag{13}
\end{equation*}
$$

where $N$ is the concentration of molecules per unit volume; $\sigma(r, \lambda)$ stands for the attenuation of radiation and is the function of the wavelength and molecule dimensions; $f(r)$ describes the function of distribution of molecules with respect to their varying dimensions. Relation (13) takes into account the dissipative processes of infrared radiation due to its absorption by the gaseous components and its scattering occurring on the fluctuations of the density of molecules (molecular scattering), on the aerosols, and on disuniformities created by the atmospheric turbulence.

For the case of average atmospheric conditions when the particle concentration amounts to $N=28 \mathrm{~cm}^{-3}, k_{\mathrm{LT}}=k_{\mathrm{TL}}$ are for $\lambda=10.6 \mu \mathrm{~m}$ equal to $19.6 \mathrm{~cm}^{-1}$ when the target is placed at the distance $L_{\mathrm{LT}}=0.2 \mathrm{~km}$ from the source-laser; with the increase of this distance $k_{\mathrm{KL}}$ increases in a manner directly proportional to $L_{\mathrm{LT}}=L_{\mathrm{TL}}$ [6].

## 2. Dimensionless formulation of the problem and its formal solution

Further on the dimensionless forms of Equations (1)-(12) will be used. The lengths along $x, y$ and $z$ coordinates are scaled in the following manner: the mirror length $L_{x}\left(\xi=x / L_{x}\right)$, the distance between the mirrors $L_{y}\left(\eta=y / L_{y}\right)$, and the mirror width $L_{z}\left(\zeta=z / L_{z}\right)$, respectively. The populations of the upper and lower vibrational laser levels are scaled by their values at the entrance $(x=0)$ of the cavity ( $\left.n_{m}=N_{m} / N_{m 0}, n_{n}=N_{n} / N_{n 0}\right)$. The photon densities ( $N_{f}^{+}, N_{f}^{-}$and $N_{b}$ ) are made dimensionless by dividing them by the number of vibrational quanta brought to the cavity inlet with the transverse flow of the molecules excited to the upper vibrational laser level ( $N_{m}$ ) giving $n_{f}^{+}=N_{f}^{+} / N_{m}, n_{f}^{-}=N_{f}^{-} / N_{m}$ and $n_{b}=N_{b} / N_{m}$. Dimensionless gain $g$ is equal to $G$ multiplied by the distance between resonator mirrors, $\left(g=G L_{y}\right)$. All coefficients entering relations (11) and (12) defining $\lambda_{r}$ for a passive and active target are dimensionless from the beginning.

Introducing dimensionless notation Equations (1)-(7) can be rewritten in the following forms:

$$
\begin{align*}
& \frac{\partial \ln (g)}{\partial \xi}=-\Pi_{c}\left(n_{f}^{+}+n_{f}^{-}\right), \quad \frac{\partial \ln \left(n_{f}^{+}\right)}{\partial \eta}=-\frac{\partial \ln \left(n_{f}^{-}\right)}{\partial \eta}=g,  \tag{14}\\
& n_{f}^{-}(\xi, 1)=R^{+} n_{f}^{+}(\xi, 1), \quad n_{f}^{+}(\xi, 0)=R^{-} n_{f}^{-}(\xi, 0), \quad g(0, \eta)=g_{0} \tag{15}
\end{align*}
$$

where [6]:

$$
\begin{aligned}
& g=\left(I_{0} n_{m}-\min I_{\mathrm{lb}} n_{n}\right) \frac{c^{2} L_{y} A_{m n}}{8 \pi v_{m n}^{2}} N_{n 0} f_{m}\left(T_{0}, J_{\max }\right) \mathscr{L}\left(v, v_{m n}\right), \\
& g_{0}=g\left(n_{m}=n_{n}=1\right), \quad I_{0}=\exp \left[\frac{\hbar}{k}\left(\frac{\omega_{n}}{T_{n 0}}-\frac{\omega_{m}}{T_{m 0}}\right)\right],
\end{aligned}
$$

$$
\min I_{\mathrm{th}}=B_{n m} f_{n}\left(T_{0}, J_{\max }^{\prime}\right) / B_{m n} f_{m}\left(T_{0}, J_{\max }\right) \simeq g_{m} / g_{n} \simeq 1
$$

In the above given relations $g_{m}, g_{n}$ are the statistical weights of the $m$ and $n$ states, $A_{m n}, B_{m n}$ and $B_{n m}$ are the Einstein coefficients of the spontaneous emission and of the stimulated emission and absorption, $\mathscr{L}\left(v, v_{m n}\right)$ is the normalized line shape function for the radiative transfer $\left(m, J_{\max }\right) \leftrightarrow\left(n, J_{\max }^{\prime}\right)$ of the central frequency $v_{m n}, \hbar \omega_{m} / k$ and $\hbar \omega_{n} / k$ are the energies of the upper and lower laser level, $T_{m}, T_{n}$ and $T$ are vibrational temperatures of the asymmetric stretching ( $v_{3}$ ), symmetric stretching $\left(v_{1}\right)$ and doubly degenerated bending ( $v_{2}$ ) modes, and the gas mixture translational temperature, respectively. The subscript " 0 " refers all quantities to their values at $\xi=0$ (cavity entrance).


