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On higher order distortion as related to wave aberration of the sagittal focus**

Basing on properties of wave aberration of the sagittal focus an analytic expression for fifth-order distortion has been derived, which included the field curvature and astigmatism of the fifth-order as well as a perturbing function. The dependence of the higher order distortion upon that of fifth order as a function of Z' has been analysed for a number of optical system types. From the investigations carried out it follows that the dependence is quasi-linear, thus it may be employed for examination of higher order aberrations. The function Z' may be also used for correcting the field aberrations of optical systems.

1. Introduction

In the paper [1] a method has been proposed, which simplifies in an essential way the analysis of the influence of the higher order aberrations on the correction of optical systems. While investigating the properties of the sagittal focus wave aberrations of fifth order it has been stated that there exists an approximately linear dependence between the aberrations of orders higher than fifth and the term F. The properties of this term have been examined for a most representative groups of photographic, telescopic and microscopic objectives. The deviation from the linearity for 90% of the field of view area proved not to exceed 8% (with the exception of 1.5% cases). Because of this practical linearity dependence the expression for F may reflect the character of aberrations of order higher than fifth. The properties of the term F encouraged us to continue the examinations for an astigmatic beam. Consequently, the respective expressions for the fifth order distortion have been derived.

2. The fifth-order distortion

Let an arbitrary ray $\bar{Q}B$ of the sagittal astigmatic beam (fig. 1) be defined by the invariant reduced coordinates $(\bar{H}_s, \bar{G}_s)$ in the object plane and the respective coordinates in the exit pupil (X, Y), whereby

$$\bar{H}_s = \frac{\sin u_s}{\sin u}\bar{H};\ \bar{H} = n\sin u\bar{\eta};$$

$$\bar{G}_s = \frac{\sin u_s}{\sin u}\bar{G};\ \bar{G} = n\sin u\bar{\xi};$$

$$X = -\frac{x}{h_s};\ Y = -\frac{y}{h_s},$$

$\bar{\eta}, \bar{\xi}$ – are the distances of the intersection point of the principal ray with Gaussian plane from the optical axis.

u – paraxial aperture angle,

u_s – paraxial sagittal aperture point,

h_s – paraxial height of the sagittal ray for an astigmatic beam.

The principal sagittal ray of the astigmatic beam is thus determined by

$$\bar{H}_s = nu_s\bar{\eta};\ \bar{G}_s = nu_s\bar{\xi};\ X, Y$$

in the object plane, and by $\bar{H}'_s, \bar{G}'_s, X', Y'$ respectively, in the image plane. Because of the lack of aberration in the diaphragm it is reasonable to assume for an astigmatic beam that in the diaphragm plane (region D) the sagittal paraxial height h_s is equal to height of incidence for an aperture paraxial ray [2]

$$(h_s)_D = h_D. \tag{2}$$

In the paper [2] it has been shown, that the object and image foci S and S' lie on one straight line passing through the centre of curvature of the surface (fig. 2). It is well known also that the ray passing through the

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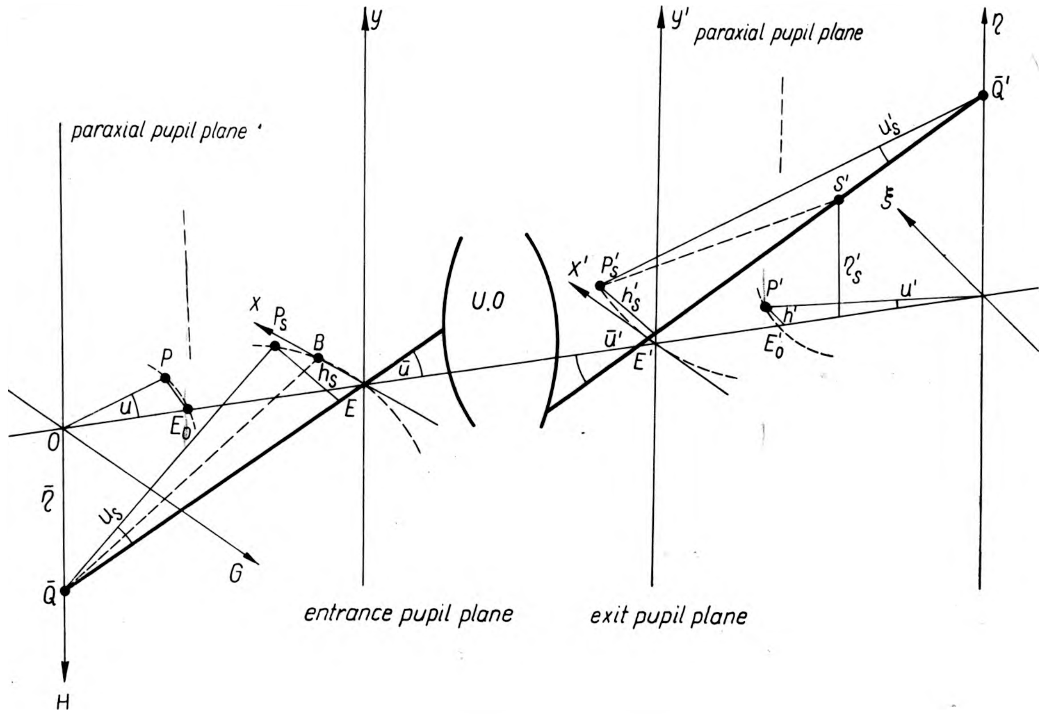


