Optica Applicata, Vol. XXI, No. 4, 1991

"Topological" interferometry with data reduction for quick measurements of Δ and α parameters of preforms and waveguides *

W. Kowalik

Institute of Physics, Technical University of Wrocław, Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, Poland.

A new approach to complex but often repeated measurements of objects of similar class has been presented. A two-stage measurement cycle has been proposed. The first stage consists in finding the theoretical model of the measurement process in the form of a direct transformation of the data to the final results. It has been shown that, as a result of this, the number of measurements as well as all the calculations may be drastically reduced, while the apparatus used may be significantly simplified. The second stage is defined by the measurements realized in real time, for instance, by shifting the examined object in a measuring setup. The described method remains valid for all possible complex measurements. The results shown below, being only an illustration of the method, concern interference measurements of phase objects (such as preforms or fiber waveguides) of both cylindric symmetry and continuous profile of the refractive index.

1. Introduction

Interference measurements of the parameters Δ and α describing the results of measurements of the refractive index profile have been employed for a long time [1], [2]. However, in many applications, the analytic results (described by parameters Δ and α) are more desired than the detailed tabulated results. Our approach is more general than those presented in the publications cited above and deals with composed (indirect) measurements. The application of the full two-stage measurement cycle suggested below is profitable only if a great number of measurements of objects of similar class are performed. This method may be used also for the measurements for which the first stage of the measurement cycle has been already performed as is the case for preforms and light waveguides in this work. The idea of the method is general enough to include a wide class of measurements, while the presented results of interference measurements of the parameters Δ and α constitute only an illustration of the method. The generality of the method enabling its application to different fields of metrology is worth emphasizing. Besides, the significant simplification of the due calculations offered by the method makes

* This work has been done under the CPBP 01.06 Project, and presented at the Czechoslovakian-Polish Optical Conference in Hradec nad Moravici, September 25-29, 1990.

possible the measurements to be made in real time. This concerns also such measurements which, when performed in a classical way, are impossible to realize in the real time because of very complex and time-consuming calculation cycle.

A two-stage measurement cycle has been proposed. The first stage (the so-called "topological" examinations) consists in finding the direct transformation between the data and the final results. It has been shown that both the measurements and the calculations may be radically reduced and, moreover, the apparatus used may be significantly simplified. The second stage consists of measurements which may be performed in the real time, for instance, by shifting the examined object in the apparatus.

The present paper is the fifth of the whole cycle concerning interference measurements of parameters Δ and α with data reduction. In paper [3], the results of "topological" examinations for interferometry with plane reference wave are presented. The papers [4] and [5] exploit the specific relations which occur in both radial and transversal shearing methods. The work [6] contains a generalization of the "topological" examinations for the shearing methods, while the present work offers a generalization of all the methods described earlier as well as proposes a specific generalization of the methodics of any complex measurements. Also, the idea of the proposed measurement methods is described, while the measurements of parameters Δ and α provide its good illustration.

2. Classical measurements

In classical indirect (composed) measurements, the apparatus (for instance, an interferometer) provides the measurement results (for instance, an interferogram) of the object examined (for instance, a preform), see the upper part of Fig. 1. The measurement results after having been subjected to the due numerical analysis (eventually via scanning and sampling of the image) constitute the detailed (fundamental) data in the data space. These data include also: data concerning the measurement conditions as defined by the apparatus used (such as the shearing parameter, location of the optical wedge edge in the interferometer, light wavelength, and so on) as well as *a priori* data concerning the object (for instance, an exact geometry of the object). The detailed data are in the numerical form. In both the measurements and the elaboration of the results, the general data are also very important, being, on the whole, difficult to express in the numerical form (for instance, the object under test may have a continuous and smooth profile of the refractive index). These data may be known *a priori* (for instance, either from known technology used to produce the preform or from a preliminary analysis of the due interferograms). All the said data belong to the data space. The transition from the data space to the space of results occurs according to a more or less complex algorithm which provides the sought results (for instance, the refractive index profile). The process of calculations is repeated for each sampling results. The cycle of repetitions, large number of data and complex cycle of calculations may become a serious load for the computer and therefore the results are usually obtained with