Fig. 2. Efficiency of a cw gasdynamic source-laser interacting with the active target for $g_{0}=0.5$
Taking into account that for practical reasons the $\boldsymbol{Z}^{+}$mirror can be treated as a totally reflecting one ( $R^{+}=1$ ), Eqs. (14) and (15) have almost identical formal structure as those investigated in [5] and [7]. Consequently, the solution defining
$n_{f}^{-}(\xi, 0)$ which allows us to calculate the cw gasdynamic source-laser output power is given [5], [7] by a properly normalized Dirac pseudo-function $\delta_{\mathrm{D}}(\xi)$, whose norm has the following form:

$$
\begin{equation*}
\left\|n_{f}^{-}(\xi, 0)\right\|=\frac{2}{\Pi_{c}\left(1+\lambda_{r} R^{-}\right)} \ln \left\{\left[\frac{\lambda_{r} R^{-}+\sqrt{\lambda_{r} R^{-}}}{1-\sqrt{\lambda_{r} R^{-}}}\right]\left[\frac{1-\exp \left(-g_{0}\right)}{\lambda_{r} R^{-}+\exp \left(-g_{0}\right)}\right]\right\} . \tag{16}
\end{equation*}
$$

The corresponding to Eq. (16) output power is equal to

$$
P_{f}=\left(1-R^{-}\right) h c v_{m n} L_{x} L_{z} N_{m 0} \int_{0}^{1}\left\|n_{f}^{-}(\xi, 0)\right\| \delta_{\mathrm{D}}(\xi) d \xi .
$$

If the maximum virtual radiation power brought to the cavity by the following medium is defined [5], [7] by the relation

$$
P_{v}=h v v_{m n} L_{y} L_{z} N_{m 0},
$$

then the optical efficiency is given by the $P_{f}$ to $P_{v}$ ratio and can be expressed by the following formula:

$$
\begin{equation*}
\eta_{c} / \Pi_{i}=\frac{1}{g_{0}} \frac{\left(1-R^{-}\right)}{\left(1+\lambda_{r} R^{-}\right)} \ln \left\{\left[\frac{\lambda_{r} R^{-}+\sqrt{\lambda_{r} R^{-}}}{1-\sqrt{\lambda_{r} R^{-}}}\right]\left[\frac{1-\exp \left(-g_{0}\right)}{\lambda_{r} R^{-}+\exp \left(-g_{0}\right)}\right]\right\} \tag{17}
\end{equation*}
$$

where newly introduced dimensionless parameter


Fig. 3. Same as in Fig. 2, but only for $g_{0}=1.0$

$$
\Pi_{i}=\frac{1}{I_{0}} \frac{I_{0}-\min I_{\mathrm{th}}}{1+\min I_{\mathrm{th}}} \lesssim \frac{1}{2}
$$

measures the difference between the boundary value of the inversion parameter $I_{0}$ and its minimum threshold value $I_{\mathrm{th}}$. Relation (17) being given, the problem can be considered as formally closed.

Relation (17) offers four constraining inequalities which ought to be fulfilled if the source-laser has to give the net output power. In the presence of the back reflected radiation field, two of them have the following forms:

$$
\begin{equation*}
\Pi_{i}>0, \quad \lambda_{r}^{-1} \exp \left(-2 g_{0}\right)<R^{-}<1 \tag{18}
\end{equation*}
$$

The first of these inequalities is not a strident one as it is always fulfilled apart from a very specific case $I_{0}=\min I_{\mathrm{th}}$ in which $P_{f}=0$. The second of inequalities (18) gives some interesting information about laser-target(s) interaction.

For an interaction with a passive target the obvious requirement that $P_{f}>0$ leads to an additional inequality

$$
\begin{equation*}
\gamma_{r}=\left(1-R_{b}\right) R_{t} \exp \left[-\left(k_{\mathbf{L T}}+k_{\mathbf{T L}}\right) L_{\mathbf{L T}} \leqslant \exp \left(-2 g_{0}\right) .\right. \tag{19}
\end{equation*}
$$



Fig. 4. Same as in Fig. 2, but only for $g_{0}=1.5$
Formally, for $\gamma_{r}>\exp \left(-2 g_{0}\right)$ the reflectivity $R^{-}$of the source-laser becomes less than zero. Interpreting in a heuristic manner the formulae obtained from distributive solution (16) it can be suggested that if $\gamma_{r}$ exceeds its critical value $\gamma_{\mathrm{cr}}=\exp \left(-2 g_{0}\right)$ the source-laser ceases to produce output power, because the intensity of radiation


Fig. 5. Efficiency of a cw gasdynamic source-laser interacting with the passive target for $g_{0}=0.5$
field takes-off the population inversion via absorption on its way between the source-laser cavity mirrors.