Fig. 1. Reduced coordinates of the sagittal ray

centre of the surface curvature is an aberrationless ray (fig. 2).

In fig. 2 the following quantities are marked as well:

s — distance of the object sagittal ray from the surface, measured along the principal ray,

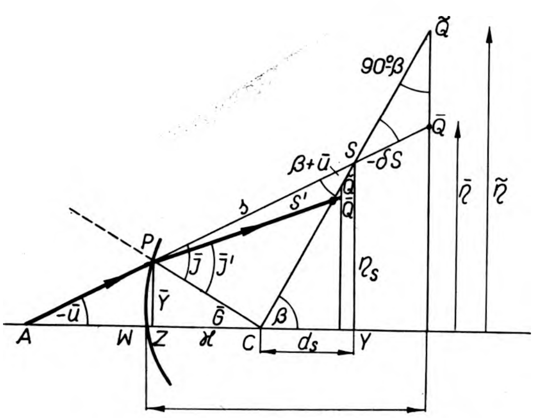


Fig. 2. Quantities characterizing distortion

s' — distance of the image sagittal ray from the surface measured along principal ray,

$\bar{I}, \bar{I}'$ — the incidence and refraction angles respectively of the principal ray,

d_s — the distance of the projection of the object sagittal focus S on the optical axis from the curvature centre in the meridional plane.

The distorsion is by definition determined by the relation

$$\frac{\delta\eta}{\tilde{\eta}} = \frac{\bar{H}-H}{H} = \frac{nu\bar{\eta}-nu\tilde{\eta}}{nu\tilde{\eta}}. \tag{3}$$

After refraction the difference of the terms defined by (3) before and after refraction is equated to

$$\Delta\frac{\delta\eta}{\tilde{\eta}} = \frac{1}{H}\Delta(\bar{H}-H) = \frac{1}{H}\Delta(nu\,\delta\eta), \tag{4}$$

where $\delta\eta = \bar{\eta}-\tilde{\eta}$.

The magnitude $\Delta(nu\delta\eta)$ should be expressed by the paraxial quantities with the accuracy up to the fifth order. From the triangle $S\tilde{Q}\bar{Q}$ it follows that

$$\frac{\bar{\eta}-\tilde{\eta}}{\sin(\beta+\bar{u})} = -\frac{\delta S}{\cos\beta}.$$

Hence after multiplying both the numerator and denominator by nuu_s it follows that:

$$\Delta(nu\,\delta\eta) = \frac{\sin(\beta+\bar{u})}{u_s\cos\beta}\Delta(nuu_s\,\delta S). \tag{5}$$

The expression

$$\frac{\sin(\beta+\bar{u})}{u_s\cos\beta}$$

may be transformed to the form including the quantities determining the principal ray and the paraxial quantities. Next it may be transformed to the form including Hopkins invariants H_s

$$\frac{\sin(\beta+\bar{u})}{u_s\cos\beta} = \frac{B_s}{A_s\cos\bar{G}+B_s h_s C_s\sin\bar{G}} = \frac{B_s\bar{Y}}{B_s h_s-\bar{H}_s\cos\bar{G}}, \qquad (6)$$

where: $A_s = ni_s$; $B_s = n\sin\bar{I}$; $i_s = h_s C_s\cos\bar{I}-u_s$.

The term appearing in the formula (5) is expressed by the quantity $\Delta(nuu_s\bar{N}\delta S)$ by which the wave aberration W_s of the sagittal focus was defined in paper [3] as a distance of two spheres with the centres located at the points S' and $\bar{Q}'$ measured along the given ray of the astigmatic beam. In the reduced coordinates it has the form

$$W_s = -\frac{1}{2}nuu_s\delta S\frac{\bar{H}_s}{\bar{H}}. \qquad (7)$$

From fig. 2 we obtain

$$l = \bar{z}+r_s\cos\bar{G}+d_s-\bar{N}\delta S, \qquad (8a)$$

$$\bar{N}\delta S = \bar{z}+r_s\cos\bar{G}+d_s-l, \qquad (8b)$$

$$\bar{N}\delta S = r+d_s-l, \qquad (8c)$$

$$d_s = s\cos\bar{u}-r_s\cos\bar{G}. \qquad (8d)$$

Substituting (8d) to the expression (8c) and multiplying both sides of (8c) by nuu_s we get

$$nuu_s\bar{N}\delta S = nuu_s(s\cos\bar{u}-r_s\cos\bar{G})-nuu_s(l-r) = \bar{D}_s u-Du_s,$$

where $D = nu(l-r)$; $D_s = nu_s d_s$.