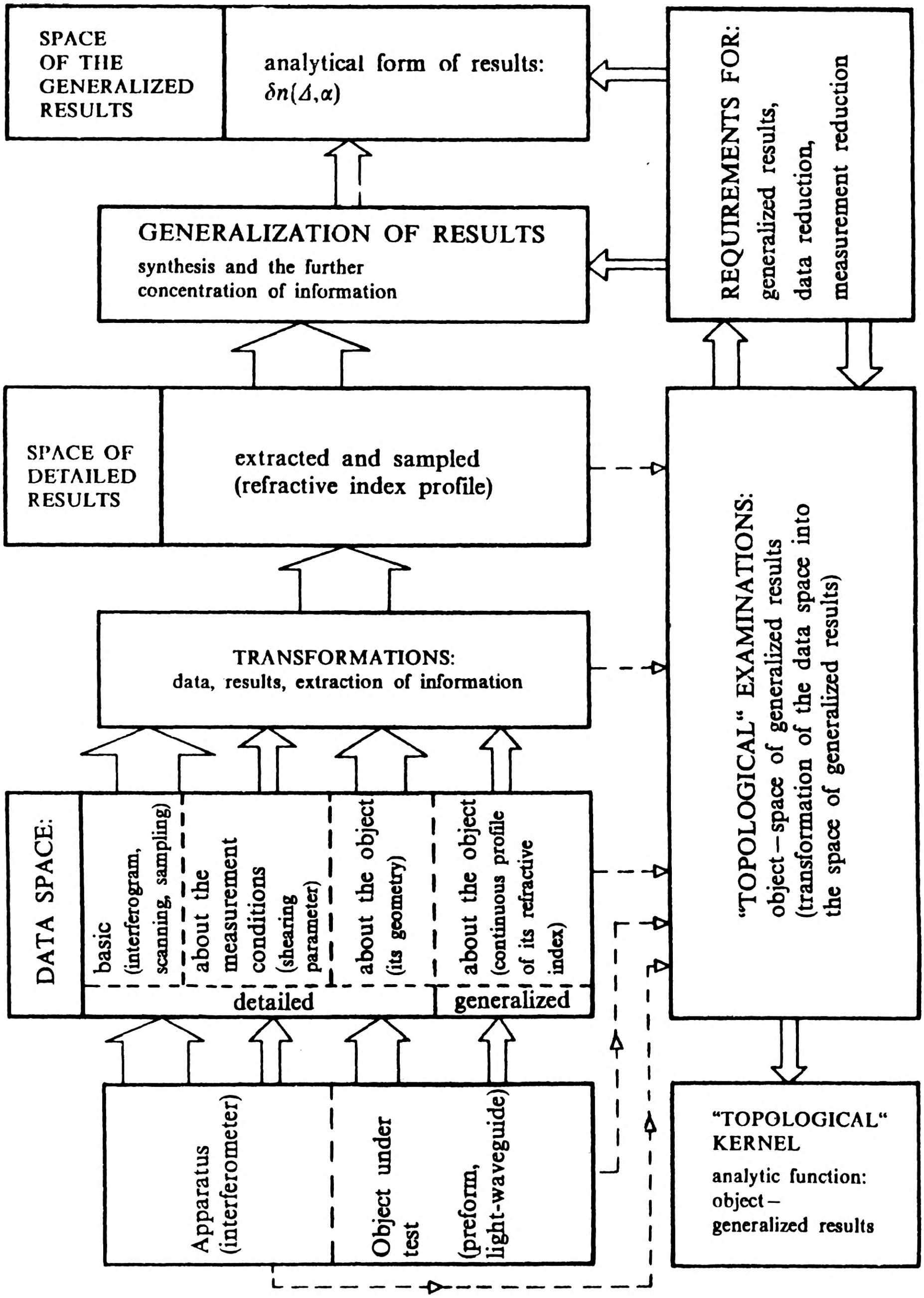

Fig. 1. Block scheme of the measurements. Classical measurements – upper part of the figure. "Topological" examinations – lower part of the figure

some delay. If we want to generalize the results, their analytic form may usually be obtained by using an approximating procedure (at the expense of some additional delay caused by the approximations). A further extraction of information occurs during this process.

3. "Topological" measurements

The term topology, in this work, is used in the most general sense and should not be mistaken for the strict mathematical definition of the term.

The idea of the topological measurement is the following. In the case of composite measurements, the direct results of measurements (called the data space) depend on many variables. If these results constitute a continuous function (of the above variables), then the sought magnitude being a continuous function depending on the data space may be found in the space of the detailed results, independently of the ways the calculations are done or the information is extracted. Thus, independently of the true transformation of the data space into the space of the results, a simplified transformation may be found which approximates the true transformation. The results are obtained by changing the values of particular variables in the data space and carrying out the true transformations. In this way, both spaces are filled up. In the course of these operations, the dependence of the results on the variables is examined (topological examination) and a substitute transformation is found. The substitute transformation is a single equation, which (for a limited range of variables) well represents very complex and time-consuming calculation algorithms (being a true transformation). The substitute transformation found in this way contains not only the information about the true transformation and thus the information about the examined object but also that about the measuring conditions, the apparatus, measuring method and the requirements imposed on the results, and so on. The method presented below is wider than the above mentioned idea and includes the description of the results in a generalized (analytic) form, i.e., the form of a function rather than as a set of tabulated results.

The topological measurements are of the two-stage type: first the topological examinations are carried out and then the measurements are made.

3.1. Topological examinations

The processes of both calculations and measurements are treated as a whole and the data from each stage are essential in topological examinations (broken line in Fig. 1, and its lower part) and in later measurements, as well.

