Letters to the Editor

Thermal lensing compensation from composite CO₂-laser windows

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Laser beam defocusing and distortion which is referred to as thermal lensing can be produced in high-power CO_2 laser systems due to a window nonuniform irradiation. An analysis of thermal lensing compensation from composite windows is given. Isotropic materials and single crystals cut along [111] plane are considered.

1. Introduction

A nonuniform laser window irradiation in high-power CO_2 -laser systems can produce a radial temperature gradient across the window that causes the window to bulge becoming thicker in the center. A temperature gradient in the refractive index is induced. As an added complication, the thermally induced stresses cause the refractive index to be different for different polarizations, that is a birefringence can be thermally produced. The resulting distortion and defocusing of the laser beam, which is referred to as thermal lensing has been studied for example in [1]–[5]. The distortion can be reduced by using a composite window consisting of two layers of transparent materials, one of which tends to diverge and the other to converge the laser beam [1], [5].

The purpose of this note is to analyse further the thermal lensing diminution in high-power CO_2 laser systems.

2. Basic formulae

Let us consider a Gaussian beam of amplitude $a(\rho, \theta) \sim \exp(-\alpha^2 \rho^2)$ incident on a thin cylindrical window, where ρ is measured in units of the window radius. The thermal lensing of the laser beam transmitted through the window is determined by the aberration function $\Phi^{\rho,\theta}$ associated with the ρ and θ polarized waves. For a thin cylindrical window the aberration function takes the form [3], [4]

$$\Phi^{\gamma} = \rho_0 S_1^{\gamma} \overline{\Delta T} + 4\rho_0 S_2^{\gamma} \rho^{-2} \int_0^{\rho} dx \,\overline{\Delta T} x \tag{1}$$

where $\gamma = \rho$ or $\theta \Delta T = \int_{-\eta_0}^{\eta_0} dz \Delta T(\rho, z, t)$, $\eta_0 = L_0/(2\rho_0)$, ρ_0 and L_0 are the window radius and thickness, respectively, z refers to the coordinate along the window

thickness, ΔT is the temperature rise in the sample, and S_i^{γ} are the material parameter coefficients. For the small time case when $\Delta T \sim t$ one obtains [3]

$$\Phi^{\gamma} = C_1^{\gamma} \exp(-2\alpha^2 \rho^2) + C_2^{\gamma} [1 - \exp(-2\alpha^2 \rho^2)] / (\alpha^2 \rho^2)$$
(2)

where $C_i^{\gamma} = S_i^{\gamma} L_0 P_0 \beta t / c'$, P_0 is the laser beam peak power, β is the bulk absorption coefficient and c' is the specific heat.

The laser beam intensity I^{γ} at a prefocal point relative to the initial value I_0^{γ} in the absence of distortions is given by [5], [6]

$$I^{\gamma}/I_{0}^{\gamma} = 1 - \tilde{\Delta}^{\gamma}, \tag{3}$$

with

$$\tilde{\mathcal{A}}^{\gamma} = k^2 [\langle (\Phi^{\gamma})^2 \rangle - \langle \Phi^{\gamma} \rangle^2]$$
⁽⁴⁾

where k is the free space wave vector, and $\langle \rangle$ is defined as

$$\langle F \rangle = \int dSa(\rho, \theta) F / \int dSa(\rho, \theta), \tag{5}$$

the integral being taken over the window plane.

In the case of a single-layer window A of thickness L_A we obtained

$$\widetilde{\mathcal{A}}_{\mathcal{A}}^{\gamma} = (kP_0 t L_{\mathcal{A}})^2 \mathcal{A}_{\mathcal{A}}^{\gamma},\tag{6}$$

with

$$\Delta_A^{\gamma} = U(\alpha) [f_1^{\gamma}(A)]^2 + R(\alpha) [f_2^{\gamma}(A)]^2 + 2Q(\alpha) f_1^{\gamma}(A) f_2^{\gamma}(A)$$
(7)

where

$$f_i^{\gamma}(A) = \beta S_i^{\gamma}(A)/c', \qquad (8)$$

$$U(\alpha) = \frac{f(5\alpha)}{f(\alpha)} - \left[\frac{f(3\alpha)}{f(\alpha)}\right]^2,$$
(9)

with

$$f(m\alpha) = [1 - \exp(-m\alpha^2)]/(m\alpha^2), \qquad (10)$$

$$R(\alpha) = \{E_1(\alpha^2) - 6E_1(3\alpha^2) + 5E_1(5\alpha^2) + f(\alpha) - 6f(3\alpha) + 5f(5\alpha)$$