Both $\bar{D}_s$ (cf. [2]) and D are invariant during refraction, thus

$$D(nuu_s\bar{N}\delta S) = \bar{D}_s\Delta(u)-D\Delta(u_s). \qquad (9)$$

The term for directional cosine $\bar{N}$ with the accuracy to τ^4 [4]:

$$\bar{N} = 1-\frac{1}{2}\left[\bar{u}^2\tau^2+\frac{\bar{u}}{H}\left(\bar{u}S_{\mathrm{V}}+u\bar{S}_{\mathrm{I}}-\frac{3}{4}H\bar{u}^3\right)\tau^4\right]+ +0(\tau^6),$$

allows to write the formula (9) in the form

$$\Delta(nuu_s\bar{N}\delta S) = \Delta(nuu_s\delta S)-\frac{1}{2}(\bar{u}^2nuu_s\delta S)\tau^2+0(\tau^4).$$

Hence

$$\Delta(nuu_s\delta S) = \bar{D}_s\Delta(u)-D\Delta(u_s)+\frac{1}{2}\tau^2\Delta(nuu_s\bar{u}^2\delta S)+0(\tau^4). \qquad (10)$$

From (7) it may be found that

$$\delta S = -\frac{2\bar{H}W_s}{nuu_sH_s}.$$

The quantities $\bar{H}$, $\bar{H}_s$, u_s and W_s have the forms:

$$\bar{H} = nu(\bar{\eta}+\delta\eta) = H\tau+0(\tau^2),$$

$$\bar{H}_s = H\tau+0(\tau^2),$$

$$u_s = u\tau+0(\tau^2),$$

$$W_s = \frac{1}{4}(S_{\mathrm{III}}+S_{\mathrm{IV}})\tau^2,$$

with the acurracy to the terms of fourth order of smallness with respect to τ, $0(\tau^4)$, while the aberration

$$\delta S_k' = \frac{1}{2n'u'^2}\sum_1^k(S_{\mathrm{III}}+S_{\mathrm{IV}})\tau^2+0(\tau^4).$$

This allows to write (10) in the form

$$\Delta(nuu_s\delta S) = [\bar{D}_s\Delta(u)-D\Delta(u_s)]- -\frac{1}{4}\tau^4\left[\bar{u}_k'^2\sum_1^k(S_{\mathrm{III}}+S_{\mathrm{IV}})-\bar{u}_k^2\sum_1^{k-1}(S_{\mathrm{III}}+S_{\mathrm{IV}})\right]+ +0(\tau^6), \qquad (11)$$

and

$$\Delta(nu\delta\eta) = \frac{B_s\bar{Y}}{B_sh_s-\bar{H}_s\cos\bar{G}}\left\langle[\bar{D}_s\Delta(u)-D\Delta(u_s)]- -\frac{1}{4}\tau^2\left[u'^2\sum_1^k(S_{\mathrm{III}}+S_{\mathrm{IV}})-\bar{u}_k^2\sum_1^{k-1}(S_{\mathrm{III}}+S_{\mathrm{IV}})\right]\right\rangle. \qquad (12)$$

The quantities B_s, $\cos\bar{G}$, $\bar{M}$, occurring in (12) should be expressed with the accuracy to $0(\tau^5)$, while $h_s/\bar{H}_s$ should be expressed with the accuracy to $0(\tau^4)$:

$$\sin\bar{I} = \bar{i}\tau-\frac{1}{2\bar{H}}(H\bar{u}^2\bar{i}-\bar{i}S_{\mathrm{V}}-i\bar{S}_{\mathrm{I}})\tau^3+0(\tau^5),$$

$$\cos \bar{G} = 1 - \frac{1}{2}\bar{g}\tau^2\left\{\bar{g} + \left[\frac{1}{4}\bar{h}^2\bar{g} - \bar{h}^2\bar{u} + \frac{1}{\bar{H}}(\bar{g}S_{\mathrm{V}} + g\bar{S}_{\mathrm{I}})\right]\tau^2\right\} + 0(\tau^5),$$

$$\bar{M} = -\bar{u}\tau + \frac{1}{2}\left[\bar{u}^3 - \frac{1}{\bar{H}}(\bar{u}S_{\mathrm{V}} + u\bar{S}_{\mathrm{I}})\right]\tau^3 + 0(\tau^5),$$

$$\frac{h_s}{\bar{H}_s} = \frac{1}{H\tau}\times \left\{h - \frac{1}{2}\left[\bar{h}\bar{g}\bar{u} - h\bar{u}^2 + \frac{1}{\bar{H}}(\bar{h}S_{\mathrm{III}} + \bar{h}S_{\mathrm{IV}}) + hS_{\mathrm{V}}\right]\tau^2\right\} + 0(\tau^4).$$

