# Letters to the Editor 

Matching of Bragg condition of phase gratings of point source holograms in $1.3-1.5 \mu \mathrm{~m}$ region*

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#### Abstract

Matching of the Bragg condition of holographic phase gratings has been calculated for an off-axis hololens recorded with the aid of the conventional holography and an on-axis hololens recorded by using computer generated holography in 1.3-1.5 $\mu \mathrm{m}$ region. The spectral bandwidths of both lenses were estimated with the aid of the holographic imagery after refractive index matching in accordance with the Bragg condition.


## 1. Introduction

The spectral diffraction efficiency of phase holograms depends on refractive index modulation, grating spacing and the thickness of a holographic grating [1]-[3]. For example, the refractive index modulation of the dichromated gelatin grating (DCG) can be increased so that the diffraction efficiency of $90 \%$ measured in Bragg angle without refractive index matching can be achieved in the recording wavelength. Use of the same holographic grating in longer wavelength region in accordance with the Bragg condition demands the refractive index matching.

In our applications, holographic phase gratings in the on-axis and off-axis hololens forms (HOL) [4], [5] are used as wavelength selective elements in conventional optical multi-/demultiplexing (mux/demux) applications [6], [7], where various wavelengths are multiplexed to the detectors needed. On the other hand, the novel applications of the HOL elements used as a monochromator in the external cavity construction of semiconductor laser according to the refractive index matching to realize the Bragg condition are theoretically investigated in the wavelength region from $1.3 \mu \mathrm{~m}$ to $1.5 \mu \mathrm{~m}$.

## 2. Computational procedure

The wavefront aberrations of object and reference point sources from the reference sphere are given by the law of hologram imagery presented by Latta [8], [9]. The aberrations are calculated in the hologram plane from the reference sphere with the coefficients of spherical aberration, coma and astigmatism. To avoid an increase in

[^0]the number of data points the areal weighted average aberration, AWAA, is used for the variation of spherical, coma and astigmatism aberrations over the hologram aperture
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\begin{equation*}
\mathrm{AWAA}=\sum_{i} \frac{A_{i}}{A} \sqrt{S_{i}^{2}+C_{i}^{2}+A s_{i}^{2}} \tag{1}
\end{equation*}
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where: $A$ - format area, $A_{i}$ - area of an annulus and $S_{i}$ - spherical, $C_{i}$ - coma, $A s_{i}$ - astigmatism aberration for an annulus mid-point.

The optimization of aberration was calculated by using refractive index matching. In these applications, zero-chromatism caused by the refractive index was less than $1 \mu \mathrm{~m}$.

The diffraction efficiency of the phase grating was calculated with the aid of the coupled wave theories [2]-[4], [10].

## 3. Applications

Matching of the Bragg condition was made for the phase gratings working in offand on-axis hololens forms. The focal length of the off-axis hololens 1 (Fig. 1a) with


Fig. 1. Off-axis (a) and on-axis (b) hololens with the focal length of 20 mm and the numerical aperture of 0.2 . The reference $(\mathrm{R})$ and the object $(\mathrm{O})$ sources are in $x z$-plane producing the respective interference pictures
grating spacing from infinity to $1.0 \mu \mathrm{~m}$ recorded at the $0.488 \mu \mathrm{~m}$ wavelength was 20 mm . The focal length of the on-axis hololens 2 (Fig. 1b) recorded at the $0.488 \mu \mathrm{~m}$ wavelength was 20 mm wifh grating spacing from infinity to $2.0 \mu \mathrm{~m}$. The numerical
aperture of the two components planned for mux/demux or external cavity operation was 0.2 at the recording wavelength. The diameter of the lens 1 and 2 was 10 mm .

## 4. Results

The areal weighted values of geometrical aberrations of the lens 1 (Fig. 1a) and the lens 2 (Fig. 1b) recorded with a $0.488 \mu \mathrm{~m}$ wavelength and reconstructed with a $1.5 \mu \mathrm{~m}$ wavelength were $205.3 \mu \mathrm{~m}$ (Fig.2a) and $31.3 \mu \mathrm{~m}$ (Fig. 2b) before the refractive


Fig. 2. Aberrations of spherical aberration (As), coma (C) and astigmatism (A) as a function of the aperture of the hololens 1 (a) and the hololens 2 (b) in $x$-direction before the refractive index matching to realize the Bragg condition
index matching. The areal weighted values of geometrical aberrations of the respective lenses were less than $0.05 \mu \mathrm{~m}$ after the refractive index matching in accordance with the Bragg condition.