$$E_1(\alpha^2) - E_1(3\alpha^2) + 5E_1(5\alpha^2) + f(\alpha) - 6f(3\alpha) + 5f(5\alpha)$$
(11)

$$-\left[E_1(3\alpha^2) - E_1(\alpha^2)\right]^2 / \left[\alpha^2 f(\alpha)\right] \right\} / \left[\alpha^2 f(\alpha)\right], \tag{11}$$

$$Q(\alpha) = \{E_1(5\alpha^2) - E_1(3\alpha^2) - [E_1(3\alpha^2) - E_1(\alpha^2)]f(3\alpha)/f(\alpha)\}/[\alpha^2 f(\alpha)]$$
(12)

where E_1 is the exponential integral [7].

For a thin composite of two layers A and B of thickness L_A and L_B , the aberration functions are additive, $\Phi^{\gamma} = \Phi^{\gamma}(A) + \Phi^{\gamma}(B)$, and one obtains

$$\widetilde{\varDelta}_{AB}^{\gamma} = (kP_0 tL_A)^2 \varDelta_{AB}^{\gamma}, \tag{13}$$

with

$$\Delta_{AB}^{\gamma} = a_1^{\gamma} \chi^2 + 2a_2^{\gamma} \chi + a_3^{\gamma} \tag{14}$$

where

$$\chi = L_B/L_A, \quad a_1^{\gamma} = \Delta_B^{\gamma}, \quad a_3^{\gamma} = \Delta_A^{\gamma}, \tag{15}$$

$$a_{2}^{\gamma} = U(\alpha)f_{1}^{\gamma}(A)f_{1}^{\gamma}(B) + R(\alpha)f_{2}^{\gamma}(A)f_{2}^{\gamma}(B) + Q(\alpha)[f_{1}^{\gamma}(A)f_{2}^{\gamma}(B) + f_{1}^{\gamma}(B)f_{2}^{\gamma}(A)].$$
(16)

As it was done in [5], we require the thickness ratio χ which minimizes Δ_{AB}^{γ} for a fixed thickness L_A

$$\chi_{\rm m}^{\rm y} = -a_2^{\rm y}/a_1^{\rm y}. \tag{17}$$

For this value of χ one obtains

$$\Delta_{ABm}^{\gamma} = a_3^{\gamma} - (a_2^{\gamma})^2 / a_1^{\gamma}.$$
⁽¹⁸⁾

The sensitivity to variations of χ about χ_m^{γ} may be measured by the parameter η [5]

$$\eta = (1/2) |\partial^2 (\Delta_{AB}^{\gamma} / \Delta_{ABm}^{\gamma}) / \partial (\chi / \chi_{\rm m}^{\gamma})^2| = |a_2^{\gamma 2} / (a_1^{\gamma} a_3^{\gamma} - a_2^{\gamma 2})|.$$
(19)

It can be noted that the relations obtained are quite different from those in [5].

3. Results

We have applied to above procedure for composite pairs of typical 10.6 μ m window materials by supposing a unit length of material A and obtaining the value of L_B/L_A which minimizes Δ_{AB}^{γ} . The seven materials investigated are NaCl, KCl, KI, KBr, GaAs, ZnSe, and CdTe. The material parameters as given in [3], [4] are considered.

3.1. Isotropic materials

In case of isotropic materials the material parameter coefficients S_i^{γ} are given by [2]–[4], [8]:

$$S_{1}^{\rho} = \frac{\partial n}{\partial T} + \bar{\alpha} n^{3} [(1 - \nu)p_{12} - \nu p_{11}]/2 + \bar{\alpha}(1 + \nu)(n - 1), \qquad (20)$$

$$S_2^{\theta} = \bar{\alpha}n^3(1+\nu)(p_{11}-p_{12})/8 = -S_2^{\theta}, \qquad (21)$$

$$S_{1}^{\theta} = \frac{\partial n}{\partial T} + \bar{\alpha}n^{3}(p_{11} - 2\nu p_{12})/2 + \bar{\alpha}(1 + \nu)(n - 1)$$
(22)

where *n* is the refractive index, $\partial n/\partial T$ is taken at zero stress, $\bar{\alpha}$ is the linear thermal expansion coefficient, *v* is Poisson's ratio and p_{ij} 's are elasto-optic coefficients.

Results are given in Table 1 for ρ -polarized waves. They are almost the same for θ -polarized waves. As one can see the composite NaCl-KI would result in substantially less lensing.