The respective transformation of the formulae (11) and (12) yields the distortion of fifth order in the form:

$$\Delta(nu\delta\eta) = Z\left[(K+S+F) - \frac{1}{4}\tau^4\left\{u_k'^2\sum_1^k(S_{\mathrm{III}}+S_{\mathrm{IV}}) - \bar{u}_k^2\sum_1^{k-1}(S_{\mathrm{III}}+S_{\mathrm{IV}})\right\}\right] + 0(\tau^6), \quad (13)$$

where K, S, F denote respectively the field curvature of fifth order, generalized astigmatism, and the perturbing term, determined in [4]. Simultanously Z is equal to

$$Z = \tau\left\langle\left\langle\frac{\bar{i}}{i} - \frac{1}{2\bar{h}i}\left\langle\bar{h}\bar{u}\bar{i}(\bar{g}+\bar{u}) + \frac{\bar{i}}{i}\times\left[\frac{H}{n}(\bar{g}^2 + V_D) - \bar{h}\bar{g}\bar{i}u\right] - \frac{1}{H}\left\{2\bar{h}\bar{i}S_{\mathrm{V}} + \bar{h}i\bar{S}_{\mathrm{I}} + \frac{\bar{i}}{i}\times[\bar{i}\bar{h}(S_{\mathrm{III}}+S_{\mathrm{IV}})]\right\}\right\rangle\tau^2\right\rangle\right\rangle. \quad (14)$$

Variables in the region of third order occur with the Gaussian approximation, for instance $H_S = H$, $\bar{H} = H$, $h_s = h$, $u_s = u$ and so on, and the equation (14) takes the well-known form [5]:

$$\Delta(nu\delta\eta) = \left\langle\frac{\bar{i}}{i}\left[-H^2\Delta\left(\frac{1}{n}\right)\right]C + \frac{\bar{i}}{i}n^2\Delta\left(\frac{1}{n}\right)\bar{i}^2h(u-i')\right\rangle. \quad (15)$$

3. Numerical results

The examination of the expression (13) has been preceded by a number of numerical calculations. The results obtained allowed to determine the quantity ΔD, i.e. a difference between the distortion calculated trigonometrically and that evaluated from (13). The dependence of this difference upon the value of perturbance Z', where

$$Z' = \frac{\bar{i}}{iH}F\tau - \frac{1}{4H}\frac{\bar{i}}{i}\left\{\bar{u}_k'^2\sum_1^k(S_{\mathrm{III}}+S_{\mathrm{IV}}) - \bar{u}_k^2\sum_1^{k-1}(S_{\mathrm{III}}+S_{\mathrm{IV}})\right\}\tau^5 + 0(\tau^7)$$

has been examined.

The exemplified results obtained have been presented in the graphs for various objectives namely:

- photographic objectives like:
  - a) symmeric aplanat of f-number $N = 3, 2$ $w = 30°$ (field angle), (fig. 3),
  - b) objective of great speed $N = 2$, and small field angle, $2\ w = 12°$ (fig. 4),
  - c) Celora, $N = 3, 5, 2\ w = 40°$ (fig. 5),
  - d) wide-angle objective, $N = 18, 2w = 100°$ (fig. 6)
- microscopic objectives of magnification 5x:
  - a) achromat (fig. 7),
  - b) planachromat (fig. 8),
- microscopic objectives of magnification 20x:
  - a) achromat (fig. 9),
  - b) planachromat (fig. 10),
- microscopic objectives of magnification 40x:
  - a) achromat (fig. 11),
  - b) planachromat (fig. 12).

The analysis carried out shows that the deviation from the extrapolated linearity in the whole field of view does not exceed 5 % for

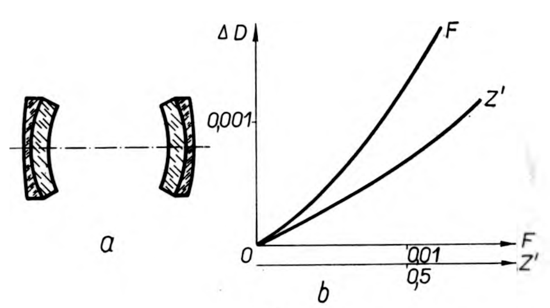


Fig. 3

a) Photographic objectives, symmetrical aplanat
b) Dependence of higher order distortion ΔD on the values of Z' and F