It is worth mentioning that for an active target, when the value of $\lambda_{r}$ is not limited, inequality (18) changes its form. One has [7]

$$
\begin{equation*}
\lambda_{r}^{-1} \exp \left(-2 g_{0}\right)<R^{-}<\lambda_{r}^{-1} . \tag{20}
\end{equation*}
$$

When $R^{-}$reaches the critical value equal to $\lambda_{\mathrm{cr}}^{-}$, the intensity of the output radiation field goes theoretically ad infinitum. In practice, heuristic again explanation of this effect indicates that source-laser becomes destroyed because its mirrors cannot withstand the sudden increase of their thermal load. From the physical point of view the requirement that $R_{\text {eff }}^{-} \xlongequal{\text { df }} \lambda_{r} R^{-}<1$ is obvious as it could be greater only if the active target will be able to supply to the source-laser cavity the infinitum amount of energy.

Inequalities (18)-(20) do, however, contain also nontrivial information which gives some introductory insight into the physical problem of the laser-target(s) interaction. The obtained solutions permit us to define the source-laser quality range and its output power as a function of the target parameters and atmospheric conditions. In particular, the recovered relations allow us to calculate the critical values of $\gamma_{\mathrm{cr}}$ (for passive target) and $\lambda_{\mathrm{cr}}$ (for active target) which knowledge is essential from the applicative viewpoint. It is worth mentioning that the results obtained remain in agreement with experimental information given in [1] and [2].


Fig. 6. Same as in Fig. 5, but only for $g_{0}=1.0$
The examples of the cw gasdynamic source-laser efficiency are given in Figs. 2-4 (for active target) and Figs. 5-7 (for passive target). The efficiency was calculated for typically encountered in $\mathrm{CO}_{2} \mathrm{cw}$ gasdynamic laser values of boundary gain, i.e., for $g_{0}=0.5,1.0$, and 1.5. The results reported in Figs. 2-7 are self-evident and do not need any special comments.

The presented theoretical considerations have some shortcomings resulting from the corpuscular light model and rate equation description of the laser-target interaction. It is known from experimental investigations that this interaction depends on such subtle physical effects as the coupling between various modes within the laser resonator, the fluctuation of the radiation intensity, and the changes of the output beam divergence. All these effects cannot be treated basing on the model accepted in this paper which has all characteristics of a "black-box" allowing to calculate the laser power characteristics without considering the sophisticated effects and the way in which they influence those characteristic ultimate values.

## 3. Summary

The aim of the paper consisted in relating cw gasdynamic laser output power (optical efficiency) to the number of parameters describing its cavity and medium state in a case in which this laser operates in hybride conditions of the generator and amplifier due to its interaction with passive or active target. This aim has been


Fig. 7. Same as in Fig. 5, but only for $g_{0}=1.5$
reached by giving the formulae (17)-(20) containing also the parameters defining the target parameters and atmospheric conditions. For practical applications, the most important was the recovery of the $\gamma_{\mathrm{cr}}$ and $\lambda_{\mathrm{cr}}$ critical values which set the limits to the possibility of the source-laser interaction with the passive and active target. To avoid the detrimental for source-laser effects which occur when $\gamma_{\mathrm{cr}}$ is overpassed (passive target) or $\lambda_{\text {cr }}$ reached (active target) the cw gasdynamic laser ought to operate in a continuous Q -switching regime.

Acknowledgements - The author feels deeply grateful to dr Z. Trzęsowski, from the Institute of Quantum Electronics of the Technical Military Academy in Warsaw, whose critical remarks and valuable suggestions helped in giving proper physical interpretation to some results reported in this paper.

Verified by Marzena Luczikiewicz

## References

[1] Panderese F. (CISE, Milan, Italy), private communication, 1991.
[2] GAY P. (FIAT Research Center, Turin, Italy), private communication, 1991.
[3] Rigord W. W., J. Appl. Phys. 36 (1965), 247.
[4] Marlow W. C., Lockheed Palo Alto Research Laboratory Report, Palo Alto, USA, 1971 (unpublished).
[5] Brunné M. P., Milewski J., Stańco J., Zieliński A., J. Appl. Phys. 7 (1976), 135.
[6] Zuyev A., Propagation of Visible and Infrared Radiation in the Atmosphere, (in Russian), Nauka, Moscow 1976.
[7] Brunne M. P., 7th Symp. on Gas-Flow and Chemical Lasers, SPIE 1031 (1988), 184.

Received March 20, 1991

## Войздействие газодинамического лазера постоянного действия с пассивной и активной целями

В настоящей работе рассмотрено изменение энергетических характеристик газодинамического лазера постоянного действия, вызванное воздействием с пассивной и активной целями. Расчеты проведены в подходе уравнений непрерывности, принимая корпускулярную модель света и принимая, что вынужденная конвекция составляет основной источник усиления, доставленного к резонатору газодинамического лазера. Определен оптический коэффициент полезного действа резонансного колодца лазера-источника. Даны критические значения параметров, зависимых от свойств газодинамического лазера, содействующих с ним целей, а также свойства атмосферы, для которых наступает замирание генерации (пассивная цель) или разрушение лазера-источника (активная цель).

Перевел Станислав Ганцаж