The first task in topological measurements is to make a choice of the optimal form of the function describing the generalized results. The form of this function should be convenient enough. In order to reduce the measurements, the number of coefficients which describe the function should be as small as possible. From the point of view of the fidelity with which the measured magnitudes should be recovered, the form of the function and the number of coefficients which describe this function should be optimal. These requirements may be met at the stage of choosing the function. On the base of the measurements of similar objects the space of measurement data is filled for each object. Instead of measurement data, the data calculated in the course of computer simulation process, if the latter is possible, may be taken into account. Such data have been obtained in the present paper. By making the corresponding calculations in a classical way, we pass from the space of measurement data to the space of detailed data. The latter constitute, in turn, the basis to determine the expected form of the function (using the approximation procedure) and the procedure of fitting this function to the results is used. By carrying out this cycle for many examined objects of the same class and basing on the fitting procedure, the optimal form of the function of results may be obtained. This process may be automatized if a suitable computer program is elaborated, for which a bank of the forms of the sought functions is created and the suitable selection criteria are elaborated.

Another task in topological examinations is to find the so-called topological kernel (of an analytic function) according to which a direct transformation of the data space into the space of the generalized results may be performed. The topological examinations are carried out under the imposed or optimally chosen requirements concerning the form of the generalized results and reduction of both data and measurements. In the process of topological examinations, the analytic function of the kernel is found with definite accuracy of approximation which my be influenced. This may assure the wanted simplicity of the transformation on the one hand, and the required accuracy – on the other. The transforming equations are obtained by analysing and synthesizing the corresponding relations. Next, the process of data reduction is carried out. From the approximation process, the corresponding approximating coefficients ($a_i(r)$, and others) are calculated to be next used in the transformation equation. These coefficients must well describe the full ranges of changes of parameters filling the spaces of both data and results. The transformation equations together with these coefficients constitute the topological kernel. The number of transforming equations should assure their unique resolution and depends on the number of the sought parameters describing the function of the generalized results. The problem is now reduced to the solution of a system of equations for the parameters of the sought function. For the nonlinear equations, the best way is to apply the standard computer methods. If the systems of equations are not too complicated, the problem may be solved analytically as it has been shown in the present work and in works [3]–[6].

3.2. Measurements

The topological measurements (see Fig. 2) must be performed under the imposed requirements concerning measurement conditions, form of the generalized results, reduction of both data and the results. Since the kernel was found for the definite measurement conditions and the corresponding class of the examined objects, the measurements must be carried out also under the same conditions and for the same class of the objects. Only if such conditions are fulfilled, the correct results are obtained and a significant reduction of both data and measurements is possible.

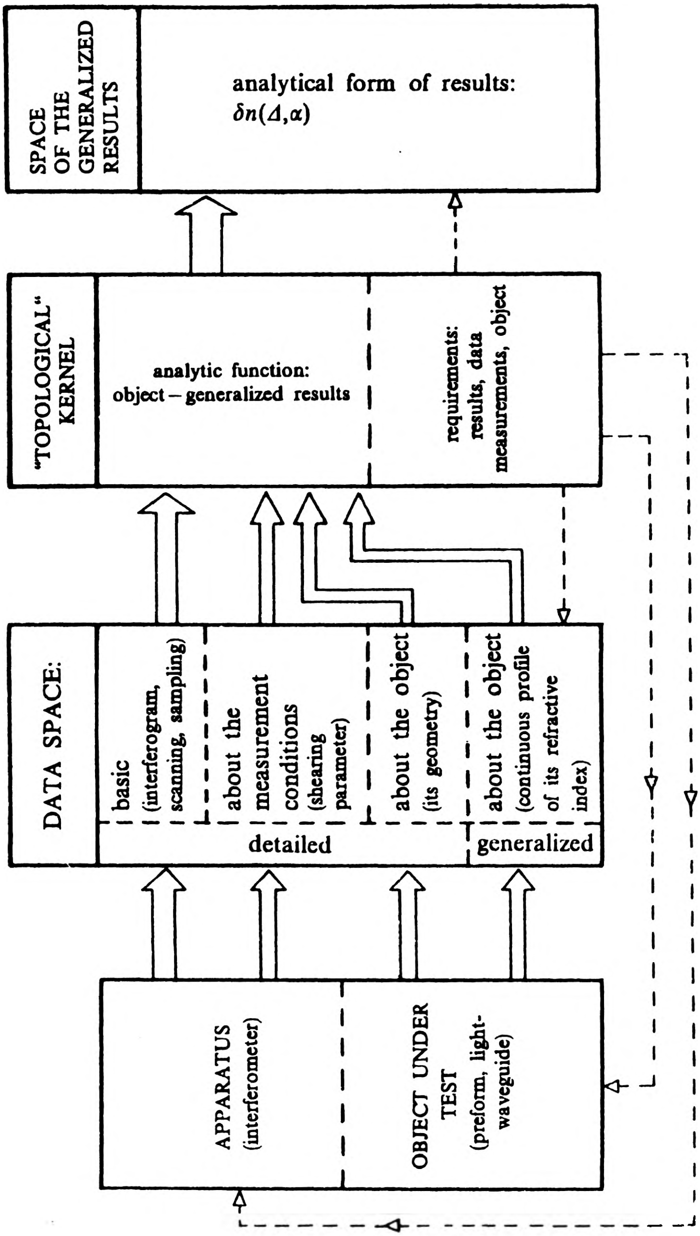

Fig. 2. Block scheme of the "topological" measurements

Since the kernel was found for definite measurement conditions and a corresponding class of the examined object the measurements must also be carried out under the same conditions and for the same class of the object. The correct results as well as very high reduction of data, measurements and calculations may be achieved only when these conditions are fulfilled. The results in a generalized (analytic) form are obtained immediately. It may be stated that the above reduction became possible due to the transition of certain processes to the stage of topological examinations. These examinations increased the general (*a priori*) data by finding the form of the function of generalized results and the transforming equation.