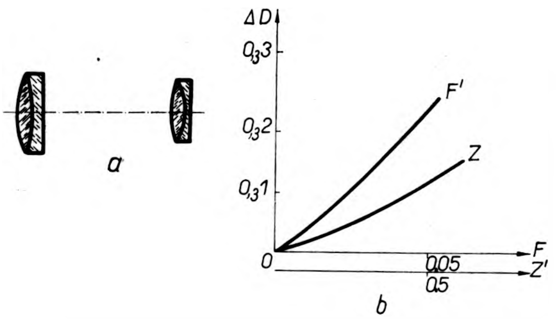


Fig. 4.
a) High speed photographic objective of small field of view
b) Dependence of higher order distortion ΔD upon the magnitudes Z' and F

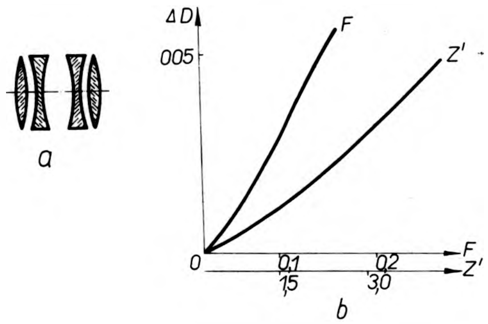


Fig. 5.
a) Celor, a photographic objective
b) Dependence of higher order distortion ΔD) upon the values of Z' and F

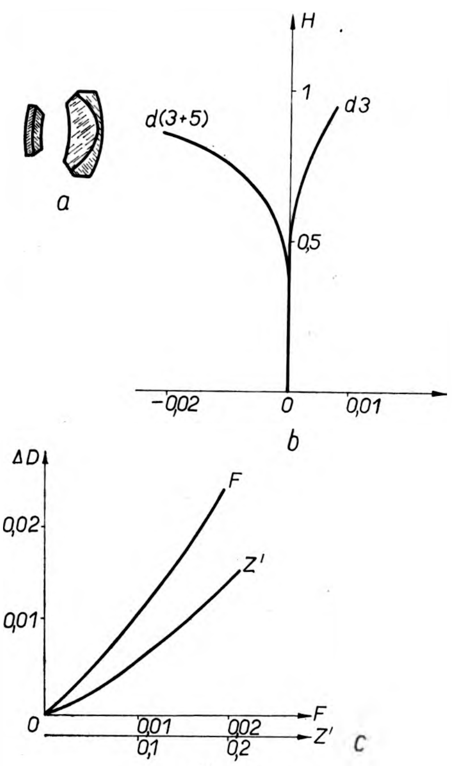


Fig. 6.
a) Wide-angle photographic objective
b) Dependence of third order distortion and that of both third and fifth orders upon the field magnitude H
c) Dependence of the higher order distortion (ΔD) upon the values of Z' and F

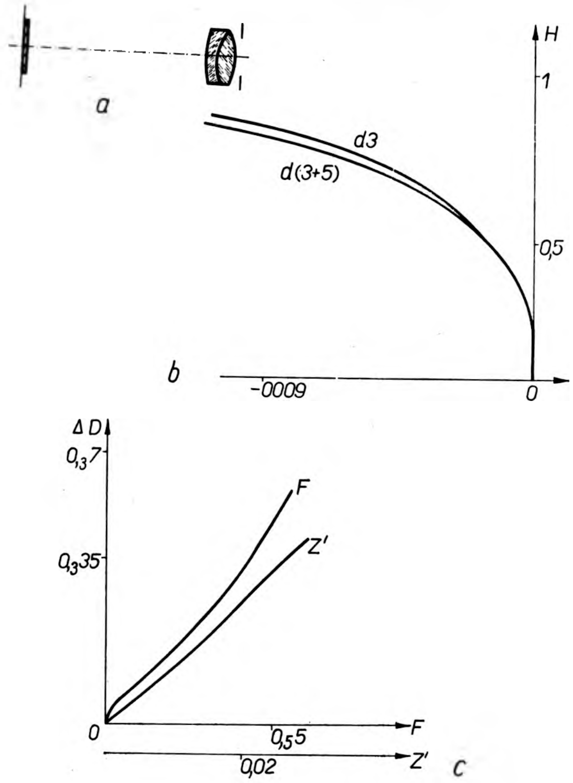


Fig. 7.
a) An achromatic microscopic objective of $5\times$ magnification
b) Dependence of third order distortion and that of both third and fifth orders upon the field magnitude H
c) Dependence of the higher order distortion (ΔD) upon the values of Z' and F