We obtained a strong dependence on the beam shape (α^2) as is shown in Table 2 for ρ -polarized waves for composite NaCl-KI. It is different from the relatively weak dependence on α^2 which is reported in [5].

3.2. Single crystals cut along [111] plane

For a single-crystal window whose plane is cut along [111] plane we obtained:

$$S_{1}^{\rho} = \frac{\partial n}{\partial T} + \bar{\alpha}n^{3} \left[(1 - 5\nu)p_{11} + (5 - 7\nu)p_{12} - 2(1 + \nu)p_{44} \right] / 12 + \bar{\alpha}(1 + \nu)(n - 1),$$
(23)

$$S_2^{\rho} = \bar{\alpha}n^3(1+\nu)(p_{11}-p_{12}+4p_{44})/24, \qquad (24)$$

$$S_{1}^{\theta} = \frac{\partial n}{\partial T} + \bar{\alpha}n^{3} [3(1-\nu)p_{11} + 3(1-3\nu)p_{12} + 6(1+\nu)p_{44}]/12 + \bar{\alpha}(1+\nu)(n-1).$$
(25)

Composite (A-B)	L_B/L_A	Δ_{AB}/Δ_{A}	η
NaCl-KI	0.969	0.039	26.3
KI-GaAs	0.514×10^{-2}	0.304	4.29
KI–ZnSe	0.838×10^{-2}	0.500	3.00
KICdTe	0.570×10^{-1}	0.393	3.55

Table 1. Aberration properties of composite windows at $10.6 \ \mu m$ for isotropic materials

Table 2. Aberration properties of NaCl-KI composite at 10.6 μ m as a function of α^2 for isotropic materials

α ²	L_B/L_A	Δ_{AB}/Δ_{A}	η
0.5	0.640	0.37×10^{-1}	28.0
1.0	0.969	0.39×10^{-1}	26.3
2.0	0.807	0.37×10^{-3}	2680.0

Table 3. Aberration properties of composite windows at 10.6 μ m for [111] plane and $\alpha^2 = 1$

Composite (A-B)	L_B/L_A	Δ_{AB}/Δ_{A}	η
KBrGaAs	0.137×10^{-3}	0.4×10^{-1}	0.26×10^{2}
KBr-ZnSe	0.201×10^{-3}	0.7×10^{-4}	0.14×10^{5}
KBr-CdTe	0.146×10^{-2}	0.2×10^{-1}	0.44×10^{2}
KCl–GaAs	0.476×10^{-2}	0.8×10^{-1}	0.13×10^{2}
KCl–ZnSe	0.689×10^{-2}	0.6×10^{-2}	0.17×10^{3}
KClCdTe	0.508×10^{-1}	0.6×10^{-1}	0.18×10^{2}
KI-GaAs	0.648×10^{-2}	0.2×10^{-2}	0.52×10^{3}
KI-ZnSe	0.985×10^{-2}	0.3×10^{-1}	0.38×10^{2}
KICdTe	0.699×10^{-1}	0.1×10^{-4}	0.93×10^{5}

These formulae are different from those given in [5]. Results are shown in Table 3 for ρ -polarized waves. A wider variety of appropriate pairs for composites there is with excellent improvements, by as much as several orders of magnitude, comparing to izotropic materials.

References

- [1] SPARKS M., J. Appl. Phys. 42 (1971), 5029.
- [2] JASPERSE J. R., GIANINO P. D., J. Appl. Phys. 43 (1972), 1686.
- [3] BENDOW B., GIANINO P. D., Appl. Opt. 12 (1973), 710.
- [4] BENDOW B., GIANINO P. D., J. Elect. Mat. 2 (1973), 87.
- [5] BENDOW B., GIANINO P. D., Appl. Opt. 14 (1975), 277.
- [6] BORN M., WOLF E., Principles of Optics, Macmillan, New York 1964.
- [7] ABRAMOWITZ M., STEGUN I., Handbook of Mathematics, NBS, Washington 1964.
- [8] BENDOW B., GIANINO P. D., TSAY Y. F., MITRA S. S., Appl. Opt. 13 (1974), 2382.

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Компенсация термического линзинга, происходящего из сложных окон лазера СО₂

Расфокусирование, а также дисторсия, называемые здесь совместно термическим линзингом, могут возникнуть в лазерных системах CO₂ большой мощности вследствие неоднородности облучения окна. Дан анализ компенсации термического линзинга, происходящего из окон. Рассуждены изотропические материалы, а также монокристаллы, срезываемые вдоль плоскости [111].

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and $\alpha^2 = 1$