In the intelligent systems of topological measurements, the automation is applicable to both the topological examinations and the measurements. During the topological examinations the objects of the same class are examined and the spaces of data and results are filled. The optimal form of the function of results is sought by taking advantage of the created bank of functions and the respective selection criteria. Next, the transforming equation is found. The found numerical coefficients of the transforming equations creating the topological kernel as well as parameters describing the measuring conditions (such as wavelength and shearing parameters, for instance) are attributed to a corresponding class of the examined object. The system memorizes these parameters and learns to measure the corresponding class of objects. During the measurement the system may recognize automatically the class and apply the procedure of calculations of the generalized results learned earlier, if the class of the examined object is not indicated *a priori*. Thanks to the high reduction of data and the simple formula the measurements in the real time become possible.

4. Interferometry with reduction of data as applied to fast measurements of the Δ and α parameters of both preforms and waveguides

The examples presented above refer to the measurement of preforms and waveguides, but they should be treated only as concrete examples of applications. It should be remembered that all composite measurements are essentially the topological examinations, which are not restricted exclusively to preforms, waveguides or interferometry. Although the objects chosen for the measurements, i.e., preforms and light waveguides, are of rotational symmetry their measurements are still composite enough even if the form of their refractive index distribution function is known *a priori*. Thanks to the last fact, the process of topological examinations has been slightly simplified, while the choice of not too complex objects gave the hope of obtaining the following information: whether and to what degree it is possible to verify the method via measurements, whether the degree of data reduction may be examined, and whether the measurement data reduction constitutes a topic worth any further consideration if the objects are more complicated.

In the case of preforms and light waveguides, the data filling the space of results have been found with the help of simulating computer examinations [3]–[6]. It has been assumed that:

1. A continuous function of the following form is a sufficiently good and exact approximation of the real function

$$n(r) = n_0\sqrt{1-2\Delta r^{\alpha}}, \quad n_0 = n_p/\sqrt{1-2\Delta} \tag{1}$$

where: $r = r'/r_0$ is its normed argument, $r_0 = d/2$ – radius of the core, d – its diameter, r' – current coordinate in the core, and n_p – refractive index of the coat. Function (1) is an optimal function describing the generalized results.

2. Visual or automatic measurement of the interference order of fringes is sufficiently insensitive to microdiscontinuities. Also, some additional assumptions have been accepted which affect the computed coefficients in this work.

3. The sought parameters range within the following limits: $\Delta \in \langle 0.009, 0.015\rangle$, and $\alpha \in \langle 1.6, 2.4\rangle$. While for the shearing methods the shearing parameters $b \in \langle 0.995, 0.98\rangle$ and $s \in \langle 0.005, 0.02\rangle$; b is the ratio of the radius of the smaller wavefront to the radius of the greater wavefront; s is the ratio of the transversal shift of the wavefront to the radius of the wavefront core.

4. The accuracy of the wavefront calculation achieved in the zero order approximation is sufficient.

5. Since the calculations have been carried out for the wavelength $\lambda = 632.8$ nm, the measurements should be performed for the same wavelength.

Simulating computer examinations consisted in:

– Calculating the wavefront $g(r)$, on the base of the relation (1), for the given Δ and α (which were changed in the above limits), and for different core diameters d.

– Calculation of interference orders δm (fringe shapes) for different values of Δ, α, d and shearing parameters b and s.

– Analysing the interference orders and statement which of the variables have no influence on the orders (which caused consequently the due reduction of the number of variables) and which affect them and to what degree.

– Expressing the interference order $\delta m(r)$ by the parameters Δ and α, i.e., finding the transforming equation and calculating its coefficients.

– Solving the system of transforming equations, i.e., finding the new functions of new numerical coefficients, which renders possible the immediate calculation of Δ and α from the measured orders of interference $\delta m(r)$.

The symbol δ denotes the difference operator. The relative interference orders $\delta m(r) = m(r) - m_p$ are the differences between the interference orders in the core and coat, respectively. For the solutions below, it has been assumed that the value of r increases if the indices at r increase as well; r without any index denotes an arbitrary fixed value of r.

4.1. Topological examinations

These examinations are presented below for three different interference methods: interference with plane reference wave, radial shearing interference and transversal shearing interference. The results presented concern the objects of cylindric symmetry of continuous and smooth profile of the refractive index.

4.1.1. Method of interfering with plane reference wave

A more accurate analysis has been given in work [3]. From topological examinations, it follows that the so-called normed wavefront is of the form

$$\delta\hat{g}(r,d,\Delta,\alpha) = \delta m(r,d,\Delta,\alpha)/(d\Delta) = a_0(r)+a_1(r)\alpha+a_2(r)\alpha^2+\partial_{\Delta}(r)(\Delta-0.012). \quad (2)$$

This is the equation transforming the data space ($\partial m(r)$, d, λ) into the space of generalized results (Δ, α). The coefficients $a_0(r)$, $a_1(r)$, $a_2(r)$ and $\partial_{\Delta}(r)$ are given in Tables 1 and 1a.