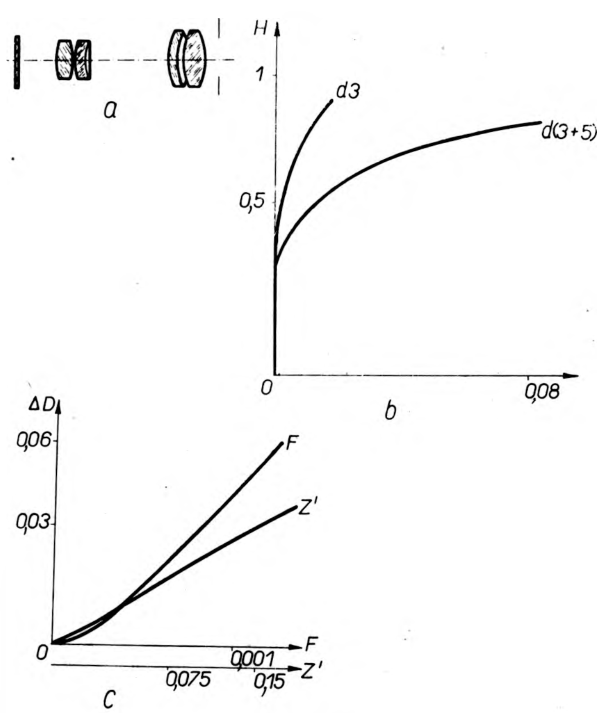


Fig. 8.
a) Planachromatic microscopic objective of $5\times$ magnification
b) Dependence of third order distorsion and that of both third and fifth orders upon the values of H
c) Dependence of higher order distortion (ΔD) upon the values Z' and F

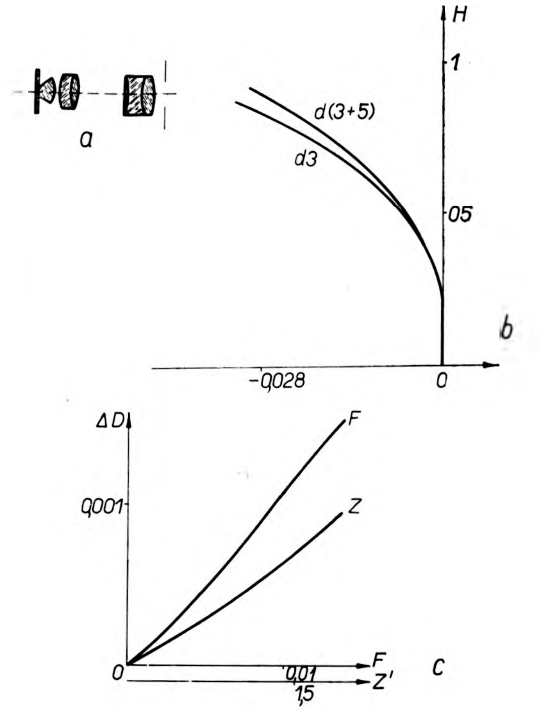


Fig. 9.

a) Achromatic microscopic objective of 20 × magnification
b) Dependence of third order distortion and that of both third and fifth orders upon the values of H
c) Dependence of higher order distortion (ΔD) upon the values of Z' and F

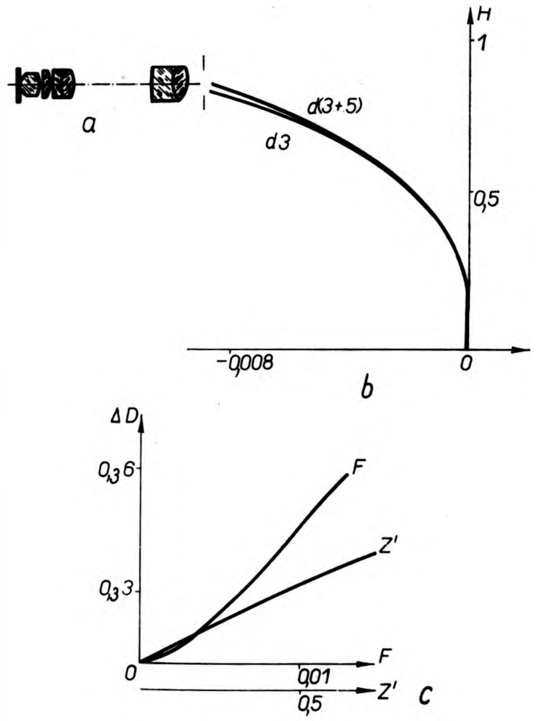


Fig. 10.

a) Planachromatic microscopic objective of 20 × magnification
b) Dependence of third order distortion and that of both third and fith orders upon the values of H
c) Dependence of higher order distortion (ΔD) upon the values of Z' and F