The calculated spectral bandwidths of the lens 1 (Fig. 3a) and lens 2 (Fig. 3b) recorded with the $0.488 \mu \mathrm{~m}$ wavelength were $0.02 \mu \mathrm{~m}$ and $2.0 \mu \mathrm{~m}$ when the respective matchings of the Bragg condition in the wavelength regions of $1.3 \mu \mathrm{~m}, 1.4 \mu \mathrm{~m}$ and 1.5 $\mu \mathrm{m}$ were made.

The calculated diffraction efficiencies of lens 1 and lens 2 in non-symmetric (Fig. 4a) and symmetric (Fig. 4b) working conditions after the refractive index matching were $82.6 \%$ and $90.8 \%$. The attenuation losses of lens 1 and lens 2 were -0.84 dB and -0.43 dB when the refractive index of the phase grating ( $n=1.54$ ) and the matchings glass accordance with the diffraction efficiency of the lenses are taken into consideration.

## 5. Conclusions

Matching of the Bragg condition was made by using the data of the dichromated gelatin gratings, where the thickness of grating was $15 \mu \mathrm{~m}$ and the refractive index modulation being 0.042 . The angle between the object and the reference point sources was fixed producing the off-axis hololens 1 (numerical aperture $\mathrm{NA}=0.2$ )


Fig. 3. Areal weighted aberrations of spherical aberration, coma and astigmatism of the hololens 1 (a) and the hololens 2 (b) recorded by using $0.488 \mu \mathrm{~m}$ wavelength after the refractive index matching in the 1.2$1.6 \mu \mathrm{~m}$ wavelength region


Fig. 4. Diffraction efficiency (DE) of the hololens 1 (a) and the hololens 2 (b) recorded by using $0.488 \mu \mathrm{~m}$ wavelength before and after the refractive index matching to realize the Bragg condition in the wavelength of $1.5 \mu \mathrm{~m}$
and the on-axis hololens $2(\mathrm{NA}=0.2)$ with the grating spacing from infinity to 1.0 $\mu \mathrm{m}$ and from infinity to $2.0 \mu \mathrm{~m}$ at the recording wavelength of $0.488 \mu \mathrm{~m}$. The results of the off-axis hololens 1 are in accordance with our earlier results recorded at the wavelength of $0.488 \mu \mathrm{~m}$ and reconstructed in the wavelength region of $1.2 \mu \mathrm{~m}$.

However, according to spectral efficiency analysis in the wavelength range from $1.3 \mu \mathrm{~m}$ to $1.5 \mu \mathrm{~m}$, the diffraction efficiency of the grating spacing less than $1 \mu \mathrm{~m}$ is more than $20 \%$ lower than the efficiency of grating spacing greater than $2 \mu \mathrm{~m}$. The calculated attenuation loss of the off-axis hololens 1 made by conventional holography was 0.4 dB higher than that of the on-axis hololens 2 made by computer generated holography. For that reason, we have investigated the applications of computer generated on-axis hololenses to slove this geometric problem in fabrication of hololens with grating spacing greater than $2 \mu \mathrm{~m}$ [5].

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## Согласование условия Брагта для фазовой решетки точечной голограммы в области 1,3-1.5 $\mu \mathrm{m}$

Рассчитано согласование условия Брагта для голографической фазовой решетки для внеаксиальной голографической линзы, зарегистрированной при помощи голографии, генерируемой мик-ро-ЭВМ, а также аксиальной голографической линзы, зарегистрированной с употреблением синтетической голографии в области $1,3-1,5 \mu \mathrm{~m}$. ІШирина спектральной полосы обеих линз оценена при помощи голографического изображения после получения согласования коэффициента преломления согласно условию Брагга.


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