Table 1. Coefficients of the transforming equation for the interference method with the plane reference wave and for specified values of r

r	$a_0(r)$	$a_1(r)$	$a_2(r)$	$\partial_{\Delta}(r)$
0.00	682.1328	618.219625	−88.3496875	2560.7667
0.08	637.20265	641.619512	−92.5555937	2540.43333
0.16	542.76965	677.205875	−97.884625	2479.21666
0.20	488.1124	690.750125	−99.2796875	2433.0333
0.24	432.2066	689.877437	−99.4133437	2377.18333
0.32	323.619	695.1725	−95.569375	2234.35
0.36	273.4685	682.391875	−91.6171875	2147.7333
0.40	277.2093	662.226625	−86.4359375	2051.4667
0.48	148.07901	601.058275	−73.07354	1830.31666
0.56	88.02931	515.82344	−57.0608187	1572.84833
0.60	64.95029	466.223815	−48.6572875	1431.61166
0.64	46.203621	413.246597	−40.3114337	1283.35

Table 1a. Coefficients a_{ij} of the power series from which the coefficients for the transforming equation may be calculated for arbitrary $r \in \langle 0, 0.64 \rangle$ and for the interference method with the plane reference wave

j	$a_i(r) = \Sigma a_{ij} r^j$			$a_{\Delta}(r) = \Sigma a_{ij} r^j$
	a_{ij}			
	$i=0$	$i=1$	$i=2$	$i=\Delta$
0	682.139503	618.214404	−88.3483429	2560.76005
1	−54.566963	50.326501	−11.8651957	3.7475586
2	−7914.96898	4393.3626	−796.009043	−3234.0295
3	21942.254	−19264.704	4096.96113	227.920387
4	−31023.8712	30189.1215	−7033.17677	−669.97893
5	25633.3403	−25185.416	5938.79785	907.675959
6	−9582.56146	9499.81224	−2183.1549	

i) Three-point method

When choosing three Eqs. (2) for different values of r a system of equations, the so-called topological kernel, is obtained from which Δ and α may be calculated

$$\alpha = (-B-\sqrt{B^2-4AC})/(2A), \quad \Delta = 1/(A_0+A_1\alpha+A_2\alpha^2), \tag{3}$$

where: $A = A_2-B_2, \quad B = A_1-B_1, \quad C = A_0-B_0,$

$A_i = (a_i'(r_1)-a_i'(r_2))/(\delta m'(r_1)-\delta m'(r_2)),$

$B_i = (a_i'(r_1)-a_i'(r_3))/(\delta m'(r_1)-\delta m'(r_3)),$

$$a_i'(r_j) = a_i(r_j)/\partial_\Delta(r_j), \quad \delta m'(r_j) = \delta m(r_j)/\partial_\Delta(r_j). \tag{4}$$

As it is visible, the system of equations has been solved in an analytic way. In order to determine the parameters Δ and α, it suffices to perform the measurement of interference order in three points on the core radius.

ii) Two-point method

The transforming equation of slightly less accuracy may be obtained by neglecting the relatively small expression of $\partial_\Delta(r)(\Delta-0.012)$, in Eq. (2). When choosing two of such simplified equations for different values of r a system of equations (topological kernel) is obtained, from which Δ and α may be calculated:

$$\alpha = (-B+\sqrt{B^2-4AC})/(2A),$$

$$\Delta = \delta m(r)/[d(a_0(r)+a_1 r)\alpha+a_2(r)\alpha^2)], \tag{5}$$

where: $A = \delta m(r_1)a_2(r_2)-\delta m(r_2)a_2(r_1),$

$B = \delta m(r_1)a_1(r_2)-\delta m(r_2)a_1(r_1),$

$$C = \delta m(r_1)a_0(r_2)-\delta m(r_2)a_0(r_1). \tag{6}$$

As it may be seen, in this method it suffices to measure the relative interference order $\delta m(r)$ in two points on the core radius, i.e., in r_1 and r_2.

4.1.2. Radial shearing interference method

A more accurate analysis is given in paper [4] and [6]. From the topological examinations, it follows that the so-called normed order of interference is:

$$\delta\hat{m}(r, d, \Delta, \alpha, b) = \delta m(r, d, \Delta, \alpha, b)/[d\Delta(1-b)] = a_0(r)+a_1(r)\alpha+a_2(r)\alpha^2$$
$$+\partial_\Delta(r)(\Delta-0.012)+\partial_{b1}(r)(b-0.99)+\partial_{b2}(r)(b-0.99)^2. \tag{7}$$

This equation transforms the data space $(\partial m(r), d, b, \lambda)$. The coefficients $a_0(r)$, $a_1(r)$, $a_2(r)$, $\partial_\Delta(r)$, $\partial_{b1}(r)$, $\partial_{b2}(r)$ are collected in Tabs. 2 and 2a. This equation may be slightly simplified when $\partial_{b1}(r)$, is replaced by $\partial_b(r)$, and it is assumed that $\partial_{b2}(r) = 0$.