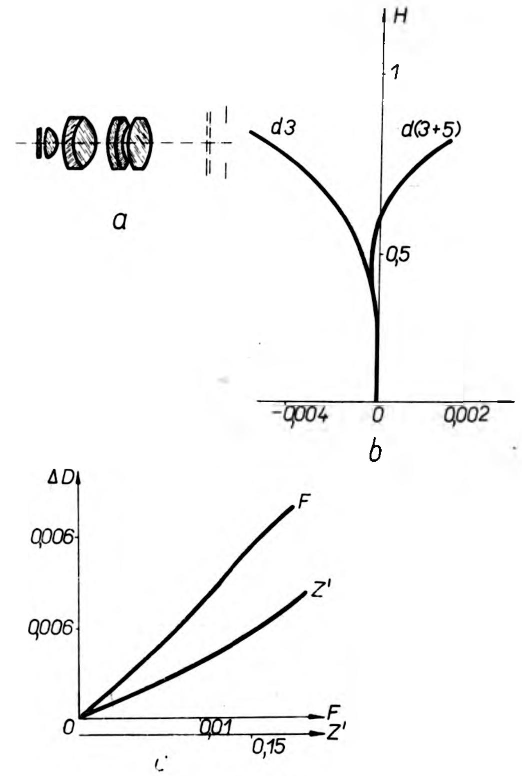


Fig. 11.

a) Achromatic microscopic objective of 40 × magnification
b) Dependence of third order distortion and that of both third and fifth orders upon the values of H
c) Dependence of higher order distortion (ΔD) upon the values of Z' and F

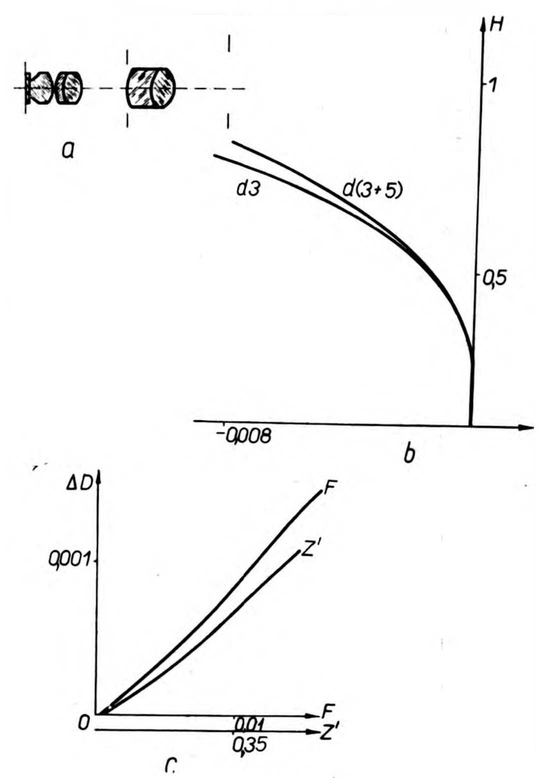


Fig. 12.

a) Planachromatic microscopic objective of magnification 40 ×
b) Dependence of third order distortion and that both third and fifth orders upon the values of H
c) Dependence of higher order distortion (ΔD) upon the values of $Z'$and F

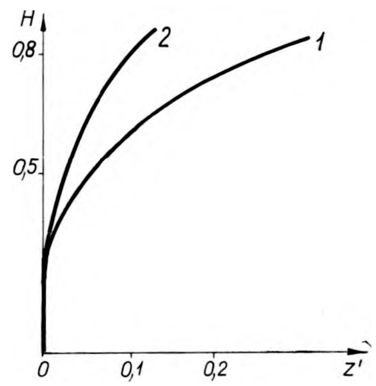
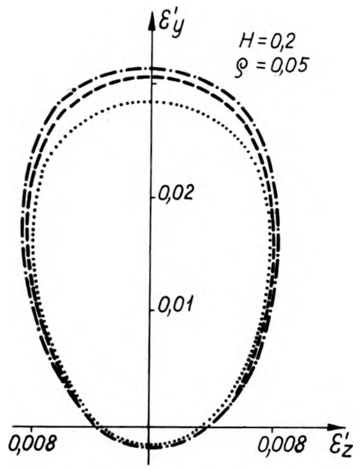


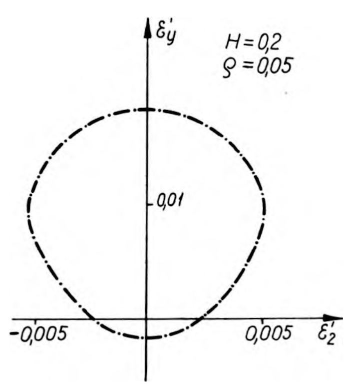
Fig. 13. Dependence of Z' and H of the systems: uncorrected (1) and corrected (2)



a

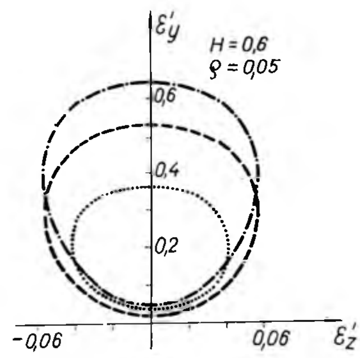
Fig. 14.