i) Three-point method

When choosing three Eq. (7) for different values of r, the sytem of equations (the so-called topological kernel) is obtained from which Δ and α may be calculated:

$$\alpha = (-B-\sqrt{B^2-4AC})/(2A),$$

$$\Delta = (-B'+\sqrt{B'^2-4A'C'})/(2A'), \tag{8}$$

where: $A = a_{2ij}/b_{ij}-a_{2kl}/b_{kl}, \quad B = a_{1ij}/b_{ij}-a_{1kl}/b_{kl},$

$C = (a_{0ij}/b_{ij}-a_{0kl}/b_{kl})+(c_{1ij}/b_{ij}-c_{1kl}/b_{kl})(b-0.99)$

$+(c_{2ij}/b_{ij}-c_{2kl}/b_{kl})(b-0.99)^2,$

Table 2. Coefficients of the transforming equation for the interference method of radial shearing for specified values of r

r	$a_0(r)$	$a_1(r)$	$a_2(r)$	$\partial_\Delta(r)$	$\partial_b(r)$	$\partial_{b1}(r)$	$\partial_{b2}(r)$
0.4	444.05824	233.749373	−56.7912156	1003.7673	276.5489	269.9647	−1316.853
0.44	438.79074	333.010552	−73.2524313	1223.2535	307.2772	305.0825	−438.94
0.48	420.94615	440.749872	−89.5080413	1419.3262	333.6147	331.4197	−439.0133
0.52	393.75232	551.69845	−104.323125	1668.0717	342.394	342.394	0.0
0.56	358.39413	662.399697	−116.874687	1919.7417	335.8087	332.5163	−658.4667
0.6	318.36029	767.504785	−126.134156	2168.4967	311.6673	307.2777	−877.9333
0.64	272.46602	865.696372	−132.101437	2428.945	241.4313	239.2357	−439.1333
0.68	224.5314	950.965047	−133.747437	2648.43	131.6893	118.5207	−2633.733
0.72	178.17651	1017.05695	−130.249437	2885.4717	−24.1433	−25.2407	−219.4666
0.76	133.53403	1060.92576	−121.812968	3107.8833	−278.745	−290.816	−2414.333
0.8	92.44601	1077.71702	−108.644187	3233.72	−625.525	−635.402	−1975.267
0.84	58.73369	1057.96324	−90.1252187	3353.7	−1150.09	−1154.48	−878.2
0.88	29.49774	998.621325	−68.5200625	3301.0283	−1953.4	−1990.71	−7462.4
0.92	12.11502	879.853222	−43.622375	3078.615	−3228.59	−3293.34	−12949.47

Table 2a. Coefficients a_{ij} of the power series from which the coefficients for the transforming equation may be calculated, for arbitrary $r \in \langle 0.4, 0.92 \rangle$ and for the interference method of radial shearing

j	$a_i(r) = \Sigma a_{ij} r^j$			$\partial_\Delta(r) = \Sigma a_{ij} r^j$	$\partial_b(r) = \Sigma a_{ij} r^j$
	a_{ij}				
	$i = 0$	$i = 1$	$i = 2$	$i = \Delta$	$i = b$
0	−368.751971	−4278.60453	99.366183	−28066.1897	36993.5877
1	4047.838702	43250.372	−715.846552	293524.1688	−472714.0084
2	−3978.5638	−186941.2209	2874.7285	−1254392.42	2569103.05
3	−5769.8499	434400.8039	−8772.3331	2847693.54	−7676389.49
4	8944.235265	−541774.7676	10738.0122	−3570406.05	13652488.06
5	−2868.526998	348744.9385	−4216.977809	2359759.68	−14453911.32
6		−93006.6461		−646661.8529	8432873.67
7					−2097095.26

$$
\begin{aligned}
a_{hmn} &= a_h(r_m)\delta m(r_n) - a_h(r_n)\delta m(r_m),\\
b_{mn} &= \partial_\Delta(r_m)\delta m(r_n) - \partial_\Delta(r_n)\delta m(r_m),\\
c_{pmn} &= \partial_{bp}(r_m)\delta m(r_n) - \partial_{bp}(r_n)\delta m(r_m),\\
A' &= \partial_\Delta(r),\\
B' &= a_0(r) + a_1(r)\alpha + a_2(r)\alpha^2 - \partial_\Delta(r)0.012 + \partial_{b1}(r)(b-0.99) + \partial_{b2}(r)(b-0.99)^2,\\
C' &= -\delta m(r)/[d(1-b)].
\end{aligned}
\tag{9}
$$

Here, for m and n the indices i, j, or k, and l are taken. The last indices assume the values $i = 3, j = 2, k = 2, l = 1$ or $i = 3, j = 1, k = 2, l = 1$. In this method, it suffices to measure the relative order of interference $\delta m(r)$ in the three different points (r) on the core radius.

ii) Two-point method

The transforming equation of slightly less accuracy may be obtained by neglecting the expression $\partial_{\Delta}(r)(\Delta-0.012)$ in Eq. (7). Choosing two such simplified equations for different values of r, a system of equations (topological kernel) is obtained from which:

$$\alpha=(-B+\sqrt{B^2-4AC})/(2A),$$
$$\Delta=\delta m(r)/[d(1-b)(a_0(r)+a_1(r)\alpha+a_2(r)\alpha^2+\partial_{b1}(r)(b-0.99)+\partial_{b2}(r)(b-0.99)^2)] \quad (10)$$

where: $A=a_2(r_2)/\delta m(r_2)-a_2(r_1)/\delta m(r_1)$,

$$B=a_1(r_2)/\delta m(r_2)-a_1(r_1)/\delta m(r_1),$$
$$C=[a_0(r_2)/\delta m(r_2)-a_0(r_1)/\delta m(r_1)]+[\partial_{b1}(r_2)/\delta m(r_2)-\partial_{b1}(r_1)/\delta m(r_1)]\times(b-0.99)+[\partial_{b2}(r_2)/\delta m(r_2)-\partial_{b2}(r_1)/\delta m(r_1)](b-0.99)^2. \quad (11)$$

It suffices to measure the relative order of interference $\delta m(r)$ in this case, in two different points r on the core radius.

4.1.3. Transversal shearing interference method

A more accurate analysis has been reported in papers [5] and [6], from which it follows that all the equations and conclusions are the same as those for the method of radial shearing (comp. Sect. 4.1.2). It suffices in all the formulae (7)–(11) to replace: the magnitude $(b-0.99)$ by $(s-0.01)$, the magnitude $(1-b)$ by s and the indices b by s. The coefficients: $a_0(r)$, $a_1(r)$, $a_2(r)$, $\partial_s(r)$ (or $\partial_{s1}(r)$ and $\partial_{s2}(r)$) have been collected in Tables 3 and 3a.

Table 3. Coefficients of the transforming equation for the interference method of the transversal shearing for specified values of r

r	$a_0(r)$	$a_1(r)$	$a_2(r)$	$\partial_{\Delta}(r)$	$\partial_s(r)$	$\partial_{s1}(r)$	$\partial_{s2}(r)$
0.4	1116.4669	572.68575	−140.331875	2510.89	1755.865	−1733.92	−4389.467
0.44	1001.6746	748.82141	−165.846906	2739.15	1624.175	−1602.23	−4389.866
0.48	885.35094	907.672122	−185.188812	2961.56	1496.874	−1499.07	439.
0.52	764.19597	1053.18972	−200.004	3189.82	1308.121	−1299.34	−1755.667
0.56	645.80723	1177.47156	−208.646031	3406.3767	1093.025	−1060.1	−6584.533
0.6	532.8169	1278.21478	−211.115343	3581.9633	904.27	−899.879	−878.2
0.64	428.78191	1351.30263	−207.000031	3763.405	610.1627	−581.628	−5706.867
0.68	333.83372	1397.06467	−197.12325	3915.5783	320.4447	−305.08	−3072.867
0.72	247.44517	1415.50122	−182.308125	4032.6333	−17.558	21.947	−877.8
0.76	176.02727	1398.7403	−161.4435	4071.44	−447.879	467.8303	−3990.333
0.8	116.19288	1349.32511	−136.566406	4074.54	−963.179	988.7253	−5109.333
0.84	70.58216	1262.0229	−108.043187	3976.9133	−1605.91	1634.71	−5759.2
0.88	36.94688	1134.243	−77.6764375	3761.9433	−2484.94	2525.391	−8089.8
0.92	15.39326	956.17524	−47.1408437	3374.6533	−3794.6	3870.014	−15082.87

4.2. Measurements

From the above analysis and the described methods, it follows that the number of measurements may be drastically reduced (only 2 or 3 measuring points of the

Table 3a. Coefficients a_{ij} of the power series from which the coefficients for the transforming equation may be calculated, for arbitrary $r \in \langle 0.4, 0.92 \rangle$ and for the transversal shearing

j	$a_i(r) = \Sigma a_{ij} r^j$			$a_\Delta(r) = \Sigma a_{ij} r^j$	$a_s(r) = \Sigma a_{ij} r^j$
	a_{ij}				
	$i=0$	$i=1$	$i=2$	$i=\Delta$	$i=b$
0	788.272942	475.6018	334.209969	5619.0525	195639.1
1	7233.195555	−11753.1317	−1681.254471	−42276.1136	−2310108.68
2	−24369.6211	57633.9605	1690.132402	168207.0086	11666021.58
3	23833.8392	−92959.3544	−2649.4027	−292438.9268	−32320011.46
4	−7481.958501	68040.4324	4777.488792	256389.0588	53049354.11
5		−21006.3065	−2464.938837	−93578.2118	−51657655.95
6					27660963.86
7					−6293828.17

relative order of interference $\delta m(r)$ on the core radius). Practically, in order to get more sure measurements, it is recommended to measure from both the left hand and right hand sides of the examined object axis. In turn, to remove the influence of the microdiscontinuities both the size and shape of the "point" detectors should be chosen suitably. The main advantage offered by the proposed measurement method is the simplification of the interferogram analysis and calculation of the results. Thanks to this, in order to automatize the measurement, it suffices to use a very primitive scanning device with point detectors located in fixed positions on the core radius (Fig. 3). In order to perform the measurements of the objects of different diameters, it suffices to change the magnification in such a way that the core image diameter remains constant in the detection plane. A simple calculation of the parameters Δ and α enables obtaining the results at once. Therefore, the measurement in the real time is possible, for instance, during shifting of the object in the interferometer.