Dependence of the transversal aberration components upon the field magnitude θ for fixed values $H = 0.2^{(0.2)}$, $\varrho = 0.05$, for an uncorrected system



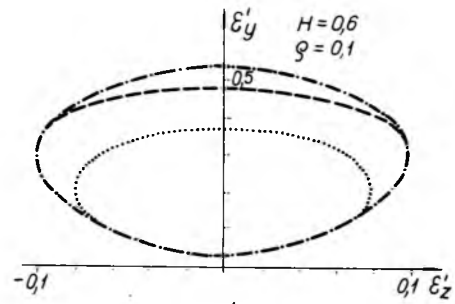
b

Dependence of the transversal aberration component upon the magnitude θ for fixed values $H = 0.2$ and $\varrho = 0.05$, for a corrected systems



c

Dependence of the transversal aberration components upon the value of Q for fixed values of $H = 0.6$ and $\varrho = 0.05$, for an uncorrected system



d

Dependence of the transversal aberration components upon the magnitude of Q for fixed values of $H = 0.6$ and $\varrho = 0.05$, for a corrected system . . . . 3, — — — 3+5, — . — . — . — 3+5+7

a wide class of optical systems. The method of calculation of field aberrations of higher order has been verified by comparing the deviations from the linearity ΔW as a function of Z', and ΔD as a function of F for all examined objectives by taking account of the properties of wave aberration of sagittal focus. A consistency up to 92% of cases has been stated. There exists a possibility of controlling the aberrations of order higher than fifth, which is offered by determining both the functions mentioned above as determined by the paraxial parameters. This possibility has been used for correction of a microscopic ocular of 8^x magnification. The tables 1 and 2 as well as fig. 13 present the parameters of the uncorrected and corrected systems as well as the dependence of Z' on the field angle. The method used has been verified by determining the values of the Buchdahl coefficients of third and fifth orders (tab. 3) and transversal aberrations including third, fifth and seventh orders, respectively, presented in figs. 14a, b, c and d. The results obtained are consistent with the aberrations calculated basing on the Buchdahl series. The

Table 1

Parameters of uncorrected system

Parameter \ Surface number	1	2	3	4	5	6
r	0.997	6.19	0.417	0.274	−0.561	
n	1	1.78446	1	1.72802	1.50371	1
d	0.095	1.108	0.077	0.118	0	

Table 2

Parameters of corrected system

Parameter \ Surface number	1	2	3	4	5	6
r	0.961	5.95	0.4	0.226	−0.64	
n	1	1.78446	1	1.72802	1.5037	1
d	0.092	1.068	0.074	0.114	0	

Table 3

Values of the third and fifth order coefficients for an uncorrected and corrected systems, respectively

	δ_1	δ_2	δ_3	δ_4	δ_5	μ_1	μ_2	μ_3
Uncorrected system	8.58	3.92	1.98	0.83	0.95	63 .3	77.5	53.0
Corrected system	3.93	2.31	1.35	0.79	0.64	+42.2	+31.1	+19.7

	μ_4	μ_5	μ_6	μ_7	μ_8	μ_9	μ_{10}	μ_{11}	μ_{12}
Uncorrected system	69.0	27.4	45.1	29.3	20.0	0.83	11.3	9.25	1.92
Corrected system	7.7	0.755	3.93	2.92	2.62	2.51	1.57	0.39	0.07

adventage of the former method is that it requires neither the calculations of Buchdahl' aberrational coefficients, nor the analysis of contributions of particular orders of aberrations made for a wide range of ϱ, H, and θ parameters. optimized this method allows an uncomplex intervention of the designer (the choice of a proper direction of changes) as well as permits to estimate the working region of the system, for which the higher order aberrations do not influence the correction.

4. Conclusions

The proposed method elaborated, based on paraxial calculations does not require either determination of the aberration series coefficients or the transversal aberrations, and therefore is useful for correction of optical systems. In the case when the optical system is to be

Дисторсия высших пядов с использованием свойств волновой аберрации сагитального фокуса

Исходя из свойств волновой абэррации сагитального фэкуса, введено аналитическое выражение на дисторсию пятого ряда, содержащее кривизну и астигматизм пятого

ряда, а также функцию искажения Z'. Исследовалась зависимость дисторсии высших рядов, чем пятый от Z' для многих типов оптических систем. Из проведенных исследований вытекает, что эта зависимость является квазилинейной. Её можно использовать для исследований аберрации высших рядов. Функция Z' может быть также использована в процессе коррекции полевых аберраций оптических систем.

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