4.3. Measurements accuracy

More complete assessment of the measurement accuracy has been given in papers [3]–[6]. Here, we restrict our attention to the fact that the error of the interference order measurement amounts to 0.2 fringe, for the interference method with plane reference wave, while for the shearing method the errors amount to 0.05 fraction of the fringe for the visual method, and 0.01 fraction of the fringe for the scanning device. The errors of shearing parameters are accepted as being $\sigma(1-b)/(1-b) = \sigma(s)/s = 0.01$, while for the coordinate measurement the error is $\sigma(r) = 0.001$. The following accuracies for Δ and α have been achieved: 4% and 4.5% for the method with plane reference wave, 4.7% and 5.5% for shearing visual methods and 3.6% and 4% for scanning device in shearing methods, respectively. These errors may be diminished by about 1.5% if the accurate calculations (Eq.(7)) instead of the zero order approximation are used for the calculations of coefficients in Tables 1–3 [7].

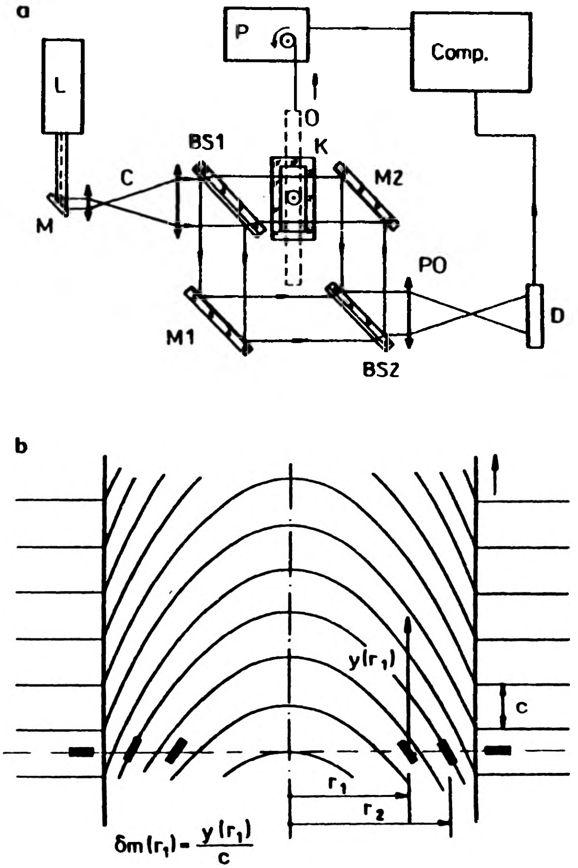

Fig. 3. One of the possible versions of the simplified method of scanning the interferogram: a — scheme of the interferometer (L — laser, C — collimator, BS1 and BS2 — beam splitters, M, M1 and M2 — mirrors, K — cuvette, O — examined object, PO — imaging objective, D — detectors, P — shifting mechanisms (stepping motor), COMP — computer and programmer of the shifting mechanism), b — scheme of the scanning interferogram (C — interfringe distance, $y(r_1)$ — deviation of a fringe from rectilinearity, block rectangulars—slits of the "point" detectors. Upward shift of the object examined)

5. Conclusions

The examinations carried out for preforms and light waveguides showed that the described method is worth using in many cases. The calculated coefficients allow us to obtain the generalized results without performing any complex calculations and for very simplified analysis of the interferograms. They may be, thus, recommended for examinations of preforms and light waveguides, if the required accuracies of measurements correspond to the above mentioned accuracies of the proposed method.

Thanks to the drastic reduction of the measurements, the analysis of the interferograms is very simplified. Therefore, a very primitive scanning device is sufficient to measure the relative order of interference in several fixed locations on the core diameter. If no scanning device is available, no recording of interferogram is necessary. It suffices to measure the fringe deformation as well as the interfringe distance with a micrometer ocular if the image is stable enough.

Since the proposed method concerns all kinds of composite measurements, it would be interesting to verify it also in other fields of metrology. The two-stage cycle of measurements allows us to reduce the data to a significant degree. The examinations of the measurement data reduction and its influence on the measurement accuracy are interesting and will be continued.

Also interesting are the examinations of intelligent measuring systems as well as their process of learning to measure similar objects. The scheme of such a system shown in this paper requires some additional examinations to be performed.

References

[1] Saunders M. J., Gardner W. B., Appl. Opt. **16**(1977), 2368.
[2] Stone F. T., Appl. Opt. **16** (1977), 2738.
[3] Kowalik W., SPIE **1085** (1989), 304.
[4] Kowalik W., ibidem, p. 309.
[5] Kowalik W., ibidem, p. 314.
[6] Kowalik W., Opt. Appl. **20** (1990), 321.
[7] Sochacki J., Appl. Opt. **25** (1986), 3473.

Received November 28, 1990